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A SIMULATION STUDY FOR SURE ESTIMATORS WITH AN APPLICATION IN AIR NAVIGATION IN SAUDI ARABIA

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Abstract

The principal objective of this paper was to compare the SURE and the GLS estimators with varying sample sizes and correlation. We have found that the SURE estimator was better than the GLS estimator, where the MSE was concerned, when the correlation between equations increased. Real air navigation data from General Authority of Civil Aviation in Saudi Arabia was applied.

1. Introduction

For a more specific discussion of the seemingly unrelated regression equations (SURE) model, we shall consider a set of individual linear multiple regression equations, each explaining some economic phenomena. This set of regression equations is said to be a *simultaneous equation model* if one or

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more of the regressors (explanatory variables) in one or more of the equations is itself the dependent (endogenous) variable associated with another equation in the full system. On the other hand, supposing that none of the variables in the system is simultaneously explanatory and dependent in nature, there may still be interactions between the individual equations if the random disturbances associated with at least some of the different equations are correlated. Zellner [17], who coined the expression "seemingly unrelated regression equations" to reflect the fact that the individual equations are in fact related to one another, even though they may superficially seem not to be discussed this.

The mathematical details of the SURE model, and the underlying assumptions, which will form at least the initial basis of our discussion, are given in the next section. Before discussing these details, it is of interest to consider some specific examples of economic phenomena and models that may give rise to a SURE specification. A better example is the one used by Zellner's [17] to illustrate his proposed SURE estimator, and subsequently discussed by Kmenta [6] and Theil [9]. Separate regression equations were specified to explain investment on the part of two large corporations, general electric and Westinghouse. In each case, real gross investment by the firm is supposed to be determined by the value of its outstanding shares at the beginning of that period, and by the opening value of the firm's real capital stock. It seems reasonable to suppose that the error terms associated with these two investment equations may be contemporaneously correlated, given the presence of common market influencing forces. For instance, if the error term in the first equation reflects the omission of some unobservable variables, then these same variables may be important determinants of the variability of the error term in the other equation. Thus, the two equations are apparent or "seemingly" unrelated regressions, in the sense described earlier, rather than independent relationships. Zellner [15] provided a Bayesian analysis for an extension of this problem involving ten corporations.

A common situation that may suggest a SURE specification is where regression equations explaining a certain economic activity in different geographical locations are to be estimated. For instance, Giles and Hampton [5] considered Cobb-Douglas production functions for five different regions of New Zealand during the period of that country's industrial development, and used a SURE framework to allow for the inter-regional correlated likely existing between the regressions' error term. Similarly, Donnelly [2] used the SURE model as the basis for estimating petrol demand equations for six different Australian states. White and Hewings [11] used the SURE model to estimate employment equations for five multi-county regions within the State of Illinois. Giles and Hampton [4] used an extended SURE model to estimate demand systems for four expenditure groups across six regions of New Zealand.

Other studies involving the SURE model are abounded see Section 6, but these examples should illustrate the wide range of empirical applications for which this model is appropriate.

2. The Model

The basic model that we are concerned with comprises M multiple regression equations as:

$$y_i = x_i\beta_i + u_i, \tag{1}$$

where y_i is a $T \times 1$ vector of observations on the ith dependent variable, x_i is a $T \times k_i$ matrix, each column of which comprises the T observations on a regressor in the ith equation of the model, β_i is a $k_i \times 1$ vector of coefficients in the ith equation, u_i is a $T \times 1$ disturbance vector and $i = 1, 2, \ldots, M$. By writing (1) as follows:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{pmatrix} = \begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_M \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{pmatrix}.$$

The model may be expressed in compact form as

$$\boldsymbol{Y} = \boldsymbol{X}\beta + \boldsymbol{U}, \tag{2}$$

where $\boldsymbol{Y}$ is $TM \times 1$, $\boldsymbol{X}$ is $TM \times K$, β is $K \times 1$, $\boldsymbol{U}$ is $TM \times 1$, and $K = \sum_{i=1}^{M} k_i$. Treating each of the M equations as classical linear regression relationships, we make the conventional assumptions about the regressors:

$$x_i \text{ is fixed with rank } \Re(x_i) = k_i \tag{3}$$

and

$$\lim_{T \to \infty} \frac{1}{T} x_i' x_i = Q_{ii}, \tag{4}$$

where $Q_{ii} (i = 1, 2, ..., M)$ is non-singular with fixed and finite elements.

Further, we assume that the elements of the disturbance vector u_i, follow a multivariate probability distribution with

$$E(u_i) = 0, \tag{5}$$

$$E(u_i u_i') = \sigma_{ii} I_T. \tag{6}$$

Here, σ_{ii} represents the variance of the random disturbance in the ith equation for each observation in the sample, and I_T is an identity matrix of order T. Considering the interactions between the equations of the model, it is assumed that:

$$\lim_{T \to \infty} \frac{1}{T} x_i' x_j = Q_{ij}, \tag{7}$$

$$E(u_i u_j') = \sigma_{ij} I_T. \tag{8}$$

Here Q_{ij} is non-singular with fixed and finite elements, and σ_{ij} represents the covariance between the disturbances of the ith and jth equations for each observation in the sample. Writing (5), (6), and (8) more compactly, we have

$$E(U) = 0 \tag{9}$$

and

$$E(UU') = \begin{pmatrix} \sigma_{11}I & \sigma_{12}I & \vdots & \sigma_{1M}I \\ \sigma_{21}I & \sigma_{22}I & \vdots & \sigma_{2M}I \\ \cdots & \vdots & \ddots & \vdots \\ \sigma_{M1}I & \sigma_{M21}I & \cdots & \sigma_{MM}I \end{pmatrix}$$

$$= (\Sigma \otimes I)$$

$$= \Omega, \tag{10}$$

where $\otimes$ denotes the usual Kronecker product, so that Ω is $MT \times MT$, and $\Sigma = [\sigma_{ij}]$ is an $M \times M$ positive definite symmetric matrix. The definiteness of the Σ precludes the possibility of any linear dependencies among the contemporaneous disturbances in the *M* equations of the model.

There are many methods of estimating of SURE model. We will consider only the most common estimator, feasible generalized least squares (FGLS), in the next section and the others will be mentioned in later sections.

3. The Feasible Generalized Least Squares Estimator

To take account of the form of the variance covariance matrix of the disturbances in (10), we may use the GLS, or Aitken, estimator of β:

$$b_G = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y$$

$$= (X'(\Sigma^{-1} \otimes I_T)X)^{-1}X'(\Sigma^{-1} \otimes I_T)Y. \tag{11}$$

It is easily verified that

$$E(b_G - \beta) = 0 \tag{12}$$

and

$$\mathrm{var}(b_G) = (X'(\Sigma^{-1} \otimes I_T)X)^{-1}. \tag{13}$$

Indeed, provided that Σ is non-stochastic and observable, it follows from Aitken's theorem[1] that b_G is the best linear unbiased estimator (*BLUE*) of β in the seemingly unrelated regression equation (*SURE*) model.

Looking at the expression for b_G in (11), it is clear that it is not an operational or a feasible estimator of β because in general $\Sigma,$ and hence $\Omega,$ will be unobservable. Recognizing this, Zellner [16] proposed an estimate of β in the *SURE* model, basing this on (11), but with Σ replaced by an observable $M \times M$ matrix *S*. In particular, the elements of *S* are chosen to be estimators of the corresponding elements of $\Sigma.$ With this replacement for $\Sigma,$ and hence for $\Omega,$ we now have a feasible generalized least square *FGLS* estimator of β in (2):

$$b_F = (X'(S^{-1} \otimes I_T)X)^{-1} X'(S^{-1} \otimes I_T)Y. \tag{14}$$

We are assuming that the matrix $S = [s_{ij}]$ is non-singular, where s_{ij} is some estimator of $\sigma_{ij}.$ Although there are many possible choices of *S*, two ways of obtaining the s_{ij} are popular. Each of these is based on residuals obtained by the application of *OLS* in one way or another:

The first approach involves the so-called "unrestricted residuals", because the restrictions on the coefficients of the *SURE* model which distinguishes it from the multivariate regression model are ignored when obtaining the residuals to be used for constructing the s_{ij}'s. Let *K* be the total number of distinct regressors in the full model (2) and let *Z* be the corresponding $(T \times K)$ observation matrix for these variables. Regressing each of the *M* dependent variables in turn on the columns of *Z*, we obtain the $(T \times 1)$ "unrestricted residuals" vectors:

$$\tilde{u}_i = \overline{M}_z y_i, \tag{15}$$

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[1]H. Theil [9].

where

$$\overline{M}_Z = I_T - Z(Z'Z)^{-1}Z'. \tag{16}$$

From these residuals, we may obtain consistent estimators of the σ_{ij} 's as follows:

$$\begin{aligned}\tilde{s}_{ij} &= \frac{1}{T}\tilde{u}_i'\tilde{u}_j \\ &= \frac{1}{T}y_i'\overline{M}_Z y_j,\end{aligned} \tag{17}$$

where $i, j = 1, 2, ..., M$. Because x_i is a sub-matrix of Z, we have

$$x_i = ZJ_i, \tag{18}$$

where J_i is a selection matrix of order $K \times K_i$, $i = 1, 2, ..., M$, with elements taking the value zero or one, as appropriate. It is easy to see that

$$\overline{M}_Z x_i = 0. \tag{19}$$

Using this result, we have

$$y_i'\overline{M}_Z y_j = u_i'\overline{M}_Z u_j. \tag{20}$$

So that, from (17) we get

$$\begin{aligned}E(\tilde{s}_{ij}) &= \frac{1}{T}\sigma_{ij}Tr\overline{M}_z \\ &= \left(1 - \frac{K}{T}\right)\sigma_{ij}.\end{aligned} \tag{21}$$

From which it follows that an unbiased estimator of σ_{ij} is obtained by replacing $1/T$ by $1/(T-K)$ in (17).

The second approach estimator of σ_{ij} may be developed by using residuals that have been obtained by taking into account the restrictions on the coefficients that effectively distinguish the *SURE* model from the multivariate regression model. The "restricted residuals" vectors are obtained by estimating each equation of (1) separately by *OLS*, yielding

$$\hat{u}_i = \overline{M}_i y_i, \tag{22}$$

where

$$\overline{M}_i = I_T - x_i (x_i' x_i)^{-1} x_i'. \tag{23}$$

So that, an alternative consistent estimator for σ_{ij} is

$$\begin{aligned} \hat{s}_{ij} &= \frac{1}{T} \hat{u}_i' \hat{u}_j \\ &= \frac{1}{T} y_i' \overline{M}_i \overline{M}_j y_j. \end{aligned} \tag{24}$$

If T in the denominator of (24) is replaced by

$$Tr\overline{M}_i \overline{M}_j = T - k_i - k_j + Tr((x_i' x_i)^{-1} x_i' x_j)((x_j' x_j)^{-1} x_j' x_i). \tag{25}$$

Then, this yields an unbiased estimator of σ_{ij}.

We now have $\tilde{S} = [\tilde{s}_{ij}]$ and $\hat{S} = [\hat{s}_{ij}]$ as two explicit choices of S to estimate Σ in the construction of an *FGLS* estimator of β in the *SURE* model. These two choices lead to the seemingly unrelated unrestricted residuals (*SUUR*) and seemingly unrelated restricted residuals (*SURR*) estimators proposed by Zellner [14, 16 and 17]:

$$\tilde{\beta}_{SU} = \left(X'(\tilde{S}^{-1} \otimes I_T)^{-1} X\right)^{-1} X'(\tilde{S}^{-1} \otimes I_T)Y \tag{26}$$

and

$$\hat{\beta}_{SR} = (X'(\hat{S}^{-1} \otimes I_T) X)^{-1} X'(\hat{S}^{-1} \otimes I_T)Y. \tag{27}$$

4. The Simulation

The main objective of this simulation is to examine the SURE estimators with the GLS estimators for small, medium, and large sample size. A correlation between the equations will be 0.1, 0.5 and 0.95. We will use the

SAS software for simulation to study the behavior of the estimators. We repeated each experiment 1000 times to know how the SURE estimators and GLS estimators perform for various sample sizes and different degree of the correlations between equations.

We shall consider three equations, each equation has three independent variables. We used SAS software to generate the values of the independent variables x_i, and the error term e_i, $i = 1, 2,$ and 3. We used the coefficients $\beta_1 = 1, 2, 3$ for the first equation, and $\beta_2 = 3, 2, 1$ for the second equation, and $\beta_3 = 0.5, 1.5, 2.5,$ for the third equation. This process is repeated 1000 times and the mean of Biased and the MSE of GLS estimators and SURE estimators is calculated. This experiment is applied for each correlation between the equations is 0.1, 0.5 and 0.95; and the sample size is 5, 10, 30, 50, 100 and 200. The summary of these results is in Tables 1-3 and represented in Figures 1-4.

Table 1. Results of SURE and GLS estimators when $n = 5$ and 10 and the Rho are different

N	Method		ρ	Equation 1			Equation 2			Equation 3		
			True β	1	2	3	3	2	1	0.5	1.5	2.5
5	Biased	GLS		-0.3220	-0.3069	-0.3094	-0.2904	-0.3511	-0.2911	-0.3252	-0.2882	-0.3203
		SURE	0.1	-0.3212	-0.3003	-0.3097	-0.2878	-0.3537	-0.2836	-0.3232	-0.2806	-0.3221
			0.5	-0.3141	-0.2715	-0.3004	-0.2707	-0.3595	-0.2483	-0.3103	-0.2463	-0.3251
			0.95	-0.2280	-0.1606	-0.2034	-0.1521	-0.2972	-0.1277	-0.2023	-0.1294	-0.2672
	MSE	GLS		0.9320	0.9087	0.8926	1.1461	1.0469	0.9039	0.9044	0.8364	0.8988
		SURE	0.1	0.9245	0.8973	0.8860	1.1357	1.0405	0.8928	0.8959	0.8252	0.8928
			0.5	0.7646	0.7178	0.7285	0.9241	0.8602	0.7217	0.7285	0.6555	0.7345
			0.95	0.3049	0.2537	0.2776	0.3370	0.3437	0.2710	0.2669	0.2270	0.2872
	Biased	GLS		-0.2850	-0.3168	-0.3127	-0.3112	-0.3036	-0.3081	-0.2949	-0.3137	-0.3100
		SURE	0.1	-0.2805	-0.3136	-0.3071	-0.3072	-0.2993	-0.3030	-0.2882	-0.3106	-0.3059
			0.5	-0.2526	-0.2873	-0.2736	-0.2792	-0.2699	-0.2726	-0.2514	-0.2865	-0.2782
			0.95	-0.0934	-0.1244	-0.0997	-0.1132	-0.1026	-0.1047	-0.0846	-0.1258	-0.1086

10	MSE	GLS		0.2346	0.2558	0.2567	0.2562	0.2462	0.2489	0.2387	0.2475	0.2455
		SURE	0.1	0.2299	0.2516	0.2510	0.2515	0.2414	0.2437	0.2327	0.2435	0.2409
			0.5	0.1738	0.1940	0.1885	0.1924	0.1832	0.1846	0.1721	0.1889	0.1848
			0.95	0.0372	0.0443	0.0394	0.0426	0.0391	0.0395	0.0354	0.0436	0.0398

Table 2. Results of SURE and GLS estimators when $n = 30$ and 50 and the Rho are different

N	Method		ρ	Equation (1)			Equation (2)			Equation (3)		
		True β		1	2	3	3	2	1	0.5	1.5	2.5
30	Biased	GLS		-0.2954	-0.3105	-0.2978	-0.3048	-0.3060	-0.2985	-0.2964	-0.3038	-0.3093
		SURE	0.1	-0.2882	-0.3059	-0.2919	-0.2992	-0.3006	-0.2915	-0.2902	-0.2983	-0.3029
			0.5	-0.2453	-0.2700	-0.2524	-0.2602	-0.2623	-0.2489	-0.2503	-0.2601	-0.2619
			0.95	-0.0582	-0.0853	-0.0687	-0.0725	-0.0761	-0.0581	-0.0658	-0.0745	-0.0748
	MSE	GLS		0.1220	0.1313	0.1240	0.1295	0.1298	0.1252	0.1226	0.1271	0.1307
		SURE	0.1	0.1172	0.1279	0.1199	0.1256	0.1259	0.1205	0.1184	0.1232	0.1261
			0.5	0.0841	0.0970	0.0880	0.0929	0.0937	0.0868	0.0866	0.0916	0.0927
			0.95	0.0091	0.0131	0.0106	0.0113	0.0118	0.0093	0.0101	0.0113	0.0114
50	Biased	GLS		-0.2993	-0.3051	-0.2972	-0.3014	-0.3034	-0.2980	-0.2976	-0.3023	-0.3077
		SURE	0.1	-0.2922	-0.2992	-0.2919	-0.2964	-0.2971	-0.2909	-0.2911	-0.2966	-0.3015
			0.5	-0.2488	-0.2590	-0.2534	-0.2584	-0.2560	-0.2473	-0.2498	-0.2567	-0.2601
			0.95	-0.0613	-0.0731	-0.0706	-0.0712	-0.0695	-0.0597	-0.0660	-0.0689	-0.0725
	MSE	GLS		0.1091	0.1128	0.1081	0.1110	0.1122	0.1090	0.1081	0.1109	0.1144
		SURE	0.1	0.1045	0.1089	0.1047	0.1076	0.1081	0.1045	0.1040	0.1072	0.1102
			0.5	0.0752	0.0806	0.0778	0.0805	0.0793	0.0750	0.0758	0.0792	0.0811
			0.95	0.0069	0.0086	0.0082	0.0084	0.0081	0.0069	0.0076	0.0079	0.0085

Table 3. Results of SURE and GLS estimators when $n = 100$ and 200 and the Rho are different

N	Method		ρ	Equation (1)			Equation (2)			Equation (3)		
			True β	1	2	3	3	2	1	0.5	1.5	2.5
100	Biased	GLS		-0.2953	-0.3006	-0.3036	-0.2987	-0.3007	-0.3027	-0.2999	-0.3011	-0.3022
		SURE	0.1	-0.2886	-0.2944	-0.2975	-0.2930	-0.2944	-0.2956	-0.2929	-0.2953	-0.2959
			0.5	-0.2457	-0.2526	-0.2560	-0.2525	-0.2518	-0.2515	-0.2494	-0.2543	-0.2538
			0.95	-0.0585	-0.0665	-0.0696	-0.0650	-0.0635	-0.0632	-0.0625	-0.0673	-0.0667
	MSE	GLS		0.0966	0.0997	0.1016	0.0988	0.1000	0.1012	0.0993	0.1001	0.1007
		SURE	0.1	0.0925	0.0959	0.0978	0.0953	0.0961	0.0968	0.0950	0.0964	0.0968
			0.5	0.0668	0.0702	0.0719	0.0703	0.0699	0.0698	0.0685	0.0711	0.0708
			0.95	0.0049	0.0059	0.0064	0.0058	0.0056	0.0055	0.0054	0.0060	0.0060
200	Biased	GLS		-0.2977	-0.3020	-0.3013	-0.3009	-0.3004	-0.3011	-0.3010	-0.2984	-0.3014
		SURE	0.1	-0.2911	-0.2954	-0.2952	-0.2948	-0.2938	-0.2944	-0.2943	-0.2922	-0.2951
			0.5	-0.2479	-0.2522	-0.2535	-0.2527	-0.2505	-0.2509	-0.2509	-0.2497	-0.2527
			0.95	-0.0613	-0.0655	-0.0681	-0.0651	-0.0629	-0.0642	-0.0647	-0.0628	-0.0664
	MSE	GLS		0.0932	0.0958	0.0954	0.0952	0.0949	0.0954	0.0952	0.0937	0.0954
		SURE	0.1	0.0893	0.0918	0.0917	0.0915	0.0909	0.0913	0.0911	0.0899	0.0916
			0.5	0.0646	0.0668	0.0674	0.0670	0.0659	0.0662	0.0661	0.0655	0.0670
			0.95	0.0045	0.0050	0.0054	0.0050	0.0047	0.0049	0.0049	0.0047	0.0051

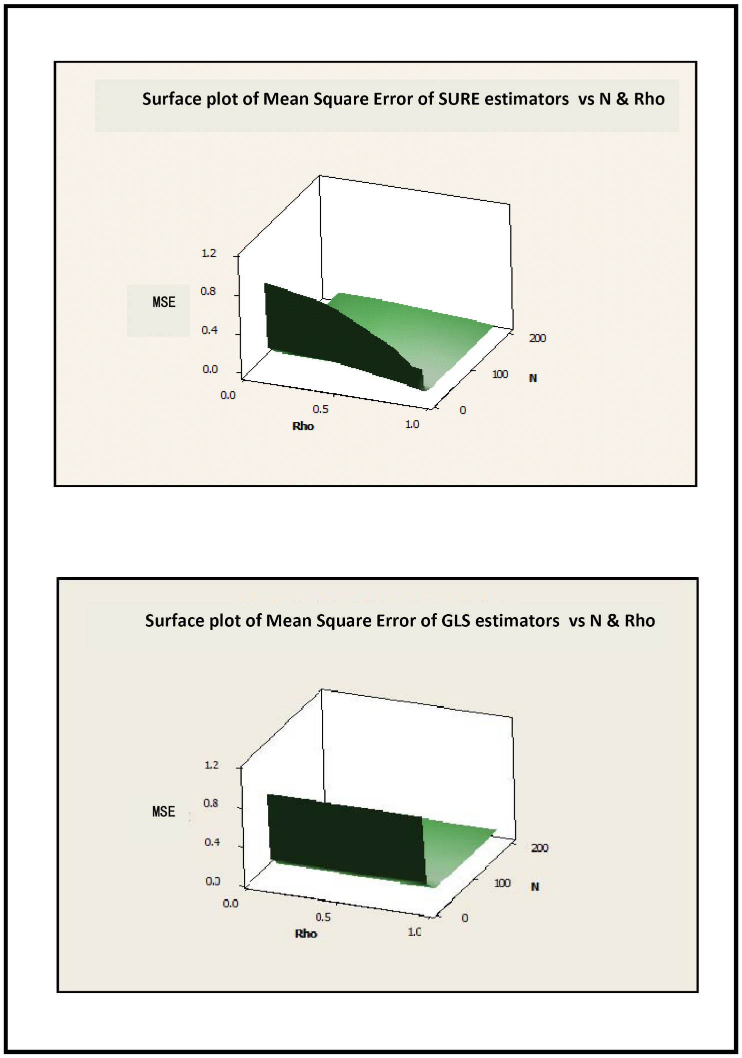

Figure 1. 3D surface plot using distance method of the mean square error vs. sample size and Rho of SURE and GLS estimators.

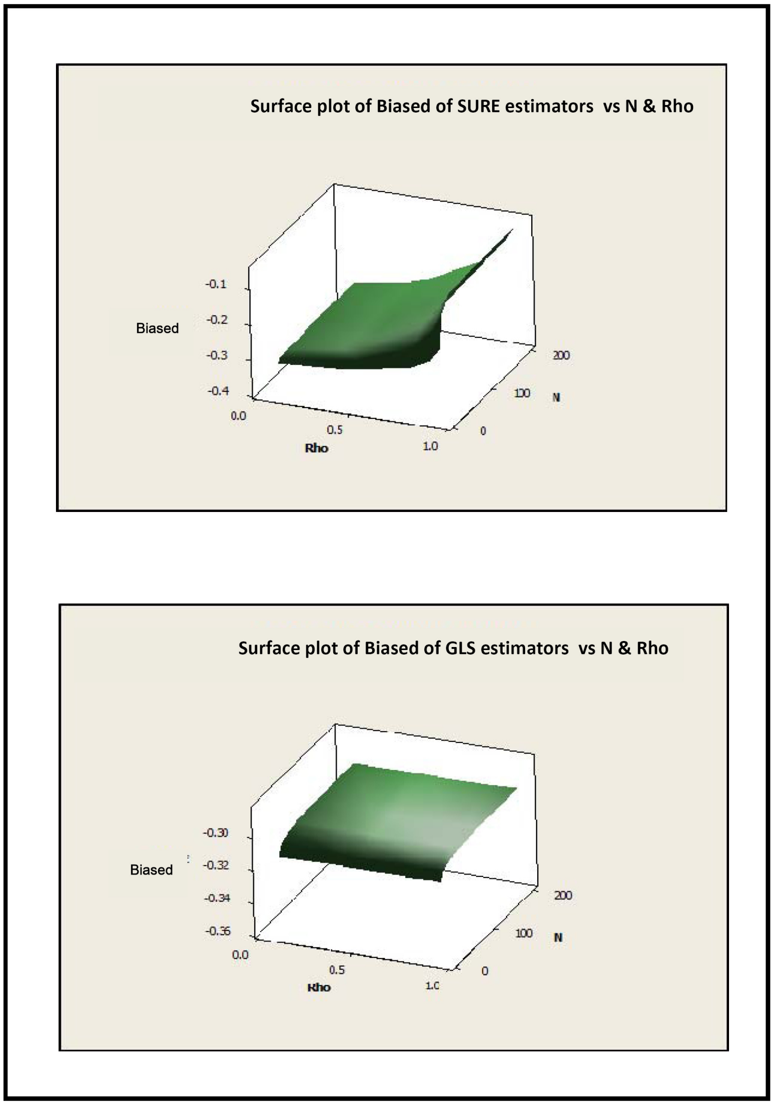

Figure 2. 3D surface plot using distance method of the biased vs. sample size and Rho of SURE and GLS estimators.

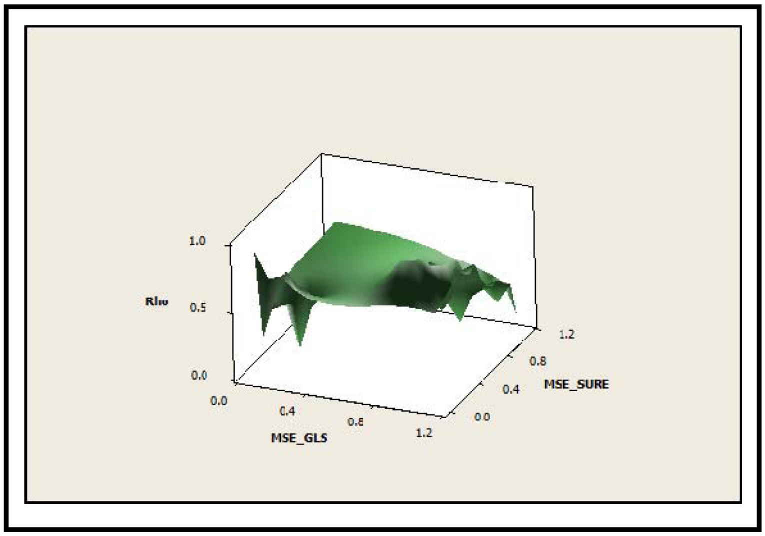

Figure 3. The mean square error of SURE and GLS estimators with different values of Rho.

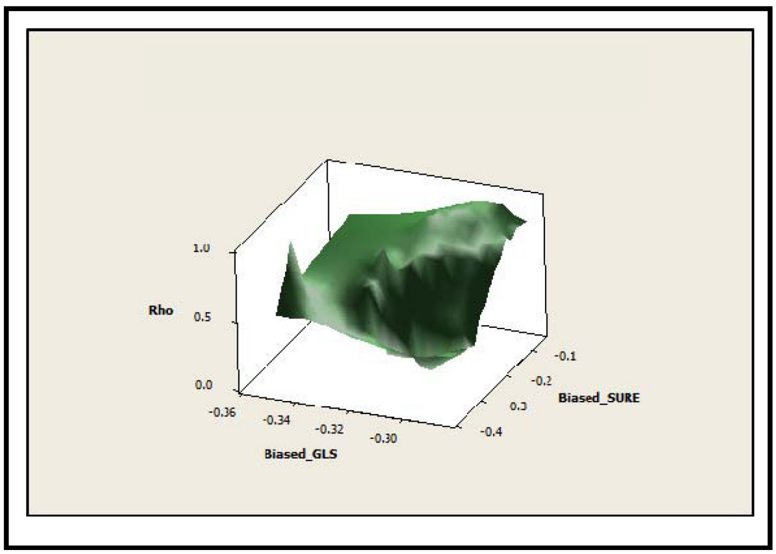

Figure 4. The biased of SURE and GLS estimators with different values of Rho.

5. The Simulation Results

From Table 3, we can see the biased and the mean square error of SURE estimators is closed to GLS estimators when the correlation between the equations was weak (0.01) and sample size is getting larger. For example, the biased of the GLS estimator, for the first parameter in the first equation and sample size 200, is (-0.2977) and the biased of SURE estimator is (-0.2911), so the difference between them is almost 0.0066. The same thing on the MSE of the first parameter in the first equation. The MSE of GLS estimator is (0.0932) and the MSE of SURE estimator is (0.0893), we note that the differences are too small.

When the correlation between the equations became (0.5), it turns out the difference between the biased and the mean square error of SURE and GLS estimators. We can note that the SURE is better, in the sense of the biased and the mean square error. For example, the first parameter in first equation when the sample size is 200 the biased of the GLS and SURE estimators are (0.2977) and (-0.2479), respectively. Therefore, we have improved the quality of the SURE estimator. The MSE of GLS estimator in the first parameter at the first equation with sample size 200, is (0.0932), while the MSE of the SURE estimator is (0.0646). The difference between the MSE of GLS estimator and SURE estimator is getting larger than before.

From Table 3, where the correlation between the equations was high (0.95), it is clear that the biased and mean square error of SURE estimators are better than GLS estimators. For example, the first parameter at the first equation with sample size 200, the biased of the GLS estimator is (-0.2977) and the biased of SURE estimator is (-0.0613). So, the difference between them is almost 0.24. The MSE of the first parameter at the first equation with sample size 200, the MSE of the GLS estimator is (0.0932) and the MSE of the SURE estimator is (0.0045). Therefore, the differences between the MSE of GLS and SURE estimators are now clear.

Finally, we can conclude that, if we have systems of equations, it is better to use SURE estimator than the GLS estimator to get less bias and MSE.

6. An Application

An application of the air navigation sector, at the General Authority for Civil Aviation in Saudi Arabia, is applied in this section. The aircraft movements have three types; international flights, domestic flights and overfly. Each of them has to pay for air navigation depending on variables; the traveled distance, the weight of the airplane and the terminal charge of landing for the international flights and the domestic flights. While, overfly movements depend only on the traveled distance and the weight of the airplane. Air navigation data from January to September 2015 is used in this study. The mean square error for SURE and GLS models in the following table:

Mean square error of SURE and GLS

Equations	Method	MSE
Overfly	GLS	105752.4
	SURE	105752.4
International	GLS	243951.4
	SURE	243951.4
Domestic	GLS	34348.23
	SURE	34348.23

The correlation between the overflight and international flight was -0.00201, and the correlation between the overflight and the domestic flight was -0.00436, while the correlation between the international flight and the domestic flight was -0.00004. It is clear that the correlations between the equations are very weak. From the simulation study, we have that the GLS and the SURE estimators are similar in the case of weak correlation between equations, see Tables 1-3. From the table above, we can see that the MSE for both GLS and SURE are the same. That is because we have a very low correlation between the aircraft movements' equations.

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