Far East Journal of Applied Mathematics

Published Online: January 2016
http://dx.doi.org/10.17654/AM094010017
Volume 94, Number 1, 2016, Pages 17-36 **ISSN: 0972-0960**

MATHEMATICAL MODELING AND NUMERICAL SIMULATION IN TRANSITORY MODE OF GROUNDWATER LEVEL IN THE PLAIN OF SOUROU (BURKINA FASO)

Wenddabo Olivier Sawadogo and Blaise Somé

Laboratory LANIBIO
University of Ouagadougou
03 B.P. 7021 Ouaga
Burkina Faso
e-mail: wenddabo81@gmail.com
some@univ-ouaga.bf

Abstract

The objective of this article is the mathematical modeling and numerical simulation in transitory mode of the groundwater level of plain of Sourou (Burkina Faso). The modeled domain is fractured. We supposed the fractures as porous media and the groundwater flow equation was resolved using the finite element method in FreeFem ++.

The evolution of the piezometric head was simulated between 1960 and 2030. The results obtained in the simulations helped to understand the variation of the groundwater level and to highlight the role of fractures in the piezometric depression.

Received: October 17, 2015; Revised: December 3, 2015; Accepted: December 5, 2015

2010 Mathematics Subject Classification: 62P12, 35A15, 65M06, 65M60.

Keywords and phrases: mathematical modeling, transitory mode, fractured porous medium, finite element method.

Communicated by K. K. Azad

1. Introduction

The water table of the plain of Sourou has already been the subject of several studies [1, 3]. The farming in this plain is becoming more intensive. But there is no even a model to monitor the evolution of the level of the water table. The objective of this work is the mathematical modeling and numerical simulation in transitory mode of groundwater level. The mathematical model was resolved by the finite element method of Galerkin. Programming was done under FreeFem ++. The geological section of the plain shows a fractured porous media [3]. The usual models programmed in software cannot simulate everything. Indeed the area must be uniform and continuous. Fractured aquifers therefore can not be modeled simply. The flow in the fractures does not meet the laws governing conventional models. Fractures influence hydraulic conductivity. In most cases, the fractures have a hydraulic conductivity greater than that of the rock. In this case, they are privileged traffic gutters. In our case, fractures have hydraulic conductivities below that of the rock. There are many methods and models to treat fractures [7, 8]. To handle the presence of fractures, we assume that the fractures are sub-domains of the overall area as in [9]. We therefore assimilate them to porous media. We used a local mesh refinement in the fractures. Each domain has been meshed separately and we imposed common nodes on the border of two neighboring sub-domain.

Our report is structured as follows: in the first part, we will present the problem to be solved. The second part is devoted to modeling and mathematical analysis of the problem. A third part is devoted to the presentation of the results of the numerical simulation.

2. Description of the Study Site and Problem to Solve

2.1. Description of the study site

Because of the limitations of data on the plain, the modeling domain is a geological section located on the eastern margin of the basin of Gondo (Figure 1). In this area many investigations were carried out [3]. This vertical reference section selected stretches from the village of Nomou located in the east in the crystalline basement to the village of Yensé located in the western sedimentary basin, it is about 25 km long. It has a height of 300 meters. In the reference section, five subvertical fractures of 500 meters thick affect both the base and the sedimentary formations.

2.2. Modeling objectives

Modeling objectives are to provide answers to the following questions:

• What has been the evolution of the groundwater level since 1960?

• What is the influence of fractures in the piezometric depression of the water table?

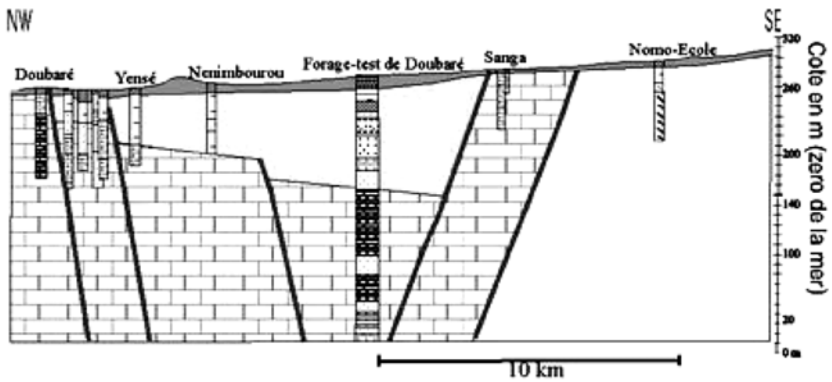

Figure 1. Domain of modeling [3].

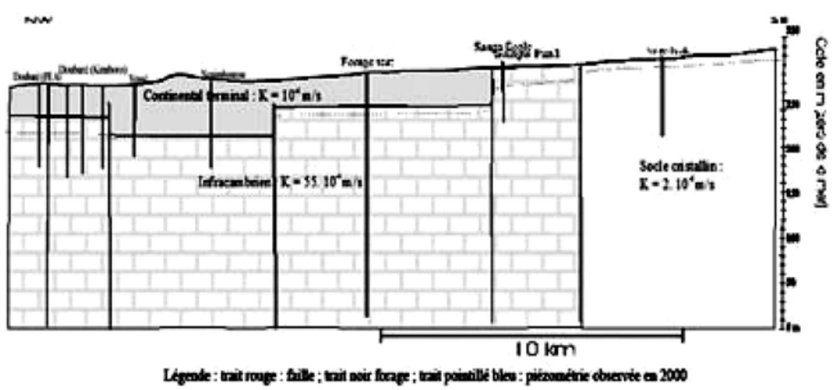

Figure 2. Simplified interpretative geological section [3].

3. Mathematical Modeling of Flow

3.1. Flow conditions

The piezometry observed upstream (upstream base region) and downstream (downstream the sedimentary area of Doubaré) the flow is prescribed as imposed potential limits (Dirichlet condition). The lower limit of the domain is a zero flux limit (homogeneous Neumann condition). We must also take into account the induced recharge by rainfall on the upper part.

The fluid is incompressible monophasic and the media is saturated porous. The piezometry initially chosen was that of 1960 which corresponds to the piezometry obtained by steady calibration.

3.2. Mathematical model

Let us subdivide the domain into several sub-domains: Ω_i $(i = 1, 2, \ldots, 13)$ with $\Omega_i \cap \Omega_j = \varnothing$ if $i \neq j$, and $\overline{\Omega} = \bigcup_{i=1}^{13} \overline{\Omega}_i$ (Figure 3). The fractures being considered as porous media.

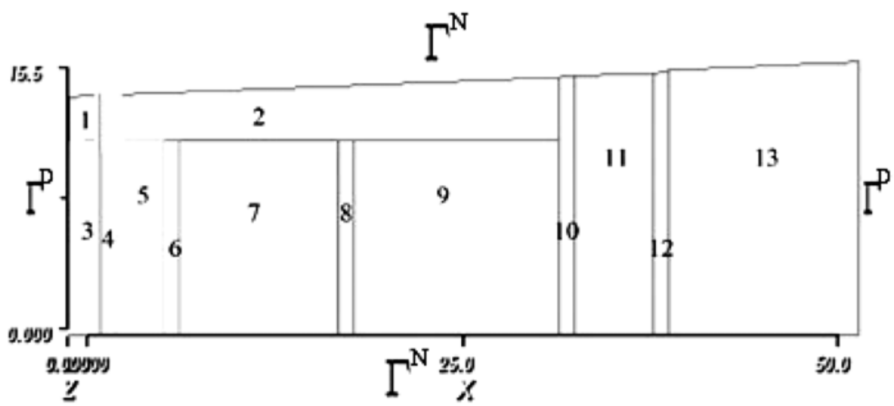

Figure 3. Subdivided domain.

Under the above mentioned flow conditions and applying the mass conservation law and the Darcy's law [11, 12], the flow in transitory mode in the domain is governed by the following system of equations:

$$\begin{cases} S\dfrac{\partial h}{\partial t} - div(K\nabla h) = f & \text{in }]0, T[\times\Omega, \\ h = h_d(t) & \text{on }]0, T[\times\Gamma^D, \\ \dfrac{\partial h}{\partial n} = 0 & \text{on }]0, T[\times\Gamma^N, \\ h(0, x) = h_0(x), & \text{in } \Omega, \end{cases} \tag{1}$$

where

- Ω is a bounded open set in $\mathbb{R}^2$, representing the domain.
- $\Gamma^D \cap \Gamma^N = \varnothing$ and $\Gamma^D \cup \Gamma^N = \partial\Omega$.
- h is the hydraulic potential.
- $f \in L^2(]0, T[; L^2(\Omega))$, is the source term and takes into account the recharge induced by the rainfall.
- $h_0 \in L^2(\Omega)$, initial condition.
- $h_d \in L^2(]0, T[; \Gamma^D)$ is the Dirichlet boundary condition, representing the imposed potential.
- $K(x)$ and $S(x)$ represent, respectively, the hydraulic coefficient and the coefficient of storage. They are piecewise constant functions.
- $L^2(]0, T[; X)$, space of the functions with values in X and whose norm in X is in $L^2(]0, T[)$. We have

$$\| u \|_{L^2(]0,T[;X)} = \left(\int_0^T \| u(t) \|_X^2\right)^{1/2}.$$

Provided with this norm, $L^2(]0, T[; X)$ is a Banach's space [5].

3.3. Mathematical analysis of the model

Let us suppose:

- $$V := \{v \in H^1(\Omega),\ v = 0 \text{ on } \Gamma^D\},$$

V is a closed subspace of $H^1(\Omega)$, it is therefore a Hilbert space for the standard norm of $H^1(\Omega)$ [4].

We recall that

$$\| v \|_V = \left(\int_\Omega | \nabla v |^2 \right)^{1/2}$$

is a norm on V, which is equivalent to the standard norm of $H^1(\Omega)$ [4].

- $\mathcal{C}^j(]0, T[; V)$, space of the functions of class $\mathcal{C}^j$ in t with value in V. It is a Banach's space for the norm

$$\| u \|_{\mathcal{C}^j(]0,T[;V)} = \sup_{t \in [0,T]} \sum_{l=0}^{j} \| d_t^l u(t) \|_V$$

with $d_t^l u$ the derivative of order l respect to t [5].

We consider h defined by

$$h :]0, T[\to H^1(\Omega),$$
$$t \mapsto h(t) = h(t, \cdot).$$

Let us

$$\gamma_d : H^1(\Omega) \to H^{1/2}(\Gamma^D) \subset L^2(\Gamma^D)$$

the trace application on Γ^D.

Let $r_d \in L^2(]0, T[; H^2(\Omega))$ with $\frac{dr_d}{dt} \in L^2(]0, T[; L^2(\Omega))$ and such that

$$\gamma_d(r_d(t)) = h_d(t), \text{ almost everywhere } t \text{ in }]0, T[.$$

Let us pose $\tilde{h}(t) = h(t) - r_d(t)$.

The problem (1) is equivalent to

$$\begin{cases} S\dfrac{\partial \tilde{h}}{\partial t} - div(K\nabla\tilde{h}) = f + S\dfrac{\partial r_d}{\partial t} - div(K\nabla r_d), & \text{in }]0, T[\times\Omega, \\ \tilde{h} = 0, & \text{on }]0, T[\times\Gamma^D, \\ \dfrac{\partial\tilde{h}}{\partial n} = 0, & \text{on }]0, T[\times\Gamma^N, \\ \tilde{h}(0, x) = h_0(x) - r_d(0), & \text{in } \Omega, \end{cases} \tag{2}$$

3.3.1. Existence and unicity of a solution of the problem (2)

Let $v \in V$ be a test function. The variational formulation associated to the problem (2) is:

$$\begin{cases} \text{Find } \tilde{h} : t \in]0, T[\to V \text{ such that} \\ \dfrac{d}{dt}\displaystyle\int_\Omega S\tilde{h}(t)vdx + \int_\Omega K\nabla\tilde{h}(t)\cdot\nabla vdx = \int_\Omega f(t)vdx \\ \qquad + \dfrac{d}{dt}\displaystyle\int_\Omega Sr_d(t)vdx - \int_\Omega div(K\nabla r_d(t))vdx \\ \forall t \in]0, T[, \forall v \in V. \end{cases} \tag{3}$$

It could be re-written

$$\begin{cases} \text{Find } \tilde{h} : t \in]0, T[\to V \text{ such that} \\ \dfrac{d}{dt}(\tilde{h}(t), v)_{L^2(\Omega)} + a(\tilde{h}(t), v) \\ \qquad = (F(t), v)_{L^2(\Omega)}, \forall t \in]0, T[, \forall v \in V \\ \tilde{h}(t = 0) = \tilde{h}_0 \end{cases} \tag{4}$$

with:

- $a(\cdot, \cdot)$ is the bilinear form defined on V by

$$a(u, v) := \int_\Omega \frac{K}{S}\nabla u\cdot\nabla vdx,$$

- $\tilde{h}_0 = h_0 - r_d(0)$,

• F is defined by

$$F(t) = \frac{1}{S}(f(t) - div(K\nabla r_d(t))) + \frac{\partial r_d(t)}{\partial t}.$$

Theorem 3.1. *The problem* (4) *admits a unique solution*

$$\tilde{h} \in L^2(]0, T[; V) \cap \mathcal{C}(]0, T[; L^2(\Omega)).$$

Moreover, there exists a constant $c > 0$ *such that*

$$\| \tilde{h} \|_{L^2(]0,T[;V)} + \| \tilde{h} \|_{\mathcal{C}(]0,T[;L^2(\Omega))}$$

$$\leq c(\| \tilde{h}_0 \|_{L^2(\Omega)} + \| F \|_{L^2(]0,T[;L^2(\Omega))}).$$

Proof. For the proof, it is enough to apply the following theorem:

Theorem 3.2. *Let* $(W, (\cdot, \cdot)_W)$ *and* $(H, (\cdot, \cdot)_H)$ *be two real Hilbert spaces such that*

• $W \subset H$ *with canonical injection.*

• *W is dense in H.*

Then the problem

$$\begin{cases} Find\ u : t \in]0, T[\to W\ such\ that, \\ \frac{d}{dt}(u, v)_H + a(u(t), v) = (f, v)_W, \forall t \in]0, T[, \forall v \in W, \\ u(t = 0) = u_0, \end{cases}$$

where $a(\cdot, \cdot)$ *is a continuous, coercive and symmetric bilinear form on W.* $u_0 \in H$ *and* $f \in L^2(]0, T[; H)$, *admits a unique solution* $u \in L^2(]0, T[; W) \cap \mathcal{C}(]0, T[; H)$. *In addition, there exists a constant* $c > 0$ *such that*

$$\| u \|_{L^2(]0,T[;W)} + \| u \|_{\mathcal{C}(]0,T[;H)} \leq c(\| u_0 \|_H + \| f \|_{L^2(]0,T[;H)}).$$

The proof of this theorem is provided in [6].

Let us apply this theorem to the problem (4) with $W = V$ and $H = L^2(\Omega)$.

• Ω being a bounded open and $V \subset H^1(\Omega)$, the theorem of Rellich [6] allows to conclude that the injection of $L^2(\Omega)$ in V is compact.

• The bilinear form a defined by

$$a(u, v) := \int_\Omega \frac{K}{S} \nabla u \cdot \nabla v dx$$

is continuous, coercive and symmetric on V. In fact, K and S are piecewise constant functions, there are constants $K_{\min}$, $K_{\max} S_{\min}$ and $S_{\max}$ strictly positives such that $K_{\min} \leq K(x) \leq K_{\max}$ and $S_{\min} \leq S(x) \leq S_{\max}, \forall x \in \Omega$ and thus $\alpha = \frac{K_{\min}}{S_{\max}} \leq \frac{K(x)}{S_x} \leq \frac{K_{\max}}{S_{\min}} = \beta$.

According to the inequality of Cauchy-Schwartz

$$a(u, v) \leq \int_\Omega \frac{K}{S} \nabla u \cdot \nabla v dx \leq \beta \| u \|_V \| v \|_V .$$

From which the continuity of a on V.

Moreover for all $u \in V$ and according to the inequality of Poincaré, there is a constant strictly positive C such that

$$a(u, u) \geq \alpha \int_\Omega | \nabla u |^2 dx \geq \alpha C \| u \|_V^2 .$$

The bilinear form a is then coercive. And it is evident that it is symmetric.

• The function F is in $L^2(]0, T[; L^2(\Omega))$ because it is the sum of functions belonging to $L^2(]0, T[; L^2(\Omega))$.

This is a proof of the existence and the unicity of the solution of (4) according to the Theorem 3.2.

Let us show now that the unique solution of the problem (4) is solution of the problem (2).

We have $\tilde{h}(t) = 0$ on Γ^D because $\tilde{h}(t) \in V$.

Moreover as $\tilde{h}(t)$ is continuous, we have $\tilde{h}(t = 0) = \tilde{h}_0$.

If the solution $\tilde{h}$ is regular enough, by integrating by part the variational formulation (3), we have

$$\int_\Omega \left(\frac{\partial \tilde{h}}{\partial t} - \Delta \tilde{h} - F \right) v dx = -\int_{\Gamma^N} \frac{\partial \tilde{h}}{\partial n} v ds, \ \forall v \in V, \ \forall t \in \,]0, T[. \tag{5}$$

If we take $v \in \mathcal{C}_c^\infty(\Omega) \subset V(\mathcal{C}_c^\infty(\Omega)$ being indefinitely derivable functions with compact support in Ω, the integral on Γ^N is nullified and we will have

$$\frac{\partial \tilde{h}}{\partial t} - \Delta \tilde{h} - F = 0 \text{ in } Ł^2(]0, T[\times \Omega). \text{ Therefore almost everywhere in }]0, T[\times \Omega.$$

Hence

$$\int_{\Gamma^N} \frac{\partial \tilde{h}}{\partial n} v ds = 0, \ \forall v \in V \text{ and almost everywhere } t \in \,]0, T[.$$

This brings

$$\frac{\partial \tilde{h}}{\partial n} = 0 \text{ almost everywhere in }]0, T[\times \Gamma^N.$$

Thus the solution of the problem (4) is solution of the problem (2). This is a proof the existence and the unicity of the solution of (1). □

3.3.2. Equivalence with a transmission problem

Proposition 3.3. *If we note* h_i, f_i *respective restrictions of h and of f to* $]0, T[\times \Omega_i$, *then the problem* (1) *is equivalent to the following transmission problem*:

$$\begin{cases} S_i \dfrac{\partial h_i}{\partial t} - K_i \Delta h_i = f_i, & in\ ]0, T[\times \Omega_i, \\ h_i = h_d(t), & on\ ]0, T[\times \Gamma_i^D, \\ k_i \nabla h_i \cdot n = k_j \nabla h_j \cdot n, & on\ ]0, T[\times \Gamma_{i,j}, \\ h_i = h_j, & on\ ]0, T[\times \Gamma_{i,j}, \\ \dfrac{\partial h}{\partial n} = 0, & in\ ]0, T[\times \Gamma_i^N, \\ h(0, x) = h_0^i(x), & in\ \Omega_i, \end{cases} \tag{6}$$

where

$$\Gamma_{i,j} = \partial\Omega_i \cap \partial\Omega_j,\ \Gamma_i^D = \partial\Omega_i \cap \partial\Gamma^D \ \ and\ \ \Gamma_i^N = \partial\Omega_i \cap \partial\Gamma^N.$$

Proof. By definition of the functions h_i, we have

$$\begin{cases} h = h_d(t) & \text{on }]0, T[\times \Gamma^D \\ \dfrac{\partial h}{\partial n} = 0 & \text{in }]0, T[\times \Gamma^N \\ h(0, x) = h_0(x) & \text{in } \Omega_i \end{cases} \Leftrightarrow \begin{cases} h_i = h_d(t) & \text{on }]0, T[\times \Gamma_i^D \\ \dfrac{\partial h_i}{\partial n} = 0 & \text{in }]0, T[\times \Gamma_i^N \\ h(0, x) = h_0^i(x) & \text{in } \Omega_i. \end{cases}$$

Let us show that (6) $\Rightarrow$ (1).

Let us $v \in V$ a test function and $h_i(t) \in H^1(\Omega_i)$ almost everywhere $t \in]0, T[$.

By multiplying each equation of (6) in Ω_i by the same test function *v* and by making the sum, we have

$$\frac{d}{dt} \sum_{i=1}^{13} S_i \int_{\Omega_i} h_i(t) v dx + \sum_{i=1}^{13} K_i \int_{\Omega_i} \nabla h_i(t) \nabla v dx$$

$$+ \sum_{i=1}^{13} \sum_{j \in J^j} \int_{\Gamma_{i,j}} \left(K_i \frac{\partial h_i(t)}{\partial n_i} + K_j \frac{\partial h_j(t)}{\partial n_j} \right) v ds$$

$$= \sum_{i=1}^{13} \int_{\Omega_i} f_i(t) v ds, \tag{7}$$

$J^i = \{j \in \{1, ..., 13\}$ such that $\partial\Omega_i \cap \partial\Omega_j \neq \emptyset\}$ for $i \in \{1, ..., 13\}$.

The integrals on Γ_i^D and on Γ_i^N nullify since $v = 0$ on Γ_i^D and $\frac{\partial h_i(t)}{\partial n_i} = 0$ on Γ_i^N.

Moreover as on $\Gamma_{i,j}$, we have $n_i = -n_j$, then

$$\int_{\Gamma_{i,j}} \left(K_i \frac{\partial h_i(t)}{\partial n_i} + K_j \frac{\partial h_j(t)}{\partial n_j} \right) = 0$$

because of the transmission boundary conditions.

The formulation (7) is then equivalent to

$$\frac{d}{dt} \int_\Omega Sh(t)vdx + \int_\Omega K\nabla h(t)\nabla vdx = \int_\Omega f(t)vdx, \ \forall v \in V \tag{8}$$

which is nothing other than the variational formulation of the problem (1).

What is left is to demonstrate that $h(t) \in H^1(\Omega)$. It is enough to apply the lemma below [6].

Lemma 3.4. *Let Ω be a regular open bounded of class $\mathcal{C}^1$. Let us suppose $(\Omega_i)_{1 \le i \le I}$ be a regular partition of Ω, i.e., that each Ω_i is regular open of class $\mathcal{C}^1$, $\Omega_i \cap \Omega_j = \emptyset$ if $i \neq j$, and $\overline{\Omega} = \bigcup_{i=1}^{I} \overline{\Omega}_i$. Let us v be a function whose the restriction v_i at every Ω_i belongs to $H^1(\Omega_i)$. If v is continuous on $\overline{\Omega}$, then v belongs to $H^1(\Omega)$.*

In our case the Ω_i are the regular polygons, they represent a regular partition of Ω. On the other part $h(t)$ being defined as $h_i(t)$ on Ω_i, it is continuous on Ω because the conditions of transmission $h_i(t) = h_j(t)$.

According to the Lemma 3.4 $h(t) \in H^1(\Omega)$. And thus (6) $\Rightarrow$ (1).

Let us show now that (1) $\Rightarrow$ (6).

Let us suppose

$$W = \{w(t) \in H^1(\Omega) \text{ such that } w(t) = h_d(t) \text{ on } \Gamma^D, \forall t \in]0, T[\}.$$

Let us say $h(t) \in W$ be a solution of (1). Let us pose $\sigma_i(t) = K_i \nabla h_i(t)$.

$h_i(t)$ being the restriction of $h(t)$ on Ω_i, the theorem of trace [6] ensures

$$h_i(t) = h_j(t) \text{ on } \Gamma_{i,j}$$

and the theorem of the divergence [6] applied to $\sigma_i(t)$ involve

$$K_i \nabla h_i(t) = K_j \nabla h_j(t) \text{ on } \Gamma_{i,j}.$$

On the other part by multiplying the problem (1) by $v \in V$ a test function and by integrating by part, we have

$$\frac{d}{dt} \int_\Omega Sh(t) v dx + \int_\Omega K \nabla h(t) \nabla v dx = \int_\Omega f(t) v dx, \ \forall v \in V$$

which is equivalent to

$$\frac{d}{dt} \sum_{i=1}^{13} S_i \int_{\Omega_i} h_i(t) v dx + \sum_{i=1}^{13} K_i \int_{\Omega_i} \nabla h_i(t) \nabla v dx = \sum_{i=1}^{13} \int_{\Omega_i} f_i(t) v dx \tag{9}$$

which is the variational formulation of (6).

Therefore (1) $\Rightarrow$ (6). □

This is the model (6) which we programmed by using the finite element method.

4. Numerical Simulation

4.1. Parameters of simulation

For the simulations, we used explicit finite difference method for the discretization of the time variable and a discretization in finite element P_1 for the space variable. The spatial mesh is triangular. We conducted a refinement of mesh at the fractures (Figure 4).

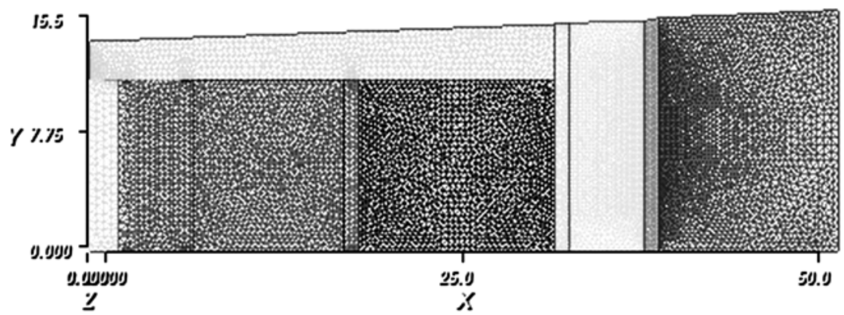

Figure 4. Domain meshing.

The source term comes from the recharge induced by precipitations, we took the average of 1960 to 2000 which is 100 mm per year.

The hydraulic coefficients are taken in [3]:

$$K_1 = 10^{-6} \mathrm{m/s},\ K_2 = 55.10^{-4} \mathrm{m/s},\ K_3 = 10^{-6} \mathrm{m/s},\ K_4 = 10^{-8} \mathrm{m/s},$$

$$K_5 = 10^{-8} \mathrm{m/s},\ K_6 = 10^{-8} \mathrm{m/s},\ K_7 = 5.10^{-8} \mathrm{m/s},\ K_8 = 2.10^{-6} \mathrm{m/s}.$$

The storage coefficients are calculated using the following formula [12]:

$$S = n \times e \times \rho \times g\left(\beta_l - \beta_s + \frac{\alpha}{n}\right),$$

where:

- n total porosity;
- e thickness;
- ρ density of water;
- α coefficient of compressibility of the porous media;
- β_l compressibility of water;
- β_s compressibility of the solid rock.

To have the initial condition we initially solved the problem in permanent mode to have the head of the piezometric of 1960, i.e., the problem

$$\begin{cases} K_i \Delta h_i = f, & \text{in } \Omega_i, \\ h_i = h_d(t), & \text{on } \Gamma_i^D, \\ k_i \nabla h_i \cdot n = k_j \nabla h_j \cdot n, & \text{on } \Gamma_{i,j}, \\ h_i = h_j, & \text{on } \Gamma_{i,j}, \\ \dfrac{\partial h}{\partial n} = 0, & \text{on } \Gamma_i^N. \end{cases} \tag{10}$$

4.2. Results and discussions

The evolution of the piezometric levels was simulated between 1960 and 2030. The Figures 5 to 7 show a comparison between the level simulated and observed in 1960, 2000 and 2005. In analyzing these figures, we see that the simulated and observed levels very close. The Figures 8 to 10 are the representations in three dimensions of the level of 1960, 2000 and 2005.

The Figure 11 is a representation of the water table level in various dates in 1960, 1970, 1980, 1990, 2000, 2005, 2020 and 2030.

Between 1960 and 1970 the level of the water table rose nearly by 8 m. From 1970 to 1980, a slight increase of Doubaré to Sanga was noticed. But from Sanga the level greatly increased compared to that of 1970. We noted that this considerable jump of the level occurred at the fracture. This confirms the results of sensitivity analysis carried out in [1] which showed that depression was partially caused by fractures.

But from 1980-2030, the level increased on average by 6m every 10 years. Under the same charging conditions, predictions in 2020 and confirmed in 2030 this tendency.

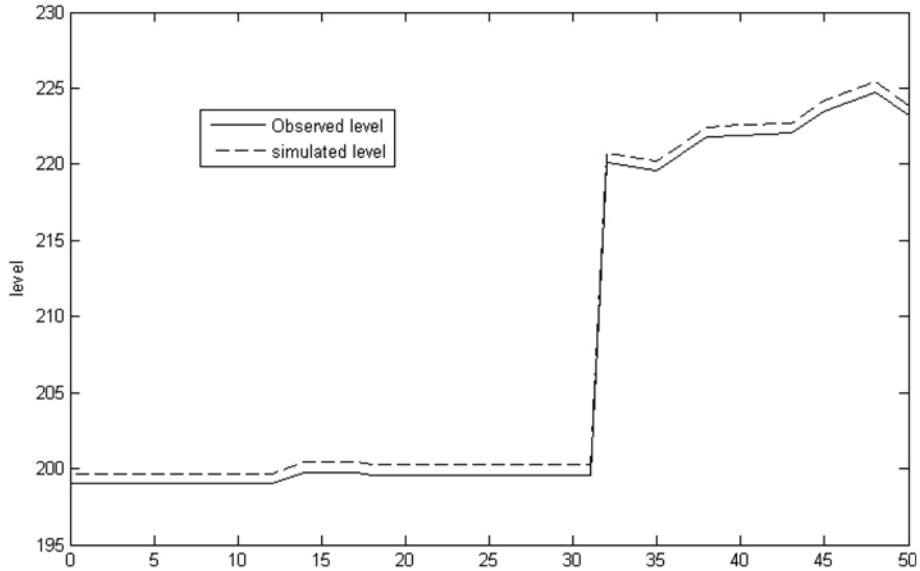

Figure 5. Comparison: level of 1960.

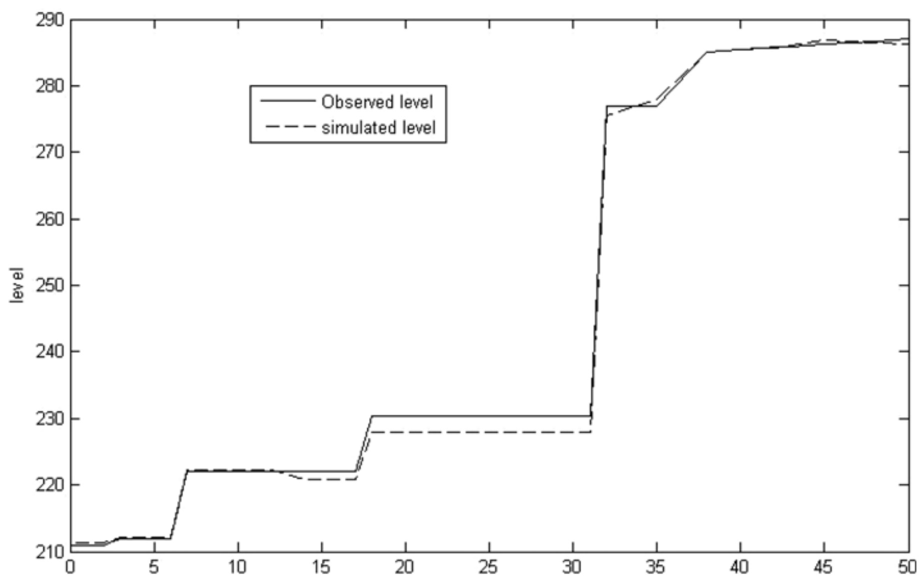

Figure 6. Comparison: level of 2000.

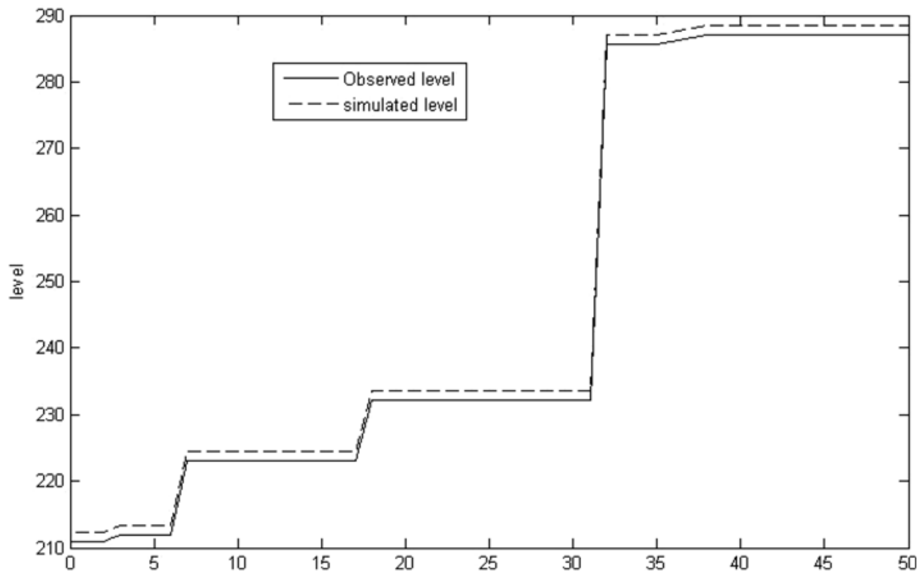

Figure 7. Comparison: level of 2005.

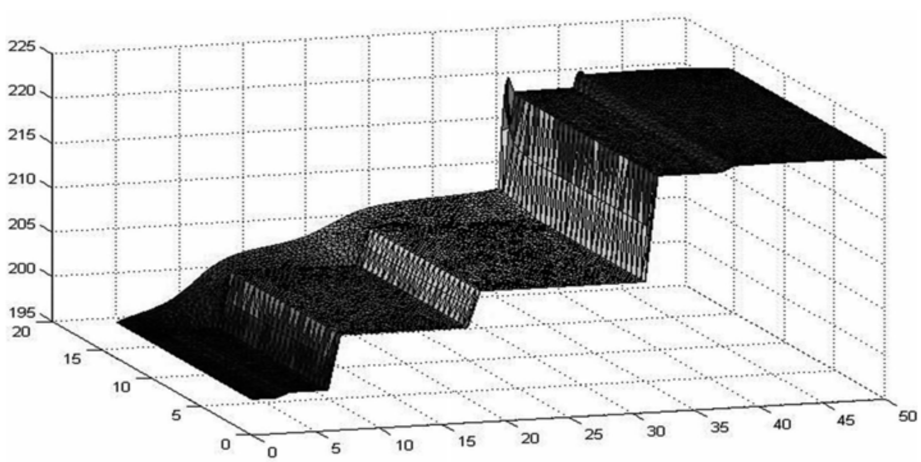

Figure 8. Representation in 3D of level of 1960.

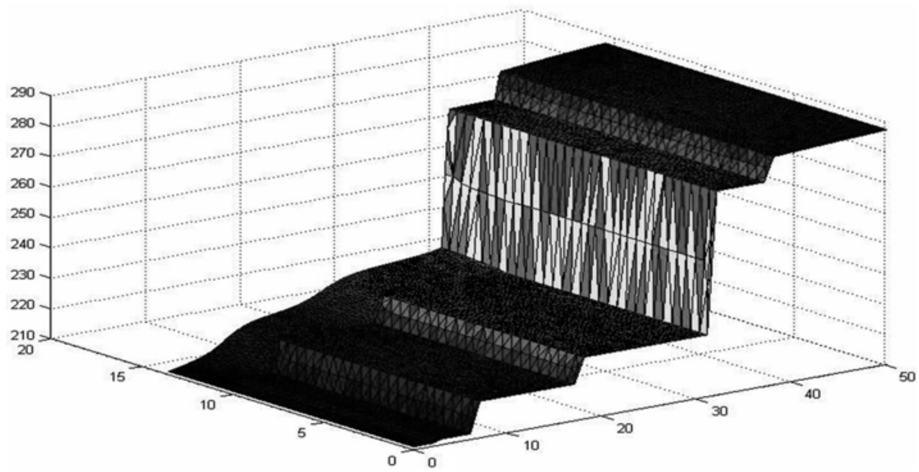

Figure 9. Representation in 3D of level of 2000.

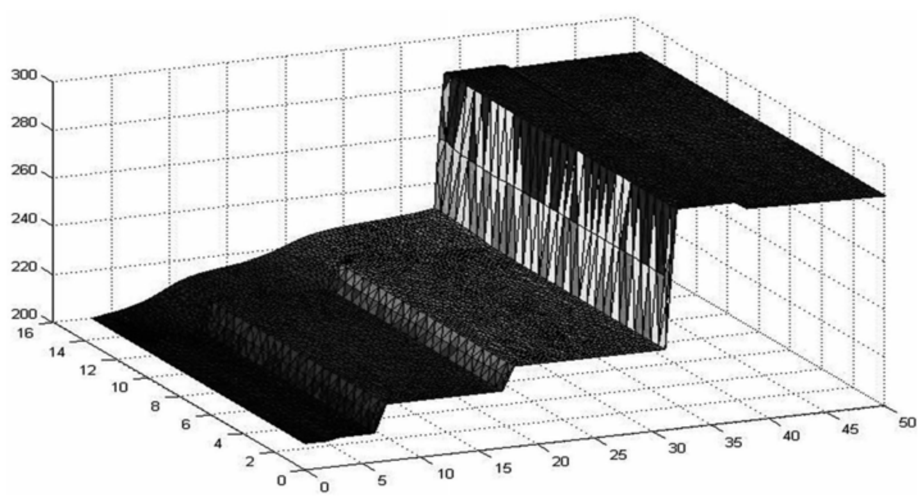

Figure 10. Representation in 3D of level of 2005.

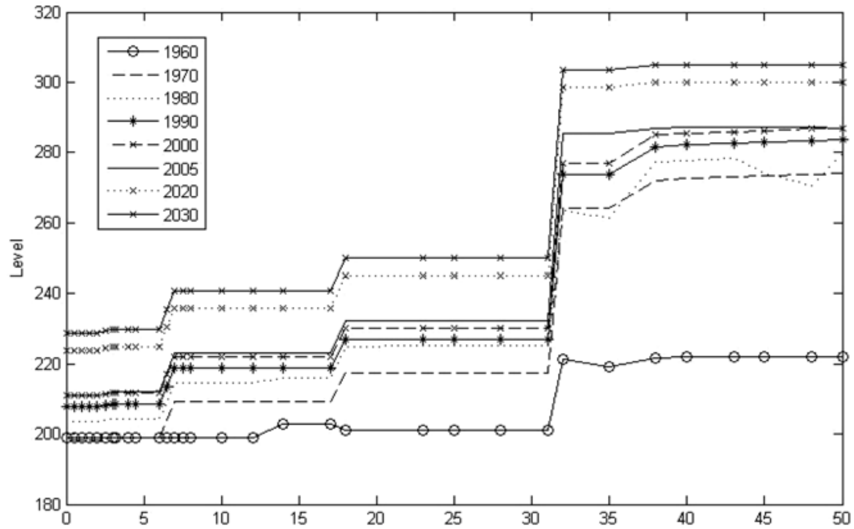

Figure 11. Level evolution from 1960 to 2030.

5. Conclusion

In this work, we proposed a mathematical modeling and numerical simulation in the transitory mode of the groundwater level in the valley of Sourou (Burkina Faso). The studied domain is fractured. After analyzing the mathematical model, a simulation of the groundwater level was conducted between 1960 and 2030. The results of this simulation show that between 1960 and 2000, the water table has risen by almost 40 m. In some places this increase exceeds 40 m. The fractures seem to be the cause of this sudden increase in the groundwater level. Further field investigations and a three-dimensional modeling will help confirm or deny this trend.

Acknowledgement

The authors thank the anonymous referees for their valuable suggestions which led to the improvement of the manuscript.

References

[1] W. O. Sawadogo, N. Alaa and B. Somé, Numerical simulation of groundwater level in a fractured porous medium and sensitivity analysis of the hydrodynamic

parameters using grid computing: application of the plain of Gondo (Burkina Faso), Int. J. Comput. Sci. 9(1) (2) (2012), 227-236.

[2] F. Hetch and O. Pironneau FreeFem ++. http://www.freefem.org.

[3] Y. Koussoubé, Hydrogéologie des séries sédimentaires de la depression piézométrique de Gondon (Bassin du Sourou) - Burkina Faso, Thése de Doctorat de l'université Pierre et Marie Curie, Spécialité: Hydrogéologie, juillet 2010.

[4] H. Brezis, Analyse fonctionnelle: théorie et applications, Masson, Paris, 1983.

[5] A. Ern and J.-L. Guermond, Eléments finis: théorie, applications, mise en oeuvre, Vol. 36, SMAI Mathématiques et Applications, Springer, 2002.

[6] G. Allaire, Analyse numrique et optimisation, Ellipses, 2006.

[7] Clarisse Alboin, Jérôme Jaffré and Jean E. Roberts, Domain decomposition for some transmission problems in flow in porous media, Numerical treatment of multiphase flows in porous media (Beijing, 1999), Volume 552 of Lecture Notes in Phys., Springer, Berlin, 2000, 2234 p.

[8] T. Arbogast, The Double Porosity Model for Single Phase Flow in Naturally Fractured Reservoirs, M. F. Wheeler, ed., Numerical Simulation in Oil Recovery, Vol. 11 of the IMA Volumes in Mathematics and its Applications, Springer Verlag, 1988, pp. 23-45.

[9] H. Mustapha, Simulation numrique de l'coulement dans des milieux fracturs tridimensionnels, Thèse de l'Université de Rennes 1, 2005.

[10] C. Bernardi, Y. Maday and A. T. Patera, Domain decomposition by the mortar element method, Asymptotic and Numerical Methods for Partial Differential Equations with Critical Parameters (Beaune, 1992), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., Vol. 384, Kluwer Acad. Publ., Dordrecht, 1993, pp. 269-286.

[11] E. Ledoux, Modèles mathématiques en hydrogéologie, Centre d'Informatique Géologique École Nationale Supérieure des Mines de Paris, 2003.

[12] G. de Marsily, Quantitative hydrogeology, Groundwater Hydrology for Engineers, Academic Press, New York, 1986.

[13] Clarisse Alboin, Deux outils mathématiques pour modéliser l'écoulement et le transport de polluants dans un milieux poreux saturé, Thèse de l'Université Paris IX Dauphine, 2000.

[14] Philippe G. Ciarlet, The Finite Element Method for Elliptic Problems, North-Holland, 1980.

[15] V. Martin, Simulations multidomaines des écoulements en milieu poreux, Thèse de l'Université Paris IX Dauphine, 2004.