Advances in Fuzzy Sets and Systems

Published Online: February 2016
http://dx.doi.org/10.17654/FS021010093
Volume 21, Number 1, 2016, Pages 93-105 **ISSN: 0973-421X**

PROPERTIES OF TRANSVERSALS OF INTUITIONISTIC FUZZY DIRECTED HYPERGRAPHS

K. K. Myithili and R. Parvathi

Department of Mathematics
Vellalar College for Women
Erode 638 012
Tamilnadu, India
e-mail: myth_maths@rediffmail.com
paarvathis@rediffmail.com

Abstract

A transversal is a line that intersects two lines, whereas in intuitionistic fuzzy directed hypergraph, the transversal is a hyperedge that intersects two or more hyperedges. In this paper, a few properties of intuitionistic fuzzy transversals of intuitionistic fuzzy directed hypergraphs (IFDHGs) are discussed. Further, properties of transversal core, simple IFDHG and fundamental sequence of IFDHG are studied.

1. Introduction

The notion of graph theory was introduced by Euler in 1736. The theory of graphs is an extremely useful tool for solving combinatorial problems in different areas such as algebra, number theory, optimization and computer science. In order to expand the application base, the notion of graph was

Received: September 22, 2015; Accepted: January 7, 2016

2010 Mathematics Subject Classification: 03E72, 05C65, 05C72.

Keywords and phrases: intuitionistic fuzzy directed hypergraph, transversals, properties of transversals of intuitionistic fuzzy directed hypergraph.

Communicated by John N. Mordeson

generalized to that of a hypergraph, that is, a set of vertices together with a collection of its subsets. In 1976, Berge [4] introduced the concepts of graph and hypergraph. In [5], the concepts of fuzzy graph and fuzzy hypergraph were introduced. Fuzzy graph theory has numerous applications in modern science and technology. In 1986, Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Also he [2] introduced the concept of intuitionistic fuzzy relations and defined intuitionistic fuzzy six types of Cartesian products. Intuitionistic fuzzy graph and intuitionistic fuzzy hypergraph (IFHG) were introduced in [6, 7]. Hypergraphs have vast applications in system analysis, data structures, circuit clustering and pattern recognition. In mathematics and computer science, hypergraphs also arise some problems, including circuit layout, Boolean satisfiability, numerical linear algebra. Directed fuzzy hypergraphs are generalization of both crisp directed hypergraphs and directed fuzzy graphs. In [8], intuitionistic fuzzy directed hypergraphs have been discussed. In [16] transversals of intuitionistic fuzzy directed hypergraphs (IFDHGs) and minimal transversals of IFDHG were introduced. Thus, the authors got motivated to study the properties of transversals of intuitionistic fuzzy directed hypergraph. In this paper, properties of transversals of intuitionistic fuzzy directed hypergraphs (TIFDHGs) are studied.

2. Notations

$H = (V, E)$	Intuitionistic fuzzy directed hypergraph with vertex set V and edge set E
$h(H)$	Height of H
$F(H)$	Fundamental sequence of H
$C(H)$	Core set of H
$I(H)$	Induced fundamental sequence of H
$H^{(r_i, s_i)}$	(r_i, s_i)-level of H

(r_i, s_i)	Membership and non-membership values of edges
$Tr(H)$	Intuitionistic fuzzy transversals (IFT) of H
V_A	Set of vertices in a hyperedge A

3. Preliminaries

In this section, basic definitions relating to intuitionistic fuzzy sets, intuitionistic fuzzy hypergraphs and IFDHGs are dealt with.

Definition 3.1 [1]**.** Let a set E be fixed. An *intuitionistic fuzzy set* (IFS) V in E is an object of the form $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle / v_i \in E\}$, where the function $\mu_i : E \to [0, 1]$ and $\nu_i : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E$, $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$.

Definition 3.2 [7]**.** An *intuitionistic fuzzy hypergraph* (IFHG) is an ordered pair $H = (V, E)$, where

(i) $V = \{v_1, v_2, ..., v_n\}$ is a finite set of intuitionistic fuzzy vertices,

(ii) $E = \{E_1, E_2, ..., E_m\}$ is a family of crisp subsets of V,

(iii) $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_j)) : \mu_j(v_i), \nu_j(v_i) \geq 0$ and $\mu_j(x_i) + \nu_j(x_i) \leq 1\}$, $j = 1, 2, ..., m$,

(iv) $E_j \neq \phi$, $j = 1, 2, ..., m$,

(v) $\bigcup_j supp(E_j) = V$, $j = 1, 2, ..., m$.

Here, the hyperedges E_j are crisp sets of intuitionistic fuzzy vertices, $\mu_j(v_i)$ and $\nu_j(v_i)$ denote the degrees of membership and non-membership of vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(e_{ij}, \mu_j(v_i), \nu_j(v_j))$. The sets V, E are crisp sets.

Notations

1. Hereafter, $\langle \mu(v_i), \nu(v_i) \rangle$ or simply $\langle \mu_i, \nu_i \rangle$ denote the degrees of membership and non-membership of the vertex $v_i \in V$, such that $0 \leq \mu_i + \nu_i \leq 1$.

2. $\langle \mu(e_{ij}), \nu(e_{ij}) \rangle$ or simply $\langle \mu_{ij}, \nu_{ij} \rangle$ denote the degrees of membership and non-membership of the edge $(v_i, v_j) \in V \times V$, such that $0 \leq \mu_{ij} + \nu_{ij} \leq 1$. That is, μ_{ij} and ν_{ij} are the degrees of membership and non-membership of *i*th vertex in *j*th edge, say e_{ij}.

Note. The *support* of E_j is defined as $supp(E_j) = \{v_i / \mu_i > 0$ and $\nu_i > 0\}$ for all $v_i \in E_j$ and also $E_j \in E$.

Definition 3.3 [8]**.** An *intuitionistic fuzzy directed hypergraph* (IFDHG) H is a pair (V, E), where V is a nonempty set of vertices and E is a set of intuitionistic fuzzy hyperarcs; an intuitionistic fuzzy hyperarc $E_i \in E$ is defined as a pair $(t(E_i), h(E_i))$, where $t(E_i) \subset V$, with $t(E_i) \neq \emptyset$, is its tail, and $h(E_i) \in V - t(E_i)$ is its head. A vertex v_s is said to be a *source vertex* in H if $h(E_i) \neq s$, for every $E_i \in E$. A vertex v_d is said to be a *destination vertex* in H if $d \neq t(E_i)$, for every $E_i \in E$.

Definition 3.4 [12]**.** Let $H = (V, E)$ be an IFDHG. The *height* of H, is defined by

$$h(H) = (\max(\max(\mu_{ij})), \max(\min(\nu_{ij})))$$

where μ_{ij}, ν_{ij} are membership and non-membership values of the hyperedge e_{ij}, for all $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$.

Definition 3.5 [12]**.** Let $H = (V, E)$ be an intuitionistic fuzzy directed hypergraph. Suppose $E_u, E_v \in E$ and $0 < \alpha \leq 1$, $0 < \beta \leq 1$. The (α, β)-*level* is defined by

$$(E_u,\ E_v)^{(\alpha,\beta)} = \{v_i \in V / \max(\mu_{ij}^{\alpha}(v_i)) \geq \alpha,\ \min(\nu_{ij}^{\beta}(v_i)) \leq \beta\}. \qquad (1)$$

Definition 3.6 [12]**.** Let $H = (V,\ E)$ be an intuitionistic fuzzy directed hypergraph with $0 \leq h_\mu(H) \leq 1$ and $0 \leq h_\nu(H) \leq 1$. Let $H^{r_i, s_i} = (V^{r_i, s_i},$ $E^{r_i, s_i})$ be the $(r_i,\ s_i)$-level intuitionistic fuzzy hypergraph of *H*. The sequence of real numbers $\{r_1,\ r_2,\ ...,\ r_n;\ s_1,\ s_2,\ ...,\ s_n\}$, such that $0 \leq r_i \leq$ $h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$, satisfying the properties:

(i) if $r_1 < \alpha \leq 1$ and $0 \leq \beta < s_1$, then $E^{\alpha,\beta} = \emptyset$,

(ii) if $r_{i+1} \leq \alpha \leq r_i$; $s_i \leq \beta \leq s_{i+1}$, then $E^{\alpha,\beta} = E^{r_i, s_i}$,

(iii) $E^{r_i, s_i} \sqsubset E^{r_{i+1}, s_{i+1}}$

is called the *fundamental sequence of* *H*, and is denoted by $F(H)$.

The core set of *H* is denoted by $C(H)$ and is defined by $C(H) = \{H^{r_1, s_1},$ $H^{r_2, s_2},\ ...,\ H^{r_n, s_n}\}$. The corresponding set of $(r_i,\ s_i)$-level hypergraphs $H^{r_1, s_1} \subset H^{r_2, s_2} \subset \cdots \subset H^{r_n, s_n}$ is called the *H* *induced fundamental sequence* and is denoted by $I(H)$. The $(r_n,\ s_n)$ level is called the *support level* of *H* and the H^{r_n, s_n} is called the *support* of *H*.

Definition 3.7 [12]**.** Suppose $H = (V,\ E)$ and $H' = (V',\ E')$ are intuitionistic fuzzy directed hypergraphs, *H* is called a *partial intuitionistic fuzzy directed hypergraph* of H' if $V' = \bigcup(supp(E_k))|\, E_k \in E'$. The partial IFDHG generated by E', and is denoted by $H \subseteq H'$. Also, $H \subset H'$ if $H \subseteq H'$ and $H \neq H'$.

Definition 3.8 [12]**.** Let $H = (V,\ E)$ be an intuitionistic fuzzy directed hypergraph. An *intuitionistic fuzzy transversal* *T* of *H* is an intuitionistic fuzzy subset of *V* with the property that $T^{(E_\mu,\ E_\nu)} \bigcap A^{(E_\mu,\ E_\nu)} \neq \emptyset$ for each

$A \in E$, where $E_\mu = \max(\mu_{ij})$ and $E_\nu = \min(\nu_{ij})$, for all $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$.

Definition 3.9 [12]**.** A *minimal intuitionistic fuzzy transversal T* for *H* is a transversal of *H* with the property that any subset T_i of *T* is not an intuitionistic fuzzy transversal of *H*.

Theorem 3.1 [12]**.** *If T is an intuitionistic fuzzy transversal of IFDHG* $H = (V, E)$, *then* $h(T) \geq h(E_j)$ *for* $E_j \in E$. *Moreover, if T is minimal intuitionistic fuzzy transversal of H, then* $h(T) = (\max(\max(\mu_{ij})), \max(\min(\nu_{ij}))) = h(H)$.

Theorem 3.2 [12]**.** $Tr(H)$ *is sectionally elementary*.

Theorem 3.3 [12]**.** *For each* $A \in Tr(H)$, A^{r_1, s_1} *is a minimal transversal of* H^{r_1, s_1}.

4. Main Results

Definition 4.1. Let $H = (V, E)$ be an IFDHG. The *intuitionistic fuzzy transversal core* (IFTC) of *H* is an IFDHG $H' = (V', E')$ which satisfies

(i) $\min Tr(H') = \min Tr(H)$,

(ii) $\bigcup \min Tr(H) = H'$,

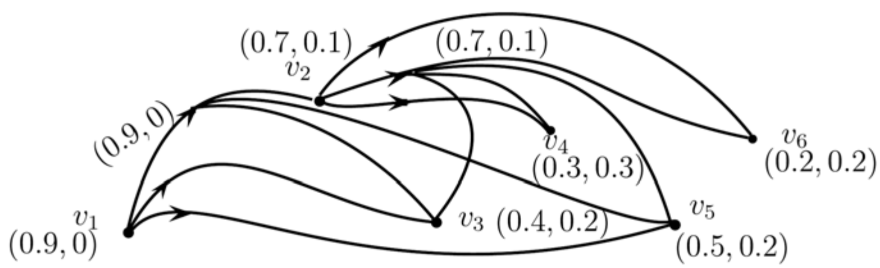
(iii) $E \backslash E'$ is exactly the set of vertices of *H* which does not belong to $Tr(H)$, where E' is the remaining hyperedge set after removing hyperedges which are properly contained in another hyperedge.

Result. The intuitionistic fuzzy transversal core exists and unique for any IFDHG without spikes. The definition does not hold good for IFDHGs with spike (an edge with single vertex) hyperedges.

Example 1. Consider an IFDHG, $H = (V, E)$ with adjacency matrix as below:

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} E_1 & E_2 & E_3 & E_4 & E_5 \end{array} \\ \begin{pmatrix} \langle 0.9, 0\rangle & \langle 0.9, 0\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.9, 0\rangle \\ \langle 0, 1\rangle & \langle 0.7, 0.1\rangle & \langle 0.7, 0.1\rangle & \langle 0.7, 0.1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.4, 0.2\rangle & \langle 0.4, 0.2\rangle & \langle 0, 1\rangle & \langle 0.4, 0.2\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.3, 0.3\rangle & \langle 0.3, 0.3\rangle & \langle 0, 1\rangle \\ \langle 0.5, 0.2\rangle & \langle 0.5, 0.2\rangle & \langle 0.5, 0.2\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.2, 0.2\rangle & \langle 0.2, 0.2\rangle & \langle 0, 1\rangle \end{pmatrix} \end{array}.$$

The corresponding graph is shown in Figure 1.



Graph of H

Figure 1. Intuitionistic fuzzy directed hypergraph.

The adjacency matrix of the IFTC H' is given below:

$$H' = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cc} E_2 & E_3 \end{array} \\ \begin{pmatrix} \langle 0.9, 0\rangle & \langle 0, 1\rangle \\ \langle 0.7, 0.1\rangle & \langle 0.7, 0.1\rangle \\ \langle 0.4, 0.2\rangle & \langle 0.4, 0.2\rangle \\ \langle 0, 1\rangle & \langle 0.3, 0.3\rangle \\ \langle 0.5, 0.2\rangle & \langle 0.5, 0.3\rangle \\ \langle 0, 1\rangle & \langle 0.2, 0.2\rangle \end{pmatrix} \end{array}.$$

Figure 2 depicts the graph of H'.

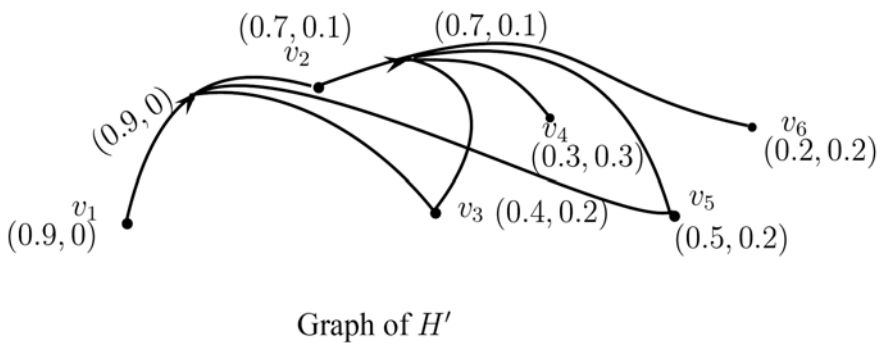


Figure 2. Transversal core of IFDHG.

Here

(i) $\min Tr(H) = \{\{v_1, v_2\}, \{v_1, v_4\}, \{v_1, v_6\}\} = \min Tr(H')$,

(ii) $\bigcup\{\{v_1, v_2\}, \{v_1, v_4\}, \{v_1, v_6\}\} = H'$,

(iii) $E \backslash E' = \{E_2, E_3\} \in H$ but not in $Tr(H)$.

Theorem 4.1. *Let* $H = (V, E)$ *be an IFDHG. Then, the following statements are equivalent*:

(i) *T is an IFT of H.*

(ii) $T^{r_i, s_i} \cap A^{r_i, s_i} \neq \emptyset$, *for each intuitionistic fuzzy hyperedge* $A \in E$ *and each* (r_i, s_i) *with* $0 < r_i \leq h_\mu(H), 0 < s_i \leq h_\nu(H)$,

(iii) T^{r_i, s_i} *is an IFT of* H^{r_i, s_i}, *for each* (r_i, s_i) *with* $0 < r_i \leq \alpha$, $0 < s_i \leq \beta$.

Proof. By Definition 3.9, the proof is direct.

Theorem 4.2. *For a simple IFDHG H,* $Tr(Tr(H)) = H$.

Theorem 4.3. *For any IFDHG H,* $Tr(Tr(H)) \subseteq H$.

Proof. By Definition 4.1, there exists a partial hypergraph H' of a simple IFDHG *H* such that $Tr(H') = Tr(H)$.

By Theorem 4.2, $Tr(Tr(H)) = Tr(Tr(H')) = H' \subseteq H$.

Theorem 4.4. *Let* $H \in (V, E)$ *be an IFDHG and suppose* $T \in Tr(H)$. *If* $H' \subseteq supp(T) \subseteq H$, *then there exists an intuitionistic fuzzy hyperedge A of H,* $(r_i, s_i) \in A$ *denote the membership and non-membership values of A such that*

(i) $(r_i, s_i) = h(A) = h(T^{r_i, s_i}) > 0$,

(ii) $T_{h(A)} \cap A_{h(A)} = H$.

Proof. Suppose that $0 < h(T^{r_i, s_i}) \leq 1$ and let E' be the set of all intuitionistic fuzzy hyperedges of H where $h(\tau^{r_i, s_i}) \geq h(T^{r_i, s_i})$ for each $\tau \in E'$.

Since T^{r_i, s_i} is a transversal of H^{r_i, s_i} and $H' \subseteq T^{r_i, s_i}$, is nonempty. Further, for each $\tau \in E'$, $h(\tau) \geq h(\tau^{r_i, s_i}) \geq h(T^{r_i, s_i})$ is true. Assume that T^{r_i, s_i} is the minimal transversal, then for each $\tau \in E'$, $h(\tau) > h(T^{r_i, s_i})$ and there exists $H_\tau \neq H$ with $H_\tau \in \tau_{h(\tau)} \cap T_{h(\tau)}$.

Define an IFDHG H_1 such that

$$H_1(U) = \begin{cases} T(U) & \text{if } U \neq H', \\ \max(h(A)/h(A) < h(T^{r_i, s_i})), \min(h(A)/h(A) < h(T^{r_i, s_i})) & \text{if } U = H'. \end{cases} \tag{2}$$

Clearly H_1 is an IFT of H and $h(H_1^{r_i, s_i}) < h(T^{r_i, s_i})$, which contradicts the minimality of *T*. Suppose each $\tau \in E'$ satisfies (i) and also contains an $H_\tau \neq H$ with $H_\tau \in \tau_{h(\tau)} \cap T_{h(\tau)}$. The process is continued and the argument of (i) provides a contradiction which completes the proof.

Theorem 4.5. *Let* $H = (V, E)$ *be an IFDHG. Then, there exists* $T \in Tr(H)$ *with* $H' \subseteq supp(T) \subseteq H$, *if and only if, for* $A \in E$, *which satisfies*

(i) $(r_i, s_i) = h(A)$,

(ii) (r_j, s_j)*-level cut of* $h(A')$ *is not a subhypergraph of the* (r_i, s_i)*-level cut of* $h(A)$, *for each* $A' \in E$ *with* $h(A') > h(A)$,

(iii) *the* (r_i, s_i)*-level cut of* $h(A)$ *does not contain any other edge of* $H_{h(A)}$, *where* (r_i, s_i) *denote the membership and non-membership values of* A.

Proof. Necessary part:

(i) Let $T \in Tr(H)$ and $0 < h(T^{r_i, s_i}) \leq 1$. Condition (i) follows from Theorem 4.4.

(ii) Assume for each A satisfying (i) there exists an $A' \in E$ such that $h(A') > h(A)$ and $A'_{h(A')} \subseteq A_{h(A)}$, then there exists $U \neq H'$, with $U \in A'_{h(A')} \cap T_{h(A')} \subseteq A_{h(A)} \cap T_{h(A)}$ which contradicts Theorem 4.4.

(iii) Suppose that for each A satisfying (i) and (ii) there exists $A' \in E$ such that $\varnothing \neq A'_{h(A)} \subset A_{h(A)}$. Since $A'_{h(A)} \neq \varnothing$ and by (ii), we have $h(A') = h(A) = (r_i, s_i)$.

If $(r_j, s_j) = h(A')$ and $A'' \in E$ such that $\varnothing \neq A''_{h(A)} \subset A'_{h(A)} \subset A_{h(A)}$. Continuing the process the chain must end in finitely many steps so without loss of generality assume $(r_i, s_i) < h(A)$. But, there exists $U \neq H'$ such that $U \in A'_{h(A)} \cap T_{h(A)} \subseteq A_{h(A)} \cap T_{h(A)}$, which contradicts Theorem 4.4.

Sufficient part:

Let $A \in E$ satisfy the conditions (i), (ii) and (iii). By condition (i), $h(A) = (r_i, s_i)$ for some member of fundamental sequence. By condition (ii) and (iii), there exists $U \in A'_{h(A')} \backslash A_{h(A)}$ for each $A' \in E$ such that $A' \neq A$ and $h(A') \geq h(A)$. Let V_A be the set of all vertices of H, such that $V_A \cap A_{h(A)} = \emptyset$.

Construct the initial sequence of transversals $\tau_t \subseteq V$ for each t, $1 \leq t < i$ and $\tau_i \subseteq V_A \cup V_i$. Clearly, $V_i \in \tau_i$, for each i. Continuing the process it leads to a minimal IFT T with $(r_i, s_i) = h(A) = h(T^{r_i, s_i})$.

Theorem 4.6. *Let* $H = (V, E)$ *be an IFDHG with* $F(H) = \{r_1, r_2, ..., r_n; s_1, s_2, ..., s_n\}$, *such that* $0 \leq r_i \leq h_\mu(H)$ *and* $0 \leq s_i \leq h_\nu(H)$. *Also,* H^{r_i, s_i} *be the elementary IFDHG containing* A' *if and only if* $h(A') = (r_i, s_i)$ *and* $supp(A')$ *is an edge of* H^{r_i, s_i}. *Then* $Tr(Tr(H))$ *is a partial IFDHG of* H^{r_i, s_i}.

Proof. By Theorem 3.3 and by construction of minimal transversal, the (r_i, s_i)-level hypergraph of $Tr(H)$ is $Tr(H^{r_i, s_i})$. That is $(Tr(H))^{r_i, s_i} = Tr(H^{r_i, s_i})$. Let $\tau \in Tr(Tr(H))$. By Theorem 4.4, $h(\tau(V_i)) > 0$ implies there exists $T \in Tr(H)$ with $h(\tau(V_i)) = h(T)$. By Theorem 3.1, $h(T) = (\max(\max(\mu_{ij})), \max(\min(\nu_{ij}))) = h(H)$ for each minimal transversal T. Hence τ is elementary with height (r_i, s_i). Since $\text{supp}(\tau) = \tau^{r_i, s_i}$, Theorem 3.3 implies $\text{supp}(\tau)$ is a minimal IFT of $(Tr(H))^{r_i, s_i}$. By Corollary 4.3, $\text{supp}(\tau)$ is an edge of H^{r_i, s_i}, hence τ is an edge of H^{r_i, s_i}.

Theorem 4.7. *Let* $H = (V, E)$ *be an IFDHG with* H^{r_i, s_i} *is simple. Then* $Tr(Tr(H)) = H^{r_i, s_i}$.

Proof. By Theorem 4.6, $Tr(Tr(H)) \subseteq H^{r_i, s_i}$. Let τ be elementary with $h(\tau) = (r_i, s_i)$ and $\text{supp}(\tau) \in H^{r_i, s_i}$. By Theorem 4.6, $\text{supp}(\tau)$ is a minimal

IFT of $(Tr(H))^{r_i, s_i}$. Since each minimal IFT of $Tr(H)$ is elementary, by algorithm of minimal IFT the process terminates at (r_i, s_i)-level and $\tau \in Tr(Tr(H))$.

Hence $H^{r_i, s_i} \subseteq Tr(Tr(H))$ which implies $H^{r_i, s_i} = Tr(Tr(H))$.

5. Conclusion

In this paper, the properties of transversals of intuitionistic fuzzy directed hypergraphs are discussed. Also, transversal core of intuitionistic fuzzy directed hypergraph is defined. It is interesting to note that the intuitionistic fuzzy transversal core exists only for non-spike intuitionistic fuzzy directed hypergraphs. There is abundant scope for future research on this topic. Further, the authors proposed to work on its applications in coloring of intuitionistic fuzzy directed hypergraphs.

Acknowledgement

The first author (K. K. Myithili) would like to thank University Grants Commission, Hyderabad, India for its financial support to the Minor Research Project F. NO: 4 - 4/2013-14 (MRP - SEM/UGC - SERO) dated 17.03.2014.

References

[1] K. T. Atanassov, Intuitionistic Fuzzy Sets - Theory and Applications, Physica-Verlag, New York, Berlin, 1999.

[2] K. T. Atanassov, On index matrix representation of the intuitionistic fuzzy graphs, Notes on Intuitionistic Fuzzy Sets 4 (2002), 73-78.

[3] K. T. Atanassov, On Intuitionistic Fuzzy Sets Theory, Springer, Berlin, 2012.

[4] C. Berge, Graphs and Hypergraphs, North-Holland, New York, 1976.

[5] John N. Mordeson and Premchand S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-Verlag, New York, 2000.

[6] R. Parvathi and M. G. Karunambigai, Intuitionistic fuzzy graphs, Proceedings of 9th Fuzzy Days International Conference on Computational Intelligence, Advances in Soft Computing: Computational Intelligence, Theory and Applications, Springer-Verlag, 20, 2006, pp. 139-150.

[7] R. Parvathi, S. Thilagavathi and M. G. Karunambigai, Intuitionistic fuzzy hypergraph, Bulgarian Acad. Sci., Cybernet. Inform. Tech. 9(2) (2009), 46-53.

[8] R. Parvathi and S. Thilagavathi, Intuitionistic fuzzy directed hypergraphs, Adv. Fuzzy Sets and Systems 14(1) (2013), 39-52.

[9] Roy H. Goetschel, Jr., Introduction to fuzzy hypergraphs and Hebbian structures, Fuzzy Sets and Systems 76 (1995), 113-130.

[10] Roy H. Goetschel, Jr., William L. Craine and William Voxman, Fuzzy transversals of fuzzy hypergraphs, Fuzzy Sets and Systems 84 (1996), 235-254.

[11] Roy H. Goetschel, Jr., Fuzzy colorings of fuzzy hypergraphs, Fuzzy Sets and Systems 94 (1998), 185-204.

[12] K. K. Myithili, R. Parvathi and M. Akram, Certain types of intuitionistic fuzzy directed hypergraph, Inter. J. Cyber. Mach. Learning (2014), 1-9.
DOI: 10.1007/s13042-014-0253-1.
http://link.springer.com/article/10.1007/s13042-014-0253-1.