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RINGS AND IDEALS IN A VAGUE SOFT SET SETTING

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Abstract

In this paper, we present a more accurate definition of the concepts pertaining to the theory of vague soft rings. We redefine the notion of vague soft rings and other concepts related to this concept such as vague soft ideals, idealistic vague soft rings and vague soft ring homomorphism in a manner which better reflects the characteristics of the concept of vague soft sets that were used to construct these concepts. The fundamental properties and structural characteristics of

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these concepts are then studied and discussed. The relationship between the concepts introduced here and the corresponding concepts in soft ring theory and classical ring theory are investigated. Lastly, we prove that the vague soft ring homomorphism introduced here preserves vague soft rings.

1. Introduction

Imprecision, uncertainties and vagueness are characteristics that are pervasive in a lot of complicated problems in many areas such as engineering, economics, environmental science, social science and medicine. These problems are due to a myriad of factors such as incomplete information, randomness and limitations of the measuring instrumentations that are used in data collection. Traditionally, mathematical tools such as fuzzy set theory, rough set theory, probability theory and interval mathematics are used to deal with and model uncertainties and vagueness. However, all these theories have their inherent difficulties as pointed put by Molodtsov [1]. In [1], Molodtsov introduced the concept of soft sets as a general mathematical tool to deal with uncertainties and vagueness.

Since the introduction of the notion of fuzzy sets by Zadeh in 1965 [2], many generalizations of this fundamental concept has been introduced and studied by various authors and researchers around the world. Examples of these generalizations include the concept of vague sets, intuitionistic fuzzy sets, fuzzy soft sets, vague soft sets and intuitionistic fuzzy soft sets. In this paper, the concept of vague soft sets introduced by Xu et al. [3] is used to develop the theory of vague soft rings in Rosenfeld's sense i.e. using Rosenfeld's concept of a fuzzy subgroup of a group [4] in the context of soft rings.

The study of the theory of soft rings was initiated by Acar et al. [5]. The notion of soft rings introduced in [5] which is a parametrized family of subrings, is a generalization of the classical ring theory and extends the notion of rings to include the algebraic structures of soft sets. Ghosh et al. [6] on the other hand, introduced the notion of fuzzy soft rings and fuzzy soft ideals and studied some of its algebraic properties. Inan and Ozturk [7] concurrently studied the notion of fuzzy soft rings and fuzzy soft ideals but they dealt with these concepts in a more detailed manner compared to [6]. In [8], Varol et al. introduced the theory of vague soft algebra by applying the concept of Rosenfeld's fuzzy subgroup of a group to vague soft set theory. They established the notion of vague soft groups, vague soft homomorphism and normal vague soft groups. Selvachandran and Salleh [9] introduced normalistic vague soft groups and its homomorphism as an extension to [8]. Selvachandran and Salleh also initiated the study of the hyperalgebraic theory of the algebraic structures associated with vague soft sets, fuzzy soft sets and soft sets [10-15] in Rosenfeld's sense. The theory of vague soft rings and its related concepts were introduced in [16] and [17]. However, these definitions were too general and did not reflect the characteristics of vague soft sets that were used to derive these concepts in an accurate manner. Furthermore, the properties and structural characteristics of vague soft rings and its related concepts were also not sufficiently studied by the authors in [16] and [17]. Furthermore, the notion of fuzzy soft rings and fuzzy soft ideals based on fuzzy soft spaces were studied in [18] and [19], respectively.

In view of this, we aim to redefine the notion of vague soft rings, vague soft ideals, vague soft ring homomorphism and idealistic vague soft rings in a more accurate manner so as to properly reflect the notion of vague soft sets. Subsequently, the fundamental properties and structural characteristics of these concepts and the structures that are preserved under this homomorphism are studied and discussed. This is done in an effort to further develop the theory of vague soft algebra in an attempt to add to the existing body of knowledge in this area of fuzzy soft algebra and its generalizations.

2. Preliminaries

In this section, some of the important definitions that were used to develop the concepts and theorems introduced in this paper are presented.

Definition 2.1 [1]**.** A pair (F, A) is called a *soft set* over *U*, where *F* is a mapping given by $F : A \to P(U)$. In other words, a soft set over *U* is a parameterized family of subsets of the universe *U*. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F, A) or as the ε-approximate elements of the soft set.

Definition 2.2 [20]**.** For a soft set (F, A), the set $\text{Supp}(F, A) = \{x \in A : F(x) \neq \varnothing\}$ is called the *support* of the soft set (F, A). Thus a *null soft* set is indeed a soft set with an empty support and a soft set (F, A) is said to be *non-null* if $\text{Supp}(F, A) \neq \varnothing$.

Definition 2.3 [21]**.** Let *X* be a space of points (objects) with a generic element of *X* denoted by *x*. A *vague set V* in *X* is characterized by a truth-membership function $t_V : X \to [0, 1]$ and a false-membership function $f_V : X \to [0, 1]$. The value $t_V(x)$ is a lower bound on the grade of membership of *x* derived from the evidence for *x* and $f_V(x)$ is a lower bound on the negation of *x* derived from the evidence against *x*. The values $t_V(x)$ and $f_V(x)$ both associate a real number in the interval $[0, 1]$ with each point in *X*, where $t_V(x) + f_V(x) \leq 1$. This approach bounds the grade of membership of *x* to a subinterval $[t_V(x), 1 - f_V(x)]$ of $[0, 1]$. Hence a vague set is a generalization of the concept of fuzzy set, albeit a more accurate form of fuzzy set.

Definition 2.4 [3]**.** A pair $(\hat{F}, A)$ is called a *vague soft set* over *U*, where $\hat{F}$ is a mapping given by $\hat{F} : A \to V(U)$ and $V(U)$ is the power set of vague sets over *U*. In other words, a vague soft set over *U* is a parametrized family of vague sets of the universe *U*. Every set $\hat{F}(e)$ for all $e \in A$, from this family may be considered as the set of *e*-approximate elements of the vague soft set $(\hat{F}, A)$. Hence the vague soft set $(\hat{F}, A)$ can be viewed as consisting of a collection of approximations of the following form:

$$(\hat{F}, A) = \{\hat{F}(x_i) : i = 1, 2, 3, \ldots\} = \left\{ \frac{[t_{\hat{F}(e_i)}(x_i), 1 - f_{\hat{F}(e_i)}(x_i)}{x_i} : i = 1, 2, 3, \ldots \right\}$$

for all $e \in A$ and $x \in U$.

Definition 2.5 [10]**.** Let $(\hat{F}, A)$ be a vague soft set over *X*. The *support* of $(\hat{F}, A)$ denoted by $\text{Supp}(\hat{F}, A)$ is defined as:

$$\text{Supp}(\hat{F}, A) = \{a \in A : \hat{F}_a(x) \neq \hat{\varnothing}, \text{ i.e., } t_{\hat{F}_a}(x) \neq 0 \text{ and } 1 - f_{\hat{F}_a}(x) \neq 0\}$$

for all $x \in X$. It is to be noted that a *null vague soft set* is a vague soft set where both the truth and false membership functions are equal to zero. Therefore, a vague soft set $(\hat{F}, A)$ is said to be *non-null* if $\text{Supp}(\hat{F}, A) \neq \varnothing$.

Definition 2.6 [8]**.** Let $(\hat{F}, A)$ be a vague soft set over *U*. Then for every $\alpha, \beta \in [0, 1]$, where $\alpha \leq \beta$, the (α, β)*-cut* or the *vague soft cut* of $(\hat{F}, A)$, is a subset of *U* which is defined as follows:

$$(\hat{F}, A)_{(\alpha, \beta)} = \{x \in U : t_{\hat{F}_a}(x) \geq \alpha \text{ and } 1 - f_{\hat{F}_a}(x) \geq \beta, \text{ i.e., } \hat{F}_a(x) \geq [\alpha, \beta]\}$$

for every $a \in A$.

Definition 2.7 [10]**.** Let $(\hat{F}, A)$ be a vague soft set over *U*. Then for every $\alpha \in [0, 1]$, the α*-cut* of $(\hat{F}, A)$, denoted as $(\hat{F}, A)_{(\alpha, \alpha)}$ is a subset of *U* which is defined as follows:

$$(\hat{F}, A)_{(\alpha, \alpha)} = \{x \in U : t_{\hat{F}_a}(x) \geq \alpha \text{ and } 1 - f_{\hat{F}_a}(x) \geq \alpha, \text{ i.e., } \hat{F}_a(x) \geq [\alpha, \alpha]\}$$

for every $a \in A$.

Definition 2.8 [10]**.** Let $(\hat{F}, A)$ be a vague soft set over *X* and *G* be a non-null subset of *X*. Then $(\hat{F}, A)_G$ is called a *vague soft characteristic set* of *G* in $[0, 1]$ and the lower bound and the upper bound of $(\hat{F}_a)_G$ are defined as follows:

$$t_{(\hat{F}_a)_G}(x) = 1 - f_{(\hat{F}_a)_G}(x) = \begin{cases} s & \text{if } x \in G, \\ w & \text{otherwise,} \end{cases}$$

where $(\hat{F}_a)_G$ is a subset of $(\hat{F}, A)_G$, $x \in X$, $s, w \in [0, 1]$ and $s > w$.

Definition 2.9 [5]**.** Let (F, A) be non-null soft set over a ring R. Then (F, A) is called a *soft ring* over R if $F(x)$ is a subring of R for all $x \in A$.

Definition 2.10 [5]**.** Let (F, A) be a soft ring over a ring R. A non-null soft set (γ, I) over R is called a *soft ideal of* (F, A), which will be denoted by $(\gamma, I) \,\tilde{\triangleleft}\, (F, A)$, if it satisfies the following conditions:

(i) $I \subset A$,

(ii) $\gamma(x)$ is an ideal of $F(x)$ for all $x \in \mathrm{Supp}(\gamma, I)$.

Definition 2.11 [22]**.** A fuzzy set μ of a ring R is called a *fuzzy subring* of R if for every $x, y \in R$, the following conditions are satisfied:

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

3. Vague Soft Rings

In this section, we introduce the notion of vague soft rings in Rosenfeld's sense. Several fundamental properties and theorems related to this concept are also studied and investigated.

From now on let $(R, +, \cdot)$ be a ring, which will be denoted simply as R, E be a set of parameters and $A \subseteq E$.

Definition 3.1. Let $(\hat{F}, A)$ be a vague soft set over R. Then $(\hat{F}, A)$ is called a *vague soft ring* over R if and only if for every $a \in A$ and $x, y \in R$,

(i) $t_{\hat{F}_a}(x+y) \geq \min\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\}$ and $1 - f_{\hat{F}_a}(x+y) \geq \min\{1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\}$, that is $\hat{F}_a(x + y) \geq \min(\hat{F}_a(x), \hat{F}_a(y))$,

(ii) $t_{\hat{F}_a}(-x) \geq t_{\hat{F}_a}(x)$ and $1 - f_{\hat{F}_a}(-x) \geq 1 - f_{\hat{F}_a}(x)$, that is $\hat{F}_a(-x) \geq \hat{F}_a(x)$,

(iii) $t_{\hat{F}_a}(xy) \geq \min\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\}$ and $1 - f_{\hat{F}_a}(xy) \geq \min\{1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\}$, that is $\hat{F}_a(xy) \geq \min(\hat{F}_a(x), \hat{F}_a(y))$.

In other words, for every $a \in A$, $\hat{F}_a$ is a *vague subring* of R in Rosenfeld's sense.

Theorem 3.2. *Let* $(\hat{F}, A)$ *be a vague soft set over* R. *Then* $(\hat{F}, A)$ *is a vague soft ring over* R *if and only if for every* $a \in A$ *and* $x, y \in R$, *the following conditions are satisfied*:

(i) $t_{\hat{F}_a}(x - y) \geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y))$ *and* $1 - f_{\hat{F}_a}(x - y) \geq \min(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y))$,

(ii) $t_{\hat{F}_a}(xy) \geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y))$ *and* $1 - f_{\hat{F}_a}(xy) \geq \min(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y))$.

Proof. For the sake of similarity, we only prove all the necessary properties for the truth membership function. The proving for the false membership function can be done in a similar manner.

($\Rightarrow$) Let $(\hat{F}, A)$ be a vague soft ring over R. Then for every $a \in A$ and $x, y \in R$,

$$\begin{aligned} t_{\hat{F}_a}(x - y) &= t_{\hat{F}_a}(x + (-y)) \\ &\geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(-y)) \\ &\geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)). \end{aligned}$$

From Definition 3.1, condition (ii) is automatically satisfied as $(\hat{F}, A)$ is a vague soft ring over R. Then both the axioms above are satisfied.

($\Leftarrow$) Let conditions (i) and (ii) be satisfied. Then for every $a \in A$ and $x, y \in R$,

$$t_{\hat{F}_a}(0) = t_{\hat{F}_a}(x - x) = t_{\hat{F}_a}(x + (-x)) \geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(-x))$$
$$\geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(x)) = t_{\hat{F}_a}(x)$$

that is $t_{\hat{F}_a}(0) \geq t_{\hat{F}_a}(x),$ where 0 is the additive identity element of ring R. Also, we have

$$t_{\hat{F}_a}(-x) = t_{\hat{F}_a}(0 - x) = t_{\hat{F}_a}(0 + (-x)) \geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(-x))$$
$$\geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(x)) = t_{\hat{F}_a}(x)$$

that is $t_{\hat{F}_a}(-x) \geq t_{\hat{F}_a}(x)$ and therefore condition (ii) of Definition 3.1 is satisfied. Furthermore,

$$t_{\hat{F}_a}(x + y) = t_{\hat{F}_a}(x - (-y)) \geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(-y)) \geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y))$$

and thus, condition (i) of Definition 3.1 is also satisfied. Condition (iii) of Definition 3.1 is automatically satisfied since condition (ii) of Theorem 3.2 is satisfied, that is $t_{\hat{F}_a}(xy) \geq \min(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)).$ As such, it is proven that $(\hat{F}, A)$ is a vague soft ring over R. □

Theorem 3.3. *Let $(\hat{F}, A)$ be a vague soft set over R. Then the necessary and sufficient condition for $(\hat{F}, A)$ to be a vague soft ring over R is for $t_{\hat{F}_a}$ and $1 - f_{\hat{F}_a}$ to be a fuzzy subring of R for every $a \in A$.*

Proof. Let $(\hat{F}, A)$ be a vague soft ring over R. Then for every $x, y \in R$ and $a \in A,$ the following conditions are satisfied:

(i) $t_{\hat{F}_a}(x - y) \geq \min\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\}$ and $1 - f_{\hat{F}_a}(x - y) \geq \min\{1 - f_{\hat{F}_a}(x),$ $1 - f_{\hat{F}_a}(y)\},$ that is $\hat{F}_a(x - y) \geq \min\{\hat{F}_a(x), \hat{F}_a(y)\},$

(ii) $t_{\hat{F}_a}(x.y) \geq \min\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\}$ and $1 - f_{\hat{F}_a}(x.y) \geq \min\{1 - f_{\hat{F}_a}(x),$ $1 - f_{\hat{F}_a}(y)\},$ that is $\hat{F}_a(x.y) \geq \min\{\hat{F}_a(x), \hat{F}_a(y)\}.$

As such, it is proven that $\hat{F}_a$ is a fuzzy subring of R by Definition 2.11. The converse is obvious from Definition 3.1. □

Example 3.4. Any soft ring is a vague subring since any characteristic function of a subring is a vague subring.

Example 3.5. Let $(\hat{F}, A)$ be a non-null vague soft set over R which is as defined below:

$$(\hat{F}, A)^+ = \{(\hat{F}_a)^+ = \{x \in R : t_{(\hat{F}_a)^+}(x) = t_{\hat{F}_a}(x) + 1 - t_{\hat{F}_a}(e)$$

$$\text{and } 1 - f_{(\hat{F}_a)^+}(x) = 1 - f_{\hat{F}_a}(x) + f_{\hat{F}_a}(e)$$

$$\text{i.e. } (\hat{F}_a)^+(x) = \hat{F}_a(x) + 1 - \hat{F}_a(e)\}.$$

Then $(\hat{F}, A)^+$ is a vague soft ring over R.

Theorem 3.6. *Let S be a non-null subset of R and $(\hat{F}, A)_S$ be a vague soft characteristic set over S. If $(\hat{F}, A)$ is a vague soft ring over R, then S is a subring of R.*

Proof. Let $(\hat{F}, A)$ be a vague soft ring over R. Then for every $a \in A$, $\hat{F}_a$ is a vague subring of R. Now let $x, y \in S$. Then $t_{(\hat{F}_a)_S}(x) = t_{(\hat{F}_a)_S}(y) = s$ and $1 - f_{(\hat{F}_a)_S}(x) = 1 - f_{(\hat{F}_a)_S}(y) = s$, which means that $(\hat{F}_a)_S(x) = (\hat{F}_a)_S(y) = s$. Since $\hat{F}_a$ is a vague subring of R, $(\hat{F}_a)_S(x - y) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\}$ $= \min\{s, s\} = s$ and also $(\hat{F}_a)_S(xy) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\} = \min\{s, s\}$ $= s$. As such, we obtain $x - y \in S$ and $xy \in S$ which implies that S is a subring of R. □

Theorem 3.7. *Let S be a non-null subset of R and $(\hat{F}, A)_S$ be a vague soft characteristic set over S. Then $(\hat{F}, A)_S$ is a vague soft ring over S if and only if S is a subring of R.*

Proof. ($\Rightarrow$) Let $(\hat{F}, A)_S$ be a vague soft ring over S. Then for every $a \in A$, $(\hat{F}_a)_S$ is the corresponding vague subring of S. Now let $x, y \in S$. The subsequent proof is similar to that of Theorem 3.6.

($\Leftarrow$) Let S be a subring of R and $x, y \in R$. There are four cases that need to be considered here.

Case 1. If $x, y \in S$, then $x - y \in S$ and $xy \in S$. Thus $t_{(\hat{F}_a)_S}(x) = t_{(\hat{F}_a)_S}(y)$ $= s$ and $1 - f_{(\hat{F}_a)_S}(x) = 1 - f_{(\hat{F}_a)_S}(y) = s$, which means that $(\hat{F}_a)_S(x) =$ $(\hat{F}_a)_S(y) = s$. Then we obtain

$$\min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\} = \min\{s, s\} = s = (\hat{F}_a)_S(x - y) = (\hat{F}_a)_S(xy).$$

Case 2. If $x \in S$ and $y \notin S$, then $t_{(\hat{F}_a)_S}(x) = 1 - f_{(\hat{F}_a)_S}(x) = s$ and $t_{(\hat{F}_a)_S}(y) = 1 - f_{(\hat{F}_a)_S}(y) = w$, which means that $(\hat{F}_a)_S(x) = s$ and $(\hat{F}_a)_S(y)$ $= w$. It is clear that $(x - y) \geq w$ and $(\hat{F}_a)_S(xy) \geq w$. However, $\min\{(\hat{F}_a)_S(x),$ $(\hat{F}_a)_S(y)\} = \min\{s, w\} = w$. This means that $(\hat{F}_a)_S(x - y) \geq \min\{(\hat{F}_a)_S(x),$ $(\hat{F}_a)_S(y)\}$ and $(\hat{F}_a)_S(xy) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\}$.

Case 3. If $x \notin S$ and $y \in S$, then $t_{(\hat{F}_a)_S}(x) = 1 - f_{(\hat{F}_a)_S}(x) = w$ and $t_{(\hat{F}_a)_S}(y) = 1 - f_{(\hat{F}_a)_S}(y) = s$, that is $(\hat{F}_a)_S(x) = w$ and $(\hat{F}_a)_S(y) = s$. It is obvious that $(\hat{F}_a)_S(x - y) \geq w$ and $(\hat{F}_a)_S(xy) \geq w$. However, $\min\{(\hat{F}_a)_S(x),$ $(\hat{F}_a)_S(y)\} = \min\{w, s\} = w$. This implies that $(\hat{F}_a)_S(x - y) \geq \min\{(\hat{F}_a)_S(x),$ $(\hat{F}_a)_S(y)\}$ and $(\hat{F}_a)_S(xy) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\}$.

Case 4. If $x, y \notin S$, then $t_{(\hat{F}_a)_S}(x) = t_{(\hat{F}_a)_S}(y) = w$ and $1 - f_{(\hat{F}_a)_S}(x) =$ $1 - f_{(\hat{F}_a)_S}(y) = w$ and this means that $(\hat{F}_a)_S(x) = (\hat{F}_a)_S(y) = w$. Once again it is clear that $(\hat{F}_a)_S(x - y) \geq w$ and $(\hat{F}_a)_S(xy) \geq w$ whereas $\min\{(\hat{F}_a)_S(x),$

$(\hat{F}_a)_S(y)\} = \min\{w, w\} = w.$ Therefore

$$(\hat{F}_a)_S(x - y) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\}$$

and

$$(\hat{F}_a)_S(xy) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\}.$$

Thus in all the four cases that were considered, it was proven that $(\hat{F}_a)_S(x - y) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\}$ and $(\hat{F}_a)_S(xy) \geq \min\{(\hat{F}_a)_S(x), (\hat{F}_a)_S(y)\}$, which implies that $(\hat{F}_a)_S$ is a vague subring of S. Hence $(\hat{F}, A)_S$ is a vague soft ring over S. □

Theorem 3.7 proves that there exists a one-to-one correspondence between vague soft rings and the classical subrings of a ring.

Theorem 3.8. *Let $(\hat{F}, A)$ be a vague soft set over R. Then $(\hat{F}, A)$ is a vague soft ring over R if and only if for every $\alpha, \beta \in [0, 1]$, $(\hat{F}, A)_{(\alpha, \beta)}$ is a soft ring over R.*

Proof. The proof is similar to that of Theorem 3.8 and is therefore omitted. □

Corollary 3.9. *Let $(\hat{F}, A)$ be a vague soft set over R. Then $(\hat{F}, A)$ is a vague soft ring over R if and only if for every $\alpha \in [0, 1]$, $(\hat{F}, A)_{(\alpha, \alpha)}$ is a soft ring over R.*

4. Vague Soft Ring Homomorphism

In this section, we present some results on the concept of vague soft ring homomorphism. This concept was first introduced in [17]. However, the definition of the image and pre-image of a vague soft set introduced in [17] is incorrect and therefore the subsequent results are also incorrect. As such, in this section we use the notion of vague soft functions and the image and pre-image of a vague soft set under a vague soft function introduced in [8] and

[9], respectively, and apply these concepts to the homomorphism introduced in [17] to prove that the vague soft ring homomorphism introduced here preserves vague soft rings.

Definition 4.1 [8]**.** Let $\varphi : X \to Y$ and $\psi : A \to B$ be two functions, where A and B are parameter sets for the classical sets X and Y, respectively. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be vague soft sets over X and Y, respectively. Then the ordered pair (φ, ψ) is called a *vague soft function* from $(\hat{F}, A)$ to $(\hat{G}, B)$ and is denoted as $(\varphi, \psi) : (\hat{F}, A) \to (\hat{G}, B)$.

Definition 4.2 [10]**.** Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be vague soft sets over X and Y, respectively. Let $(\varphi, \psi) : (\hat{F}, A) \to (\hat{G}, B)$ be a vague soft function.

(i) The *image* of $(\hat{F}, A)$ under the vague soft function (φ, ψ), which is denoted as $(\varphi, \psi)(\hat{F}, A)$, is a vague soft set over Y, which is defined as

$$(\varphi, \psi)(\hat{F}, A) = (\varphi(\hat{F}), \psi(A)),$$

where

$$\varphi(\hat{F}_a)(\varphi(x)) = (\varphi(\hat{F}))_{\psi(a)}(y)$$

for all $a \in A$, $x \in X$ and $y \in Y$.

(ii) The *pre-image* of $(\hat{G}, B)$ under the vague soft function (φ, ψ) which is denoted as $(\varphi, \psi)^{-1}(\hat{G}, B)$, is a vague soft set over X, which is defined as

$$(\varphi, \psi)^{-1}(\hat{G}, B) = (\varphi^{-1}(\hat{G}), \psi^{-1}(B)),$$

where

$$\varphi^{-1}(\hat{G}_b) = (\varphi^{-1}(y)) = (\varphi^{-1}(\hat{G}))_{\psi^{-1}(b)}(x)$$

for all $b \in B$, $x \in X$ and $y \in Y$.

If φ and ψ are injective (surjective), then the vague soft function (φ, ψ) is said to be *injective* (*surjective*).

Definition 4.3 [17]**.** Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be vague soft rings over R and S, respectively, and $(\varphi, \psi) : (\hat{F}, A) \to (\hat{G}, B)$ be a vague soft function. Then

(a) (φ, ψ) is a *vague soft ring homomorphism* if the following conditions are satisfied:

(i) $\varphi : R \to S$ is a ring homomorphism,

(ii) $\psi : A \to B$ is a mapping,

(iii) $\varphi(\hat{F}(x)) = \hat{G}(\psi(x))$ for every $x \in A$.

Then $(\hat{F}, A)$ is said to be *vague soft homomorphic* to $(\hat{G}, B)$ and this relationship is denoted as $(\hat{F}, A) \sim (\hat{G}, B)$.

(b) (φ, ψ) is a *vague soft ring isomorphism* if the following conditions are satisfied:

(iv) $\varphi : R \to S$ is a ring isomorphism,

(v) $\psi : A \to B$ is a bijection,

(vi) $\varphi(\hat{F}(x)) = \hat{G}(\psi(x))$ for every $x \in A$.

Then $(\hat{F}, A)$ is said to be *vague soft isomorphic* to $(\hat{G}, B)$ and this relationship is denoted as $(\hat{F}, A) \cong (\hat{G}, B)$.

Theorem 4.4. *Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be vague soft rings over R and S, respectively, and $(\varphi, \psi) : (\hat{F}, A) \to (\hat{G}, B)$ be a vague soft ring homomorphism. Then*

(i) $(\varphi, \psi)(\hat{F}, A)$ *is a vague soft ring over S.*

(ii) $(\varphi, \psi)^{-1}(\hat{G}, B)$ *is a vague soft ring over R.*

Proof. (i) Let $k \in \psi(A)$ and $y_1, y_2 \in S$. If $\varphi^{-1}(y_1) = \varnothing$ or $\varphi^{-1}(y_2)$

$= \varnothing$, the proof is trivial. Given that $(\varphi, \psi) : (\hat{F}, A) \to (\hat{G}, B)$ is a vague soft ring homomorphism. Then $(\varphi, \psi)(\hat{F}, A) = (\varphi(\hat{F}), \psi(A))$ for every $k \in \psi(A)$ and $y \in S$. Now suppose that there exist $x_1, x_2 \in R$, such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Then for every $y_1, y_2 \in S$ and $k \in \psi(A)$, we have

$$\begin{aligned}
\varphi(\hat{F}_k)(y_1 - y_2) &= \varphi(\hat{F}_k(y_1 - y_2)) \\
&\geq \varphi(\min(\hat{F}_k(y_1), \hat{F}_k(y_2))) \\
&\geq \min(\varphi(\hat{F}_k(y_1)), \varphi(\hat{F}_k(y_2))) \\
&= \min(\varphi(\hat{F}_k)(y_1), \varphi(\hat{F}_k)(y_2))
\end{aligned}$$

and

$$\begin{aligned}
\varphi(\hat{F}_k)(y_1.y_2) &= \varphi(\hat{F}_k(y_1.y_2)) \\
&\geq \varphi(\min(\hat{F}_k(y_1), \hat{F}_k(y_2))) \\
&\geq \min(\varphi(\hat{F}_k(y_1)), \varphi(\hat{F}_k(y_2))) \\
&= \min(\varphi(\hat{F}_k)(y_1), \varphi(\hat{F}_k)(y_2)).
\end{aligned}$$

As such $\varphi(\hat{F}_k(y))$ is a vague subring of S. Hence $(\varphi, \psi)(\hat{F}, A)$ is a vague soft ring over S.

(ii) The proof is similar to the proof of part (i) and is therefore omitted. □

Theorem 4.4 proves that the homomorphic image and pre-image of a vague soft ring is also a vague soft ring. Hence it can be concluded that the vague soft ring homomorphism preserves vague soft rings.

Theorem 4.5. *Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be vague soft rings over R and S, respectively, and $(\hat{F}, A)$ be vague soft isomorphic to $(\hat{G}, B)$. If $\hat{F}(x)$ is a vague subring of R, then $\hat{G}(\psi(x))$ is a vague subring of S and $\hat{F}(x) \cong \hat{G}(\psi(x))$.*

Proof. Since $(\hat{F}, A)$ is vague soft isomorphic to $(\hat{G}, B)$, by Definition 4.3, we obtain $\hat{F} \cong \hat{G}$ and $\varphi(\hat{F}(x)) = \hat{G}(\psi(x))$ for every $x \in A$. Moreover, since $\hat{F}(x)$ is a vague subring of R, then $\varphi(\hat{F}(x))$ is a vague subring of S. This means that $\hat{G}(\psi(x))$ is also a vague subring of S since $\varphi(\hat{F}(x)) = \hat{G}(\psi(x))$ for every $x \in A$. Subsequently, since $\varphi : R \to S$ is an isomorphism, $\hat{F}(x)$ is a vague subring of R and $\varphi(\hat{F}(x))$ is a vague subring of S, this implies that $\hat{F}(x)$ is vague soft isomorphic to $\varphi(\hat{F}(x))$. As such, it is proven that $\hat{F}(x)$ is vague soft isomorphic to $\hat{G}(\psi(x))$, that is $\hat{F}(x) \cong \hat{G}(\psi(x))$. □

Corollary 4.6. *Let* $(\hat{F}, A)$ *and* $(\hat{G}, B)$ *be vague soft rings over R and S, respectively. If* $(\hat{F}, A)$ *is a vague soft subring of* $(\hat{G}, B)$ *and* $(\hat{F}, A) \cong (\hat{G}, B)$, *then* $(\varphi(\hat{F}), \psi(A)) \cong (\varphi(\hat{G}), \psi(B))$.

5. Vague Soft Ideals

In this section, the concepts of vague soft left ideal, vague soft right ideal and vague soft ideal of a ring are re-introduced and the algebraic properties of these concepts are studied and discussed.

Definition 5.1. Let $(\hat{I}, A)$ be a non-null vague soft set over R. Then $(\hat{I}, A)$ is called a *vague soft right* (*left*) *ideal* of R if and only if for every $a \in \operatorname{Supp}(\hat{I}, A)$ and $x, y \in R$,

(i) $t_{\hat{I}_a}(x - y) \geq \min\{t_{\hat{I}_a}(x), t_{\hat{I}_a}(y)\}$ and $1 - f_{\hat{I}_a}(x - y) \geq \min\{1 - f_{\hat{I}_a}(x), 1 - f_{\hat{I}_a}(y)\}$, that is $\hat{I}_a(x - y) \geq \min\{\hat{I}_a(x), \hat{I}_a(y)\}$,

(ii) $t_{\hat{I}_a}(x.y) \geq t_{\hat{I}_a}(x)$ and $1 - f_{\hat{I}_a}(x.y) \geq 1 - f_{\hat{I}_a}(x)$, that is $\hat{I}_a(x.y) \geq \hat{I}_a(x)$,

($t_{\hat{I}_a}(x.y) \geq t_{\hat{I}_a}(y)$ and $1 - f_{\hat{I}_a}(x.y) \geq 1 - f_{\hat{I}_a}(y)$, that is $\hat{I}_a(x.y) \geq \hat{I}_a(y)$).

Thus for every $a \in \operatorname{Supp}(\hat{I},\ A),\ \hat{I}_a$ is a *vague right* (*left*) *ideal* of R in Rosenfeld's sense.

Definition 5.2. Let $(\hat{I},\ A)$ be a non-null vague soft set over R. Then $(\hat{I},\ A)$ is called a *vague soft ideal* of R if and only if for every $a \in \operatorname{Supp}(\hat{I},\ A)$ and $x,\ y \in R$,

(i) $t_{\hat{I}_a}(x - y) \geq \min\{t_{\hat{I}_a}(x), t_{\hat{I}_a}(y)\}$ and $1 - f_{\hat{I}_a}(x - y) \geq \min\{1 - f_{\hat{I}_a}(x), 1 - f_{\hat{I}_a}(y)\}$, that is $\hat{I}_a(x - y) \geq \min\{\hat{I}_a(x), \hat{I}_a(y)\}$,

(ii) $t_{\hat{I}_a}(x.y) \geq \max\{t_{\hat{I}_a}(x), t_{\hat{I}_a}(y)\}$ and $1 - f_{\hat{I}_a}(x.y) \geq \max\{1 - f_{\hat{I}_a}(x), 1 - f_{\hat{I}_a}(y)\}$, that is $\hat{I}_a(x.y) \geq \max\{\hat{I}_a(x), \hat{I}_a(y)\}$.

Therefore for every $a \in \operatorname{Supp}(\hat{I},\ A),\ \hat{I}_a$ is a *vague ideal* of R in Rosenfeld's sense.

Theorem 5.3. *Let* $(\hat{I},\ A)$ *be a non-null vague soft set over* R. *Then the necessary and sufficient condition for* $(\hat{I},\ A)$ *to be a vague soft ideal over* R *is for* $(\hat{I},\ A)$ *to be a vague soft left ideal and a vague soft right ideal of* R.

Proof. It is evident that condition (ii) of Definition 5.2 is the combination of the second conditions of Definition 5.1. As such, it can be concluded that $(\hat{I},\ A)$ is a vague soft ideal of R if $(\hat{I},\ A)$ is a vague soft left ideal and vague soft right ideal of R. □

Theorem 5.4. *Let* $(\hat{F},\ A)$ *be a non-null vague soft set over* R. *Then* $(\hat{F},\ A)$ *is a vague soft ideal of* R *if and only if for every* $t \in [0, 1],\ (\hat{F},\ A)_{(t,t)}$ *is a soft ideal of* R.

Proof. ($\Rightarrow$) Let $(\hat{F},\ A)$ be a vague soft ideal of R. Then $(\hat{F},\ A)$ must be a vague soft left ideal and a vague soft right ideal of R. Then for every $a \in$

$\mathrm{Supp}(\hat{F}, A)$, the corresponding vague subset $\hat{F}_a$ is a vague left ideal and vague right ideal of R. Now let $t \in [0, 1]$ and $x, y \in (\hat{F}_a)_{(t,t)}$. Since $\hat{F}_a$ is a vague ideal of R, it follows that:

$$t_{\hat{F}_a}(x - y) \geq \min\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\} \geq \min\{t, t\} = t$$

and

$$1 - f_{\hat{F}_a}(x - y) \geq \min\{1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\} \geq \min\{t, t\} = t,$$

i.e.,

$$\hat{F}_a(x - y) \geq \min\{\hat{F}_a(x), \hat{F}_a(y)\} \geq \min(t, t) = t.$$

Moreover, since $\hat{F}_a$ is a vague left ideal of R, $t_{\hat{F}_a}(r.x) \geq t_{\hat{F}_a}(x) \geq t$ and $1 - f_{\hat{F}_a}(r.x) \geq 1 - f_{\hat{F}_a}(x) \geq t$, that is $\hat{F}_a(r.x) \geq \hat{F}_a(x) \geq t$ and also since $\hat{F}_a$ is a vague right ideal of R, $t_{\hat{F}_a}(x.r) \geq t_{\hat{F}_a}(x) \geq t$ and $1 - f_{\hat{F}_a}(x.r) \geq 1 - f_{\hat{F}_a}(x) \geq t$, that is $\hat{F}_a(x.r) \geq \hat{F}_a(x) \geq t$ for every $r \in R$. This implies that $x - y \in (\hat{F}_a)_{(t,t)}$, $r.x \in (\hat{F}_a)_{(t,t)}$ and $x.r \in (\hat{F}_a)_{(t,t)}$. Therefore $(\hat{F}_a)_{(t,t)}$ is a left ideal and a right ideal of R for every $t \in [0, 1]$. Hence $(\hat{F}_a)_{(t,t)}$ is an ideal of R, which means that $(\hat{F}, A)_{(t,t)}$ is a soft ideal of R.

($\Leftarrow$) Let $(\hat{F}, A)_{(t,t)}$ be a soft ideal of R. Then $(\hat{F}_a)_{(t,t)}$ is an ideal of R for every $t \in [0, 1]$ and the corresponding $a \in \mathrm{Supp}(\hat{F}, A)$. Next let $x, y \in R$. Then $t_{\hat{F}_a}(x - y) < \min\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\} = t$ and $1 - f_{\hat{F}_a}(x - y) < \min\{1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\} = t$, which means that $\hat{F}_a(x - y) < \min\{\hat{F}_a(x), \hat{F}_a(y)\} = t$. This implies that $x, y \in (\hat{F}_a)_{(t,t)}$ but $x - y \notin (\hat{F}_a)_{(t,t)}$. However, this is contradictory to the assumption that $(\hat{F}_a)_{(t,t)}$ is an ideal of R. Thus

$\hat{F}_a(x-y) \geq \min\{\hat{F}_a(x), \hat{F}_a(y)\}$. Now suppose that $t_{\hat{F}_a}(x.y) < t_{\hat{F}_a}(y) = t$ and $1 - f_{\hat{F}_a}(x.y) < 1 - f_{\hat{F}_a}(y) = t$, which means that $\hat{F}_a(x.y) < \hat{F}_a(y) = t$. This implies that $y \in (\hat{F}_a)_{(t,t)}$ but $x.y \notin (\hat{F}_a)_{(t,t)}$. This shows that $(\hat{F}_a)_{(t,t)}$ is not a left ideal of R which contradicts the fact that $(\hat{F}_a)_{(t,t)}$ is an ideal of R. Therefore it can be concluded that $\hat{F}_a(x.y) \geq \hat{F}_a(y)$. Similarly, suppose that $t_{\hat{F}_a}(x.y) < t_{\hat{F}_a}(x) = t$ and $1 - f_{\hat{F}_a}(x.y) < 1 - f_{\hat{F}_a}(x) = t$, which means that $\hat{F}_a(x.y) < \hat{F}_a(x) = t$. This implies that $x \in (\hat{F}_a)_{(t,t)}$ but $x.y \notin (\hat{F}_a)_{(t,t)}$. As such, $(\hat{F}_a)_{(t,t)}$ is not a right ideal of R. However, this is a contradiction since $(\hat{F}_a)_{(t,t)}$ is an ideal of R which means that $(\hat{F}_a)_{(t,t)}$ is a left ideal and a right ideal of R. Therefore $\hat{F}_a(x.y) \geq \hat{F}_a(x)$. Hence it has been proven that $\hat{F}_a(x-y) \geq \min(\hat{F}_a(x), \hat{F}_a(y))$, $\hat{F}_a(x.y) \geq \hat{F}_a(y)$ and $\hat{F}_a(x.y) \geq \hat{F}_a(x)$. Thus it can be concluded that $\hat{F}_a$ is a vague left ideal and a vague right ideal of R for every $a \in \mathrm{Supp}(\hat{F}, A)$. As such, $\hat{F}_a$ is a vague ideal of R for every $a \in \mathrm{Supp}(\hat{F}, A)$. Then $(\hat{F}, A)$ is a vague soft ideal of R. □

Theorem 5.5. *Let S be a non-null subset of R and* $(\hat{F}, A)_S$ *be a vague soft characteristic set over R. Then* $(\hat{F}, A)_S$ *is a vague soft left* (*resp*., *right*) *ideal of R if and only if S is a left* (*resp*. *right*) *ideal of R*.

Proof. The proof is similar to that of Theorem 5.4 and is therefore omitted. □

Corollary 5.6. *Let S be a non-null subset of R and* $(\hat{F}, A)_S$ *be a vague soft characteristic set over S. Then* $(\hat{F}, A)_S$ *is a vague soft ideal of R if and only if S is an ideal R*.

Theorem 5.4 proves that there exists a one-to-one correspondence between the vague soft ideal and the soft ideal of a ring. A similar relationship is observed between vague soft left (resp., right) ideals and the classical left (resp. right) ideals as well as between vague soft ideals and the classical ideals of a ring as implied in Theorem 5.5 and Corollary 5.6, respectively.

As a consequence to the notion of vague soft rings and vague soft ideals introduced earlier in this section, we propose the concept of idealistic vague soft rings as an extension to both these concepts.

Definition 5.7. Let $(\hat{F}, A)$ be a non-null vague soft ring over R. Then $(\hat{F}, A)$ is called the *idealistic vague soft ring* over R if $\hat{F}_a$ is a vague subring of R for every $a \in A$ and a vague ideal of R for every $a \in \mathrm{Supp}(\hat{F}, A)$. Then the following conditions must be satisfied:

(i) $t_{\hat{F}_a}(x - y) \geq \min\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\}$ and $1 - f_{\hat{F}_a}(x - y) \geq \min\{1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\}$, that is $\hat{F}_a(x - y) \geq \min(\hat{F}_a(x), \hat{F}_a(y))$,

(ii) $t_{\hat{F}_a}(x.y) \geq \max\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\}$ and $1 - f_{\hat{F}_a}(x.y) \geq \max\{1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\}$, that is $\hat{F}_a(x.y) \geq \max(\hat{F}_a(x), \hat{F}_a(y))$,

for every $x, y \in R$ and $a \in \mathrm{Supp}(\hat{F}, A)$.

By this definition, it is clear that every idealistic vague soft ring over R is a vague soft ring over R, although the converse is generally not true.

Theorem 5.8. *Let* $\varphi : R \to S$ *be a ring epimorphism. If* $(\hat{F}, A)$ *is an idealistic vague soft ring over* R, *then* $(\varphi(\hat{F}), \mathrm{Supp}(\hat{F}, A))$ *is an idealistic vague soft ring over* S.

Proof. Let $(\hat{F}, A)$ be an idealistic vague soft ring over R. Then $(\hat{F}, A)$ is non-null and thus $(\varphi(\hat{F}), \mathrm{Supp}(\hat{F}, A))$ must be non-null too. Now suppose that for every $a \in \mathrm{Supp}(\hat{F}, A)$ and $y_1, y_2 \in S$, there exist $x_1, x_2 \in R$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Then we have

$$\begin{aligned}\varphi(\hat{F}_a)(y_1 - y_2) &= \varphi(\hat{F}_a(y_1 - y_2)) \\ &\geq \varphi(\min(\hat{F}_a(y_1)), (\hat{F}_a(y_2))) \\ &\geq \min(\varphi(\hat{F}_a(y_1)), \varphi(\hat{F}_a(y_2))) \\ &= \min(\varphi(\hat{F}_a)(y_1), \varphi(\hat{F}_a)(y_2))\end{aligned}$$

and

$$\begin{aligned}\varphi(\hat{F}_a)(y_1.y_2) &= \varphi(\hat{F}_a(y_1 \cdot y_2)) \\ &\geq \varphi(\max(\hat{F}_a(y_1), \hat{F}_a(y_2))) \\ &\geq \max(\varphi(\hat{F}_a(y_1)), \varphi(\hat{F}_a(y_2))) \\ &= \max(\varphi(\hat{F}_a)(y_1), \varphi(\hat{F}_a)(y_2)).\end{aligned}$$

As such, it has been proven that $\varphi(\hat{F}_a)$ is a vague ideal of S for every $a \in \mathrm{Supp}(\hat{F}, A)$. Hence $(\varphi(\hat{F}), \mathrm{Supp}(\hat{F}, A))$ is an idealistic vague soft ring over S. □

Theorem 5.9. *Let* $(\hat{G}, B)$ *be an idealistic vague soft ring over* S *and* $\varphi : R \to S$ *be a ring homomorphism. Then* $(\varphi^{-1}(\hat{G}), \mathrm{Supp}(\hat{G}, B))$ *is an idealistic vague soft ring over* R.

Proof. The proof is similar to that of Theorem 5.8. □

6. Conclusion

In this paper, we presented a study on the theory of vague soft rings. We redefined the concepts of vague soft rings, vague soft subrings and vague soft ideals to better reflect the concept of vague soft sets which were used as the base to derive these concepts. It was proven that there exists a one-to-one correspondence between the concepts introduced here and the corresponding concepts in soft ring theory and classical ring theory. Lastly, we proved that the vague soft ring homomorphism preserves vague soft rings.

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