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COMMON FIXED POINT THEOREMS OF WEAKLY COMPATIBLE MAPPINGS WITH IMPLICIT FUNCTION IN METRIC SPACES

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Abstract

In this paper, some common fixed point theorems in metric spaces are obtained using implicit function. These contributions generalize a theorem in [7]. An example to support the validity of our results is given.

1. Introduction

Recently, Popa [6] used implicit functions rather than contraction conditions to prove fixed point theorems in metric spaces whose strengths lie in its unifying power as an implicit function can cover several contraction conditions at the same time which includes known and unknown contraction

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conditions. This fact is evident from examples furnished in Popa [6]. They also proved a generalized version of Banach contraction mapping theorem which was applied to obtain fixed point semantics for logic programs. The pair of weakly compatible mappings [3] is studied, and this pair has a point of coincidence. In this paper, some common fixed point theorems in metric spaces are obtained using implicit function. These contributions generalize the theorem in [7]. An example to support the validity of our results is given.

2. Preliminary

Definition 2.1. Let T and S be self maps of a set X. Maps T and S are said to be *commuting* if $(ST)(x) = (TS)(x)$ for all $x \in X$.

Definition 2.2. Let T and S be self maps of a set X. If $w = Tx = Sx$ for some $x \in X$, then x is called a *coincidence point* of T and S, and w is called a *point* of coincidence of T and S.

Example 2.3. Take $X = [0, 1]$, $Sx = x^2$, $Tx = x/2$. It is clear that $\{0, 1/2\}$ is the set of coincidence points of S and T and 0 is the unique common fixed point.

Definition 2.4 [3]**.** The mappings S and T are said to be *weakly compatible* if and only if they commute at their coincidence points.

Known result [1]**.** If a weakly compatible pair (S, T) of self maps has a unique point of coincidence, then the point of coincidence is a unique common fixed point of S and T.

Definition 2.5. A function $f : X \to R$ is said to be *lower semi-continuous* at x_0 if

$$\liminf f(x) \geq f(x_0) \text{ as } x \to x_0.$$

Definition 2.6. A function $f : X \to R$ is said to be *upper semi-continuous* at x_0 if

$$\liminf f(x) \leq f(x_0) \text{ as } x \to x_0.$$

Definition 2.7 (Implicit function)**.** Let $\mathcal{F}$ denote the family of all real semi-continuous functions $\phi : [0, 1]^6 \to R$ satisfying the following conditions:

(F1) ϕ is non-increasing in fifth and sixth variables. There exists $h \in (0, 1)$ such that for every $u, v \geq 0$ with $\phi(u, v, v, u, u + v, 0) \geq 0$ or $\phi(u, v, u, v, 0, u + v) \geq 0$, then $u < hv$.

(F2) $\phi(u, u, 0, 0, u, u) < 0$, for all $u > 0$.

Example 2.8. Define $\phi : [0, 1]^6 \to R$ as

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \delta(\max\{t_2, t_3, t_4, t_5, t_6\}),$$

where $\delta : [0, 1] \to [0, 1]$ is a lower semi-continuous function such that $\delta(s) > s$ for $0 < s < 1$. Then

(F1) $\phi(u, v, v, u, u + v, 0) = u - \delta(\max\{v, v, u, u + v, 0\}) \geq 0.$

If $u \geq hv$, then $u - \delta(u + v) \geq 0$ implies $u \geq \delta(u + v) > u + v$, a contradiction. Hence $u < hv$.

(F2) $\phi(u, u, 0, 0, u, u) = u - \delta(\max\{u, 0, 0, u, u\}) = u - \delta(u) < 0$, for all $u > 0$.

3. Main Results

Theorem 3.1. *Let T_1, T_2, T_3 and T_4 be four self-mappings of a metric space (X, d) satisfying the condition*:

$$\phi(d(T_1x, T_2y), d(T_3x, T_4y), d(T_2y, T_3x),$$

$$d(T_1x, T_4y), d(T_1x, T_3x), d(T_2y, T_4y)) \geq 0 \qquad (1)$$

for all distinct $x, y \in X$, where $\phi \in \mathcal{F}$.

If $T_1(X) \subset T_4(X)$, $T_2(X) \subset T_3(X)$ and one of $T_1(X), T_2(X), T_3(X)$ or $T_4(X)$ is a complete subspace of X, then pairs (T_1, T_3) and (T_2, T_4) have a point of coincidence. Moreover, if the pairs (T_1, T_3) and (T_2, T_4) are weakly compatible, then T_1, T_2, T_3 and T_4 have a unique common fixed point.

Proof. Let x_0 be an arbitrary point in X. Construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_{2n} = T_4x_{2n+1} = T_1x_{2n}$ and $y_{2n+1} = T_3x_{2n+2} = T_2x_{2n+1}$.

The sequences $\{x_n\}$ and $\{y_n\}$ in X are such that $x_n \to x$, $y_n \to y$ imply $d(x_n, y_n) \to d(x, y)$. Now, it gives that

$$\phi(d(T_1x_{2n}, T_2x_{2n+1}), d(T_3x_{2n}, T_4x_{2n+1}),$$
$$d(T_2x_{2n+1}, T_3x_{2n}), d(T_1x_{2n}, T_4x_{2n+1}),$$
$$d(T_1x_{2n}, T_3x_{2n}), d(T_2x_{2n+1}, T_4x_{2n+1})) \geq 0$$

or

$$\phi(d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n}), d(y_{2n+1}, y_{2n-1}),$$
$$d(y_{2n}, y_{2n}), d(y_{2n}, y_{2n-1}), d(y_{2n+1}, y_{2n})) \geq 0.$$

In view of (F1) in Definition 2.7, it follows that $d(y_{2n}, y_{2n+1}) < d(y_{2n-1}, y_{2n})$.

Thus, $\{d(y_{2n}, y_{2n+1})\}$, $n > 0$ is a non-increasing sequence of positive real numbers in [0, 1] and hence it tends to 0. Therefore, using analogous arguments we show that $\{d(y_{2n+1}, y_{2n+2})\}, n \geq 0$ is a sequence of positive real numbers in [0, 1] that converges to 0. So $d(y_n, y_{n+1}) < d(y_{n-1}, y_n)$ and $d(y_n, y_{n+1}) \to 0$ for every $n \in N$. Now $d(y_n, y_{n+p}) \leq d(y_n, y_{n+1}) + \cdots + d(y_{n+p-1}, y_{n+p})$ for any positive integer p, and it follows that $d(y_n, y_{n+p}) \to 0$ which shows that $\{y_n\}$ is a Cauchy sequence in X. Now given that $S(X)$ is a complete subspace of X. Then it implies that the subsequence $\{y_{2n+1}\}$ must converge in $S(X)$. Call this limit to be u. Then $Sv = u$.

Since $\{y_n\}$ is a Cauchy sequence containing a convergent subsequence $\{y_{2n+1}\}$, the sequence $\{y_n\}$ converges implying thereby the convergence of $\{y_{2n}\}$ being a subsequence of the convergent sequence $\{y_n\}$.

Suppose $T_1v \neq T_3v$. Then on setting $x_{2n} = v$ and $y = x_{2n+1}$, it gets that

$$\phi(d(T_1v, T_2x_{2n+1}), d(T_3v, T_4x_{2n+1}), d(T_2x_{2n+1}, T_3v), d(T_1v, T_4x_{2n+1}),$$

$$d(T_1v, T_3v), d(T_2x_{2n+1}, T_4x_{2n+1})) \geq 0$$

which on letting $n \to \infty$ reduces to

$$\phi(d(T_1v, u), d(T_3v, u), d(u, T_3v), d(T_1v, u), d(T_1v, T_3v), d(u, u)) \geq 0,$$

$$\phi(d(T_1v, u), d(T_3v, u), d(u, T_3v), d(T_1v, u), d(T_1v, u) + d(u, T_3v), 0) \geq 0.$$

Therefore, $d(T_1v, T_3v) < 0$, a contradiction. Hence, $T_1v = T_3v$ which shows that the pair (T_1, T_3) has a point of coincidence.

Since $T_1(X) \subset T_4(X)$ and $T_1v = u$, $u \in T_4(X)$. Let $w \in T_4^{-1}u$. Then $T_4w = u$. Suppose that $T_4w \neq T_2w$.

Using (1),

$$\phi(d(T_1x_{2n}, T_2w), d(T_4w, T_4w), d(T_2w, T_4w), d(T_1x_{2n}, T_4w),$$

$$d(T_1x_{2n}, T_3x_{2n}), d(T_2w, T_4w)) \geq 0$$

which on letting $n \to \infty$ reduces to

$$\phi(d(T_4w, T_2w), d(T_4w, T_4w), d(T_2w, T_4w), d(T_4w, T_4w),$$

$$d(T_4w, T_4w), d(T_2w, T_4w)) \geq 0,$$

$$\phi(d(T_4w, T_2w), 0, d(T_2w, T_4w), 0, 0, d(T_2w, T_4w)) \geq 0$$

which implies $d(T_4w, T_2w) < 0$, a contradiction. Hence $T_4w = T_2w$.

Thus, $u = T_1 v = T_3 v = T_2 w = T_4 w$ and thus both the pairs have a point of coincidence.

Since $T(X)$ is to be complete, analogous arguments establish this claim. The remaining two cases pertain essentially to the previous cases.

Since $T_1(X)$ is complete, $u \in T_1(X) \subset T_4(X)$. If $T_2(X)$ is complete, then $u \in T_2(X) \subset T_3(X)$.

Moreover, the pairs (T_1, T_3) and (T_2, T_4) are weakly compatible at v and w, respectively, implies that $T_1 u = T_1(T_3 v) = T_3(T_1 v) = T_3 u$ and $T_2 u = T_2(T_4 w) = T_4(T_2 w) = T_4 u$.

If $T_1 u \neq u$, then

$$\phi(d(T_1 u, T_2 w), d(T_3 u, T_4 w), d(T_2 w, T_3 u),$$

$$d(T_1 u, T_4 w), d(T_3 u, T_1 u), d(T_2 w, T_4 w)) \geq 0,$$

$$\phi(d(T_1 u, u), d(u, u), d(u, u), d(T_1 u, u), d(u, T_1 u), d(u, u)) \geq 0,$$

$$\phi(d(T_1 u, u), 0, 0, d(T_1 u, u), d(u, T_1 u), 0) \geq 0$$

which contradicts (F2). Hence $T_1 u = u$. Similarly, $T_2 u = u$ is easily obtained.

Thus, u is a common fixed point of T_1, T_2, T_3 and T_4. The uniqueness of common fixed point follows easily. Also u remains the unique common fixed point of both the pairs separately. This completes the proof.

Corollary 3.2. *Let S and T be two self-mappings of a metric space (X, d) satisfying the condition: $\phi(d(Sx, Sy), d(Tx, Ty), d(Sy, Tx), d(Sx, Ty), d(Sx, Tx), d(Sy, Ty)) \geq 0$ for all $x, y \in X$, where $\phi \in \mathcal{F}$. If $S(X) \subset T(X)$ and one of $S(X)$ and $T(X)$ is complete subspace of X, then the pair (S, T) has a point of coincidence. Moreover, if the pair (S, T) is weakly compatible, then S and T have a unique common fixed point.*

Proof. The proof of this corollary follows by setting $T_2 = T_1 = S$ and $T_4 = T_3 = T$ in the above theorem.

Definition 3.3 (Implicit function)**.** Let $\mathcal{F}$ denote the family of all continuous functions $\psi : [0, 1]^6 \to R$ satisfying the following conditions:

$G_1 : \Psi$ is non-increasing in the fifth and sixth variables; there exists $h \in (0, 1)$ such that for every $u, v \geq 0$ with $\psi(u, v, v, u, u + v, 0) \geq 0$ or $\psi(u, v, u, v, 0, u + v) \geq 0$ we have $u < hv$.

$G_2 : \Psi(u, u, u, 0, 0, u) \geq 0$ for all $u > 0$.

Example 3.4. Define $\Psi : [0, 1]^6 \to R$ as $\psi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \delta(\max\{t_2, t_3, t_4, t_5, t_6\})$, where $\delta : [0, 1] \to [0, 1]$ is a continuous function such that $\delta(s) > s$ for $0 < s < 1$. Then

$G_1 : \Psi(u, v, v, u, u + v, 0) \geq 0 = u - \delta(\max\{v, v, u, u + v, 0\}) \geq 0.$ If $u \geq hv,$ then $u - \delta(u + v) \geq 0$ implies $u \geq \delta(u + v) > u + v$ a contradiction. Hence $u < hv.$

$G_2 : \Psi(u, u, u, 0, 0, u) = u - \delta(\max\{u, 0, 0, u, u\}) = u - \delta(u) < 0$ for all $u > 0.$

Theorem 3.5. *Let* (X, d) *be a complete metric space and* T_1, T_2, T_3 *and* T_4 *be the self-mappings of X satisfying the following conditions*:

(1) $T_1(X) \subset T_4(X)$ *and* $T_2(X) \subset T_3(X)$.

(2) *The pairs* (T_1, T_4) *and* (T_2, T_3) *are weakly compatible*.

(3) $T_3(X)$ *or* $T_4(X)$ *is complete*.

(4) *There exists* $k \in (0, 1)$ *such that*

$$\Psi\left(\begin{matrix} \frac{d(T_1x, T_2y)}{k}, \frac{d(T_3x, T_1x) + d(T_1x, T_4y)}{2}, d(T_1x, T_3x), \frac{d(T_2y, T_4y)}{k}, \\ d(T_2y, T_4y) + d(T_4y, T_3x), d(T_1x, T_4y) \end{matrix}\right) \geq 0 \tag{2}$$

for some $\Psi \in \mathcal{F}$, *and* $x, y \in X$. *Then* T_1, T_2, T_3 *and* T_4 *have a unique common fixed point in X.*

Proof. Let x_0 be any arbitrary point. Since $T_1(X) \subset T_4(X)$ and $T_2(X) \subset T_3(X)$, there must exist points $x_1, x_2 \in X$ such that $T_1x_0 = T_4x_1$ and $T_2x_1 = T_3x_2$.

In general, sequences $\{y_n\}$ and $\{x_n\}$ in X are obtained in such a way that

$y_{2n} = T_4x_{2n+1} = T_1x_{2n}$ and $y_{2n+1} = T_3x_{2n+2} = T_2x_{2n+1}$,

$$\Psi\begin{pmatrix} \frac{d(T_1x_{2n}, T_2x_{2n+1})}{k}, \frac{d(T_3x_{2n}, T_1x_{2n}) + d(T_1x_{2n}, T_4x_{2n+1})}{2}, d(T_1x_{2n}, T_3x_{2n}), \\ \frac{d(T_2x_{2n+1}, T_4x_{2n+1})}{k}, d(T_2x_{2n+1}, T_4x_{2n+1}) + d(T_4x_{2n+1}, T_3x_{2n}), \\ d(T_1x_{2n}, T_4x_{2n+1}) \end{pmatrix} \geq 0,$$

$$\Psi\begin{pmatrix} \frac{d(y_{2n}, y_{2n+1})}{k}, \frac{d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n})}{2}, d(y_{2n}, y_{2n-1}), \\ \frac{d(y_{2n}, y_{2n+1})}{k}, d(y_{2n+1}, y_{2n}) + d(y_{2n}, y_{2n-1}), d(y_{2n}, y_{2n}) \end{pmatrix} \geq 0$$

imply Ψ is non-increasing function, and it gives that

$$\Psi\left(\frac{d(y_{2n}, y_{2n+1})}{k}, \frac{d(y_{2n-1}, y_{2n})}{2}, d(y_{2n}, y_{2n-1}), \right.$$

$$\left. \frac{d(y_{2n}, y_{2n+1})}{k}, d(y_{2n+1}, y_{2n}) + d(y_{2n}, y_{2n-1}), 0 \right) \geq 0,$$

$$d(y_{2n}, y_{2n+1}) \leq \frac{d(y_{2n-1}, y_{2n})}{k}.$$

Similarly, $d(y_{2n+1}, y_{2n+2}) \leq \frac{d(y_{2n+1}, y_{2n})}{k}$.

Therefore, $d(y_n, y_{n+1}) \leq \frac{d(y_{n-1}, y_n)}{k}$ is obtained for all n.

Hence $d(y_n, y_{n+1}) \leq \frac{d(y_{n-1}, y_n)}{k} \leq \frac{d(y_{n-2}, y_{n-1})}{k^2} \leq \cdots \leq \frac{d(y_0, y_1)}{k^n}$.

Thus $\lim_{n\to\infty} d(y_n, y_{n+1}) = 0$.

Now, for any positive integer *p*, it gives that

$$d(y_n, y_{n+p}) \leq d(y_n, y_{n+1}) + \cdots + d(y_{n+p-1}, y_{n+p}).$$

It follows that $d(y_n, y_{n+p}) \to 0$ which shows that $\{y_n\}$ is a Cauchy sequence in *X* which is complete. Therefore, $\{y_n\}$ converges to *z*, for some $z \in X$.

So it implies that $\{T_1x_{2n}\}$, $\{T_3x_{2n}\}$, $\{T_2x_{2n+1}\}$ and $\{T_4x_{2n+1}\}$ also converge to *z*.

Now if $T_4(X)$ is complete, then $z \in T_4(X)$. So there exists $u \in X$ such that $z = T_4u$,

$$\Psi\left(\begin{matrix}\frac{d(T_1x_{2n}, T_2u)}{k}, \frac{d(T_3x_{2n}, T_1x_{2n}) + d(T_1x_{2n}, T_4u)}{2}, d(T_1x_{2n}, T_3x_{2n}) \\ \frac{d(T_2u, T_4u)}{k}, d(T_2u, T_4u) + d(T_4u, T_3x_{2n}), d(y_{2n}, y_{2n})d(T_1x_{2n}, T_4u)\end{matrix}\right) \geq 0.$$

As $n \to \infty$, it becomes

$$\Psi\left(\frac{d(z, T_2u)}{k}, \frac{d(z, z) + d(z, z)}{2}, d(z, z),\right.$$

$$\left.\frac{d(T_2u, z)}{k}, d(T_2u, z) + d(z, z), d(z, z)\right) \geq 0,$$

$$\Psi\left(\frac{d(z, T_2u)}{k}, 0, 0, \frac{d(T_2u, z)}{k}, d(T_2u, z), 0\right) \geq 0.$$

So $d(z, T_2u) \leq 0$. Hence $z = T_2u = T_3u = T_4u$.

Now (T_2, T_3) is weakly compatible, so $T_2T_3u = T_3T_2u$, and thereby $T_2z = T_3z$.

Putting $x = x_{2n}$ and $y = z$ in (2), we get

$$\Psi\begin{pmatrix} \frac{d(T_1x_{2n},\, T_2z)}{k},\, \frac{d(T_3x_{2n},\, T_1x_{2n}) + d(T_1x_{2n},\, T_4z)}{2},\, d(T_1x_{2n},\, T_3x_{2n}), \\ \frac{d(T_2z,\, T_4z)}{k},\, d(T_2z,\, T_4z) + d(T_4z,\, T_3x_{2n}),\, d(T_1x_{2n},\, T_4z) \end{pmatrix} \geq 0.$$

Taking limit as $n \to \infty$, we have

$$\Psi\Bigg(\frac{d(z,\, T_2z)}{k},\, \frac{d(z,\, z) + d(z,\, T_4z)}{2},\, d(z,\, z),$$

$$\frac{d(T_2z,\, T_4z)}{k},\, d(T_2z,\, T_4z) + d(T_4z,\, z),\, d(z,\, T_4z)\Bigg) \geq 0,$$

$$\Psi\left(\frac{d(z,\, T_2z)}{k},\, \frac{d(z,\, T_4z)}{2},\, 0,\, \frac{d(T_2z,\, T_4z)}{k},\, d(T_2z,\, z),\, d(z,\, T_4z)\right) \geq 0.$$

Since $z = T_4z,\ z \in T_4(X)$,

$$\Psi\left(\frac{d(z,\, T_2z)}{k},\, \frac{d(z,\, z)}{2},\, 0,\, \frac{d(T_2z,\, z)}{k},\, d(T_2z,\, z),\, d(z,\, z)\right) \geq 0,$$

$$\Psi\left(\frac{d(z,\, T_2z)}{k},\, 0,\, 0,\, \frac{d(T_2z,\, z)}{k},\, d(T_2z,\, z),\, 0\right) \geq 0.$$

Since Ψ is non-increasing, $T_2z = T_4z = z$, as $T_2(X) \subset T_3(X)$.

Suppose there exists $v \in X$, such that $T_2z = T_3v = z$.

Taking $x = v,\ y = z$ in (2),

$$\Psi\begin{pmatrix} \frac{d(T_1v,\, T_2z)}{k},\, \frac{d(T_3v,\, T_1v) + d(T_1v,\, T_4z)}{2},\, d(T_1v,\, T_3z),\, \frac{d(T_2z,\, T_4z)}{k}, \\ d(T_2z,\, T_4z) + d(T_4z,\, T_3v),\, d(T_1v,\, T_4z) \end{pmatrix} \geq 0,$$

$$\Psi\Bigg(\frac{d(T_1v,\, z)}{k},\, \frac{d(z,\, T_1v) + d(T_1v,\, z)}{2},\, d(T_1v,\, z),$$

$$\frac{d(z,\, z)}{k},\, d(z,\, z) + d(z,\, z),\, d(T_1v,\, z)\Bigg) \geq 0,$$

$$\Psi\left(\frac{d(T_1v,\ z)}{k},\ d(z,\ T_1v),\ d(T_1v,\ z),\ 0,\ 0,\ d(T_1v,\ z)\right) \geq 0.$$

This implies that $T_1v = z$. As $T_1(X) \subset T_4(X)$, $z = T_1v \in T_4(X)$. Thus $z = T_1v = T_4v$. Furthermore, since (T_1, T_4) is weakly compatible, $T_1T_4v = T_4T_1v$ and $T_1z = T_4z$.

Combining all the results, $T_1z = T_4z = T_2z = T_3z = z$.

Similarly, we obtain that $T_1z = T_4z = T_2z = T_3z = z$ by taking $T_3(X)$ to be complete. Thus, z is the common fixed point of T_1, T_2, T_3 and T_4.

Uniqueness. Let p and z be the two common fixed points of maps T_1, T_2, T_3 and T_4. Putting $x = z$ and $y = p$ in condition (1), we have

$$\Psi\begin{pmatrix}\dfrac{d(T_1v,\ T_2z)}{k},\ \dfrac{d(T_3v,\ T_1v) + d(T_1v,\ T_4z)}{2},\ d(T_1v,\ T_3z),\ \dfrac{d(T_2z,\ T_4z)}{k}, \\ d(T_2z,\ T_4z) + d(T_4z,\ T_3v),\ d(T_1v,\ T_4z)\end{pmatrix} \geq 0,$$

$$\Psi\begin{pmatrix}\dfrac{d(T_1z,\ T_2p)}{k},\ \dfrac{d(T_3z,\ T_1z) + d(T_1z,\ T_4p)}{2},\ d(T_1p,\ T_3z),\ \dfrac{d(T_2p,\ T_4p)}{k}, \\ d(T_2p,\ T_4p) + d(T_4p,\ T_3z),\ d(T_1z,\ T_4p)\end{pmatrix} \geq 0,$$

$$\Psi\left(\frac{d(z,\ p)}{k},\ \frac{d(z,\ z) + d(z,\ p)}{2},\ d(p,\ z),\right.$$

$$\left.\frac{d(p,\ p)}{k},\ d(p,\ p) + d(p,\ z),\ d(z,\ p)\right) \geq 0,$$

$$\Psi\left(\frac{d(z,\ p)}{k},\ \frac{d(z,\ p)}{2},\ d(p,\ z),\ 0,\ d(p,\ z),\ d(z,\ p)\right) \geq 0.$$

Hence $p = z$. So z is the unique common fixed point of T_1, T_2, T_3 and T_4.

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