Far East Journal of Applied Mathematics
© 2015 Pushpa Publishing House, Allahabad, India
Published Online: November 2015
http://dx.doi.org/10.17654/FJAMDec2015_247_268
Volume 93, Number 3, 2015, Pages 247-268 **ISSN: 0972-0960**

CORE-COLLAPSE SUPERNOVA EXPLOSIONS: AN ANALYTICAL ONE-DIMENSIONAL ANALYSIS

Ho-Young Kwak

Mechanical Engineering Department
Chung-Ang University
Seoul 156-756, Korea
e-mail: kwakhy@cau.ac.kr

Abstract

Core-collapse supernova (Type II, for example, Cassiopeia A) implosion and the subsequent supernova explosion due to core bounce were studied analytically by solving the continuity, Euler, Poisson and energy equations by taking the star as an "ideal gas system". Outward shock propagation and inward accretion shock due to the supernova implosion were calculated by solving the continuity and momentum equations for the shell layer outside of the core with the Kirkwood-Bethe hypothesis. The collapse time of the core-mass of $1.5M_{\odot}$ having radius of a 3000km to a protoneutron star with radius of 108km is calculated to be approximately 1.2s. The heat transport during the evolution turns out to depend on the macroscopic parameters such as gravitational potential, the mass and the collapsing and expanding velocity of the supernova which indicates that the heat transfer from/to a system is determined by the evolution process for an ideal gas system, and vice versa. In detail, the heat escaped from the star during

Received: August 11, 2015; Accepted: September 21, 2015
2010 Mathematics Subject Classification: 35Q31, 35Q79, 35Q85, 76L05.
Keywords and phrases: heat transport, explosion, implosion, supernova, ideal gas system.
Communicated by K. K. Azad

the collapse is about 5.9×10^{51} ergs and the heat absorption during the expansion after the bounce is about 10^{52} ergs which are comparable to the values obtained by detailed computation.

1. Introduction

It is known that the core-collapse supernova explosions enriched the galaxy with various elements such as oxygen, iron, calcium and silicon and influenced the birth of new stars and that the core-collapse supernova might be the source of the γ-ray burst [1]. It is also known that a supernova (Type II, for example, Cassiopeia A) may occur when the iron core $(1.5M_\odot)$ of a massive progenitor star $(15 \sim 20M_\odot)$ with radius of 10^8 km collapses into nuclear densities. At such extreme density, the core bounces to generate a strong shock front and leads to launching an explosion [2]. It is well known that before the bounce shock, it stalls into an accretion shock so that neutrino-heating mechanism has been proposed [2] to revive the stalled shock into explosion. Based on such scenario [1], simulations of hydrodynamical equations coupled with neutron transport were performed to unveil the supernova explosion by many researchers [3-8]. An artist view of the scenario for the supernova explosion given in Burrows [1] is shown in Figure 1.

(a) (b) (c) (d)

Figure 1. Artist views of evolution process for Cassiopeia A [22]: implosion (a), collapse (b) and subsequent core bounce (c) and explosion (d).

Radiation hydrodynamics code coupled with neutron transport [9] and sophisticated two-dimensional, time dependent, multi-group and multi-angle radiation hydrodynamics numerical scheme [4] were developed for the core-collapse supernova simulation. Buras et al. [6] used mass, momentum and energy equations in spherical coordinates and azimuthal symmetry in their hydrodynamic simulations for the core-collapse supernovae. Newtonian gravity term and neutrino interaction term for momentum transfer were included in the momentum equation. Instead of the heat flow term in the energy equation, neutron source terms for energy exchange were included.

Murphy and Burrows [7] used continuity, Euler and energy equations in their simulation. Newtonian gravity term was included in the momentum equation. However, the term representing heat flow in the energy equation was replaced as the neutrino heating and cooling terms which are dependent on the electron neutrino temperature and the neutron and proton fractions. The closure for these hydrodynamic equations was obtained with the equation of state by Shen et al. [10].

However, the core-collapse supernova mechanism has never been studied by hydrodynamic equations with keeping the heat flow term in the energy equation, which may provide simple and quantitative theory of the supernova explosion. In this study, core-collapse supernova explosion was investigated by hydrodynamic equations based on the analytical method used for explosives detonation [11] with considering the star as an "ideal gas system". Heat flow from the collapsing star and heat gain during the expansion process after explosion were obtained analytically and estimated. Also, the outgoing shock wave and the accretion shock due to the implosion of the core and the outgoing shock due to the explosion of the protoneutron star were studied analytically with the Kirkwood-Bethe hypothesis.

2. Analytical Solutions for the Hydrodynamical Equations

Exact analytic solutions of the hydrodynamical equations (continuity, Euler and Newtonian gravitational equations) first proposed by Sir James Jeans (1902) as theory of galaxy formation, were obtained to demonstrate the stellar stability, spherical oscillation and gravitational collapse of Newtonian stars such as Sun, Jupiter and Saturn by Jun and Kwak [12]. Their previous studies also found that a "bound state" exists for a star with γ (specific heat ratio) $> 4/3$ but there is no stable "bound" state when $\gamma < 4/3$. This means that a star with $\gamma > 4/3$ starts to contract from its initial radius, R_{max}, until it reaches a minimum radius, R_{min}, and then it oscillates. On the other hand, a star with $\gamma < 4/3$ will collapse. Brief summary of the previous works is as follows.

The equation of continuity, the Euler equation for an irrotational fluid and Newtonian gravitational equation are given below, respectively,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0, \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla P - \nabla \phi, \tag{2}$$

$$\nabla^2 \phi = 4\pi G \rho, \tag{3}$$

where ρ is the density, P is the pressure, $\vec{u}$ is the fluid velocity, $\phi(r, t)$ is gravitational potential and G is the gravitational constant.

From the continuity equation, the following linear velocity and quadratic density profiles can be obtained as a lowest-order approximation. The fully derivations of these equations can be found in the literature [13, 14]:

$$u = \dot{R} r / R, \tag{4}$$

$$\rho(r, t) = \frac{b}{R^3(t)} + \frac{a r^2}{R^5(t)}, \tag{5}$$

where R is the radius of star, $\dot{R}$ is the velocity of star and "*a*" and "*b*" are constants which will be identified later. The meaning of the linear velocity profile given in equation (4) is that a star collapses or expands homologously, in other words, every mass point during the collapse or expansion may be traced back to a single point, the center of star.

With the density profile given in equation (5) and the velocity u in (4), the gravitational potential $\phi(r, t)$ and the pressure $P(r, t)$ can be obtained by solving the Poisson and Euler equations. These are in terms of 'a' and 'b' given as follows:

$$\phi(r, t) = \phi_o(t) + \left(\frac{2\pi Gb}{3}\frac{r^2}{R^3} + \frac{\pi Ga}{5}\frac{r^4}{R^5}\right), \tag{6}$$

$$P(r, t) = P_0(t) - \left[\left(\frac{b}{2}\frac{r^2}{R^2} + \frac{a}{4}\frac{r^4}{R^4}\right)\frac{\ddot{R}}{R^2} + \left(\frac{2\pi Gb^2}{3}\frac{r^2}{R^2} + \frac{8\pi Gab}{15}\frac{r^4}{R^4} + \frac{2\pi Ga^2}{15}\frac{r^6}{R^6}\right)\frac{1}{R^4}\right]. \tag{7}$$

At the center of the star, the above equation can be arranged to the following after applying the boundary condition of, $P = 0$ and $\phi = -GM/R$ at $r = R$:

$$\phi_o = -\frac{GM}{R} - \frac{\pi G(3a + 10b)}{15}\frac{1}{R} \tag{8}$$

and

$$P_0(t) = \left(\frac{a + 2b}{4}\right)\frac{\ddot{R}}{R^2} + \frac{2\pi G(a^2 + 4ab + 5b^2)}{15}\frac{1}{R^4}. \tag{9}$$

To determine the values of "a" and "b" in equation (5), an equation of state for the gas inside the star is required. Excluding the viscous dissipation term, the energy equation with internal and enthalpy representation can be written as follows [13, 15]:

$$\rho C_v \frac{DT}{Dt} = -P\nabla \cdot \vec{u} - \nabla \cdot \vec{q}, \tag{10}$$

$$\rho C_p \frac{DT}{Dt} = \frac{DP}{Dt} - \nabla \cdot \vec{q}. \tag{11}$$

In the above equations, T is the temperature and C_v and C_p are the specific heats at constant-volume and constant-pressure, respectively, and $\vec{q}$ is the heat flux.

From equations (10) and (11), one can obtain the following equation by eliminating the term DT/Dt:

$$(\gamma - 1)\nabla \cdot \vec{q} = -\frac{DP}{Dt} - \frac{3\gamma P\dot{R}}{R}, \tag{12}$$

where γ is the specific heat ratio.

The above equation indicates that the heat transport inside the star can be obtained if one knows the evolution of the star without detailed mechanism of heat transfer, which can be possible only for the “ideal gas system”. On the other hand, eliminating the term, $\nabla \cdot \vec{q}$, in equations (10) and (11), one can obtain the temperature at the center of star in terms of the pressure at the center and radius:

$$bR_g T_o(t) = P_o(t)R^3(t), \tag{13}$$

where R_g is the gas constant and the subscript “o” denotes the property at the center.

Assuming that the “polytrope” only holds true at the center, the following equation can be applied:

$$P_0 = \kappa\rho_0^{\gamma}. \tag{14}$$

The assumption of the polytrope at the center is equivalent to the assumption that the center is neither a heat source nor a heat sink, i.e., $\nabla \cdot \vec{q} = 0$ at the center. This assumption, of course, is not valid when nuclear reaction occurs inside the star.

With the velocity, density and pressure profiles obtained given in equations (4), (5) and (7), one can solve the energy conservation given in equations (10) and (11) with the boundary condition equation (14). These are

$$a = -b = -15M/8, \tag{15}$$

$$T(r,\,t) = T_o - \left(\frac{R\ddot{R}}{4R_g} + \frac{2\pi Gb}{5R_g R}\right)\left(\frac{r}{R}\right)^2 + \frac{2\pi Gb}{15R_g R}\left(\frac{r}{R}\right)^4, \tag{16}$$

where

$$T_o = \frac{R\ddot{R}}{4R_g} + \frac{4\pi Gb}{15R_g R}. \tag{17}$$

With the assumption of polytrope at the center and the explicit value of '*a*' and '*b*', the profiles for pressure, temperature, density and gravitational potential in the star can be obtained [12]. These are

$$P(r,\,t) = \frac{15M}{32\pi}\left(\frac{32\pi\kappa'}{15M}\frac{1}{R^{3\gamma-4}} - GM\left(\frac{r}{R}\right)^2\right)\left(1 - \frac{r^2}{R^2}\right)^2\frac{1}{R^4}, \tag{18}$$

$$T(r,\,t) = \frac{1}{4R_g}\left(\frac{32\pi\kappa'}{15M}\frac{1}{R^{3\gamma-4}} - GM\left(\frac{r}{R}\right)^2\right)\left(1 - \frac{r^2}{R^2}\right)\frac{1}{R}, \tag{19}$$

$$\rho(r,\,t) = \frac{15M}{8\pi}\left(1 - \frac{r^2}{R^2}\right)\frac{1}{R^3}, \tag{20}$$

where $\kappa' = (15M/8\pi)^\gamma\,\kappa$.

It is noted that the above profiles for the pressure, temperature and density satisfy the mass, momentum and energy conservation equations and the equation of state for ideal gas law. The requirement that the pressure and temperature should be greater than zero everywhere results in the following constraint for κ':

$$\frac{32\pi\kappa'}{15M} \geq GMR^{3\gamma-4}. \tag{21}$$

Such ideal gas model of Newtonian stars provides the upper bound value of Chandrasekhar mass for white dwarfs and the central densities, pressures and temperatures of the stars such as Sun, Jupiter and Saturn [16].

3. Core-collapse Implosion: Early Supernova

With the polytrope assumption at the center of star, the equation for the pressure can be reduced to the following equation of motion for the star:

$$\frac{d^2}{dt^2}R(t) = \frac{32\pi\kappa'}{15M}\frac{1}{R^{3\gamma-2}} - \frac{2GM}{R^2}. \tag{22}$$

Thus, the continuity, Euler, Poisson and energy equations with the polytrope assumption only at the center can be reduced to equation (22), which can be solved using the standard "energy" method if equation (22) is converted into the following form:

$$\frac{1}{2}\dot{R}^2 + V(R) = \varepsilon, \tag{23}$$

where ε is a constant and the potential $V(R)$ is given by equation (24):

$$V(R) = \frac{32\pi\kappa'}{45M(\gamma-1)}\frac{1}{R^{3\gamma-3}} - \frac{2GM}{R}. \tag{24}$$

The solution of equation (22) and hence equation (23) can then be given by the elementary integral shown in equation (25):

$$\int_{R_{\max}}^{R} \frac{dR}{\sqrt{2[\varepsilon - V(R)]}} = \pm t, \tag{25}$$

where + and – correspond to the case of expansion and contraction, respectively.

From equation (24), it is clear that there is a bound state for $\gamma > 4/3$ but there is no stable bound state for $\gamma < 4/3.$ For collapsing star, the potential should be less than zero so that the constant κ' satisfies the following constraint:

$$GMR^{3\gamma-4} \le \frac{32\pi\kappa'}{15M} \le 6(\gamma-1)GMR^{3\gamma-4}. \tag{26}$$

With the upper bound value of $\kappa', V(R_{\max}) = 0,$ the equation of motion for the collapsing star, equation (22) can be rewritten as

$$\frac{d^2}{dt^2} R(t) = -(4 - 3\gamma) \frac{2GM}{R^2}. \tag{27}$$

One can immediately obtain the following solutions for equation (27):

$$t = \frac{\psi + \sin \psi}{2k}, \tag{28}$$

$$R = \frac{1}{2} R_{\max}(1 + \cos \psi), \tag{29}$$

where

$$k = \sqrt{-2\varepsilon(R_{\max})} / R_{\max} \tag{30}$$

and

$$\varepsilon(R_{\max}) = -(4 - 3\gamma) 2GM / R_{\max}. \tag{31}$$

Note that the solutions given in equations (28), (29), (30) and (31) coincide with those of Oppenheimer-Snyder's [17] equations of gravitational collapse. This coincidence stems from their oversimplified assumption of uniform density and zero pressure even though they started with the general relativistic equations. Their general relativistic equation for the equation of motion for star is given by

$$\frac{d^2 R}{dt^2} = -\frac{GM}{R^2}. \tag{32}$$

Comparing equation (32) with equation (22), equation (32) has no inner pressure force acting against the gravitational collapse so that the star governed by equation (32) will collapse infinitely.

4. Supernova Explosions

The collapse of the core continues until the density of the core becomes the nuclear density. At the nuclear density, the core may explode if the inner pressure force term is greater than the gravitational force term in equation (22). This condition may be written as

$$\frac{2GM}{R_{\min}} \le \frac{32\pi\kappa'}{15MR_{\min}^{3\gamma-3}}. \tag{33}$$

With the upper bound value of $2GM/R_{\min}$, the potential energy becomes

$$V(R_{\min}) = \frac{(4-3\gamma)}{3(\gamma-1)} \frac{32\pi\kappa'}{15M} \frac{1}{R_{\min}^{3\gamma-3}} = \frac{(4-3\gamma)}{3(\gamma-1)} \frac{2GM}{R_{\min}}. \tag{34}$$

The potential energy given above is the total energy deposition during the core-collapse [2]. The equation of motion of star corresponding to this potential energy is given by

$$\frac{d^2}{dt^2} R(t) = (4-3\gamma) \frac{32\pi\kappa'}{15M} \frac{1}{R^{3\gamma-2}}. \tag{35}$$

The RHS in equation (33) is always positive if γ is less than $4/3$ so that equation (35) is an equation of motion for an expanding star. Consequently, the potential given in equation (34) may be considered as energy per unit mass, which is needed for the explosion of a protoneutron star.

The solution of equation (35) can be obtained using the aforementioned energy method. The solution for the case of $\gamma = 7/6$, which provides an explicit form of analytical solution for an expanding star, is given by

$$R = R_{\min} \cosh^4 \theta, \tag{36}$$

$$t = \frac{R_{\min}}{\sqrt{2V(R_{\min})}} \left(\frac{\sinh 4\theta}{8} + \sinh 2\theta + \frac{3\theta}{2} \right). \tag{37}$$

The explosion velocity of the protoneutron star can be obtained from equations (36) and (37). That is

$$\frac{dR}{dt} = \sqrt{2V(R_{\min})} \tanh \theta = \sqrt{\frac{(4-3\gamma)}{3(\gamma-1)} \frac{2GM}{R_{\min}}} \tanh \theta. \tag{38}$$

The explosion velocity whose asymptotic limit is $\sqrt{2GM/R_{\min}}$ generates strong outgoing shock.

5. Heat Transport Equation for the Core-collapse Supernova Explosions

The heat flow from a sphere of radius R in a star during the expansion as well as contraction can be obtained from equation (10) with the obtained pressure, temperature and density profiles given in equations (18), (19) and (20), respectively, which is given by

$$-\nabla \cdot \vec{q} = \frac{(4-3\gamma)}{4(\gamma-1)} GM \frac{15M}{8\pi} \left(1 - \frac{r^2}{R^2}\right)^2 \left(\frac{r^2}{R^2}\right) \frac{\dot{R}}{R^5}. \tag{39}$$

The above equation which is consequence of equation (12) vanishes at $r = 0$ as is assumed. During collapsing phase of the star with $\gamma < 4/3,$ the RHS in equation (39) becomes negative so that the heat flows radially outward. Thus, the heat flow from the whole volume of star at given time, which may be transported by radiation, is given by

$$-\int_0^R (\nabla \cdot \vec{q}) 4\pi r^2 dr = 0.048 \frac{(4-3\gamma)}{(\gamma-1)} \frac{GM}{R} \frac{M\dot{R}}{R}. \tag{40}$$

The above equation reveals that the heat flow rate is linearly dependent on the collapsing rate of the star when the core-mass and the specific heat ratio are given. The heat may radiate away by the neutrino flux, which may leave the star without interaction. The heat gain during the expansion phase may occur by the absorption of neutrinos [1].

To solve the heat transport equation for the supernova, given in equation (10), is very hard task [3-8] because one can hardly know the processes inside the supernova during the core-collapsing explosions. However, we can calculate the time rate change of heat flow from the core-collapsing supernova through equation (40).

6. Shock Wave Propagations

The rapidly collapsing of the iron core of the progenitor star may emit shock waves outward and inward simultaneously. The heavier mantle having a specific heat ratio less than $4/3$, just outside the iron core, generates inward shock. On the other hand, the lighter layer of helium and hydrogen generates outgoing shock wave during the implosion. It has been confirmed experimentally and theoretically that the collapsing micro bubble wall with acceleration of $10^{12}\,\mathrm{m/s^2}$ generates outgoing shock waves [18]. An outgoing strong shock wave is also formed due to the explosion of the protoneutron star.

The velocity and the pressure fields in gas medium, caused by the detached shock waves can be obtained using the Kirkwood-Bethe hypothesis. This hypothesis assumes that the invariant quantity Y propagates in the medium with the characteristics velocity $c + u$ [19]:

$$\left[\frac{\partial}{\partial t} + (c+u)\frac{\partial}{\partial r}\right]Y = 0. \tag{41}$$

The invariant Y is defined as follows:

$$Y = r\left(h + \frac{u^2}{2}\right) = R\left(H + \frac{U^2}{2}\right), \tag{42}$$

where H and U are the enthalpy and velocity values at the wall of the collapsing star.

A relationship between enthalpy to the local sound velocity and pressure is obtained by the following relationship for the isentropic relation:

$$h = \int_{p_\infty}^{p} \frac{dp}{\rho}. \tag{43}$$

Thus, the continuity equation for a spherical coordinate can be written as

$$-\frac{1}{c^2}\frac{Dh}{Dt} = \frac{\partial u}{\partial r} + \frac{2u}{r}. \tag{44}$$

The shock characteristics can be obtained using equation (42) and the continuity equation for fluid [19]. These characteristics for gas medium [11] are given as

$$\left(\frac{dr}{dt}\right)_{char} = c + u, \tag{45}$$

$$\left(\frac{du}{dt}\right)_{char} = \frac{1}{c-u}\left[(c+u)\frac{Y}{r^2} - \frac{2c^2u}{r}\right], \tag{46}$$

$$\left(\frac{dp}{dt}\right)_{char} = \frac{\rho_\infty}{r(c-u)}\left(\frac{p}{p_\infty}\right)^{1/\gamma_m}\left[2c^2u^2 - \frac{c(c+u)}{r}Y\right]. \tag{47}$$

The values of Y, which can be obtained from the instantaneous motion of the core-collapse supernova are necessary to solve the aforementioned shock characteristics.

In gas medium, the compression during the development of the shock wave was assumed to be adiabatic, that is, the following relation holds:

$$p\rho^{\gamma_m} = const. \tag{48}$$

Sound velocity C and the enthalpy H at the star wall are given as follows [11]:

$$C = c_\infty\left(\frac{P}{p_\infty}\right)^{\frac{\gamma_m - 1}{2\gamma_m}}, \tag{49}$$

$$H = \frac{\gamma_m}{\gamma_m - 1}\frac{p_\infty}{\rho_\infty}\left[\left(\frac{P}{p_\infty}\right)^{\frac{\gamma_m - 1}{\gamma_m}} - 1\right], \tag{50}$$

where γ_m is the specific heat ratio and c_∞ is the sound speed for the gas medium of the shell layer outside the core-mass.

The mass flow rate accompanying the shock may be calculated by the following equation:

$$\dot{m} = 4\pi r^2 \rho_\infty \left(\frac{dr}{dt}\right)_{char}. \tag{51}$$

7. Calculation Results and Discussion

Figure 2 shows the time dependent radius and the collapse velocity of iron core having a mass of $1.5M_{\odot}$. In this calculation, initial radius and the specific heat ratio for the iron core were taken as 3000km [1] and 1.2, respectively, and the density and temperature of the medium outside the core were taken as $10^6 \mathrm{kg/m^3}$ and $10^6 \mathrm{K}$, respectively. The central density of $10^{10} \mathrm{kg/m^3}$ and central temperature of $6\times10^{10}\mathrm{K}$ (~509keV) at which Fe-He transition can occur were taken for calculation. The initial pressure estimated by the Wheeler equation of state for the progenitor star is about $10^{23}\mathrm{Pa}$ [2]. However, an initial pressure value of $1.2\times10^{24}\mathrm{Pa}$ for the star was taken to satisfy the inequality given in equation (26) in this study.

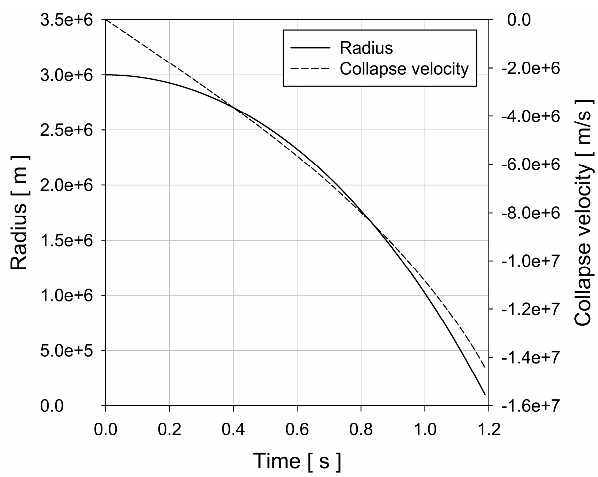

Figure 2. Time dependent radius and collapse velocity of iron core.

If the density of the mantle layer outside the iron core is less than $0.00001\mathrm{kg/m^3}$ and the specific heat ratio of the medium is greater than 1.6, then there is no accretion flow to the iron core. Time dependent radius and collapse velocity of the iron core for the case without the accretion flow are almost same as those for the case with the accretion flow as shown in

Figure 2. It takes 1.2s for the iron core to collapse to a protoneutron star having radius of 108km at which the collapse velocity reaches –14300km/s as shown in Figure 2. However, the absolute magnitude of the peak velocity is less than those (~50000km/s) obtained by various numerical models [6].

For the case without accretion flow, the particle velocity in the shock drops rapidly near the point where the velocity has its minimum value, as shown in Figure 3. This point may be taken as the turning point at which bounce of the protoneutron star starts at which the radius of star is about 108km. For the case of lower density and higher specific heat ratio for the medium gas, the outgoing shock which blows materials in the shell layer of the progenitor star into space propagates with velocity of 60000km/s as shown in Figure 3. In fact, the shock propagation which blows the materials in the shell layer into the space during the implosion of the iron core was observed for the case of Cassiopeia A as shown in Figure 1(a).

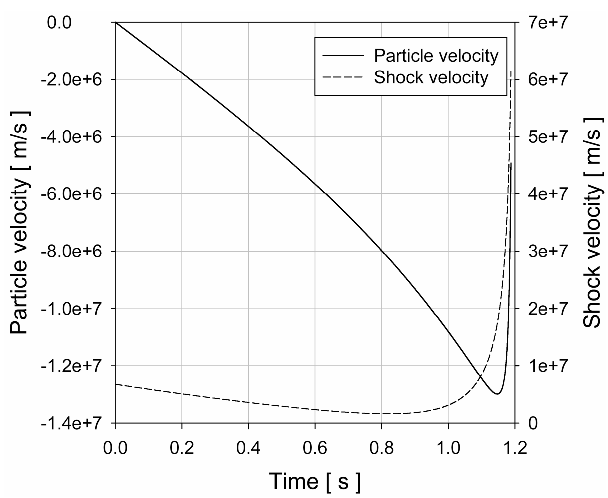

Figure 3. Time dependent particle and shock velocities obtained for the case without the accretion mass flow.

Table 1. Collapse time and heat from the star depending on the specific heat ratio with initial iron core-radius of 3000km. The center properties are obtained at the radius of 108km for the protoneutron star. The numerical values in parentheses are the case with the accretion mass flow

Specific heat ratio (γ)	Collapse time (s)	Heat from star (joules)	Center density (kg/m^3)	Center pressure (Pa)	Center temperature (K)
1.1	1.05	2.60×10^{45}	3.68×10^{16}	5.57×10^{28}	1.04×10^{10}
1.2	1.20	5.89×10^{44}	3.86×10^{16}	1.60×10^{29}	2.83×10^{10}
	(1.20)	(6.21×10^{44})	(4.52×10^{16})	(1.93×10^{29})	(2.83×10^{10})
1.3	1.59	7.52×10^{43}	4.06×10^{16}	4.63×10^{29}	7.79×10^{10}

In Table 1, the collapse time and the heat from the iron core during the collapse for several specific heat ratios of the iron core are shown for the cases without the accretion flow. The collapse time and the heat flow are dependent on the specific heat ratio of the iron core. Shorter collapse time and much heat flow from the star are obtained for smaller value of the specific heat ratio. At a radius of 108km of protoneutron star, the central density, temperature and pressure are shown in Table 1. The central density and pressure of the iron core increase by 4~6 orders of magnitude during the collapse. The maximum density obtained at bounce is $3.6\times10^{17}\,\mathrm{kg/cm^3}$ with a soft nuclear equation of state and is $2.7\times10^{16}\,\mathrm{kg/cm^3}$ with a hard equation of state in one-dimensional calculation [20]. While our calculation result with ideal equation of state for the gas, which is about $3\times10^{16}\,\mathrm{kg/cm^3}$ as shown in Table 1, is lower by one order of magnitude. On the other hand, the pressure, temperature and density profiles inside the iron core at the post bounce are uniform although numerical results [6] show several orders of magnitude differences in density between the center and the surface of the iron core.

The temperature slightly decreases or increases depending on the specific heat ratio value, which may be due to the heat flow from the iron core. The maximum temperatures obtained in this study, which are order of $10^{10}\,\mathrm{K}$ are lower than those obtained by various numerical models $(10^{11}\mathrm{K})$ by an order of magnitude [6]. For the case with accretion flow, the central density, temperature and pressure are the same as those obtained for the case without the accretion flow as shown in parentheses in Table 1. Certainly, the accretion flow exists during the implosion of the iron core due to the heavy mantle layer of iron core.

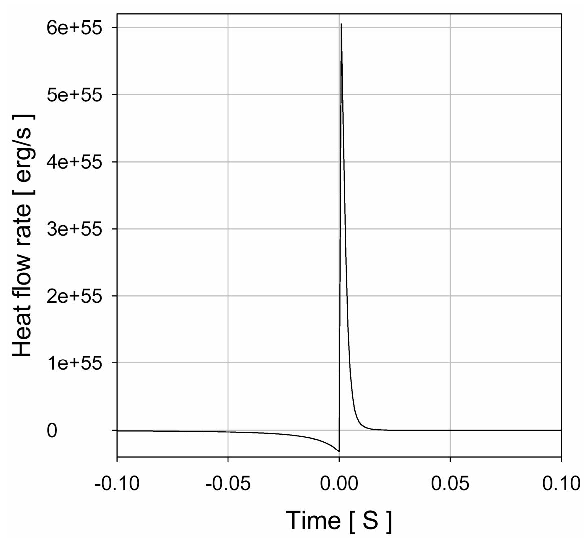

Figure 4. Heat flow rate during collapsing (left) and explosion (right) phase of iron core.

The calculated total amount of heat flow from the iron core is as much as $5.9 \times 10^{51}\mathrm{ergs}$ during the collapse period of 1.2s. At the final stage of the collapse, the calculate heat flow rate from the star is about $3.6 \times 10^{52}\mathrm{ergs/s}$. This magnitude is higher than the result obtained by Liebendoerfer et al. [5] by one order of magnitude. The amount of the heat flow rate from the star is almost equal to the neutrino energy issued from the collapse, which is about $10^{51}\mathrm{ergs}$ [8]. As can be seen in Figure 4, the calculated energy input rate to the star at earlier post-bounce is approximately $6.1 \times 10^{53}\mathrm{ergs/s}$, which corresponds to the energy of electron-type neutrinos emerged during the expansion phase [8]. This value obtained by our calculation is higher than those obtained by numerical calculation by an order of magnitude [8]. The calculated total amount of energy input during the expansion phase is about 10^{52} ergs. The heat flow into the expanding protoneutron star may occur by the absorption of neutrinos [1]. Without any detailed information for the heat flow mechanism due to neutrino transport, the hydrodynamic equations including the energy equation predict the magnitude of the heat flow rate during the evolution of star remarkably.

One can imagine that the heavy mantle outside the iron core moves inwardly during the implosion of the iron core. The time dependent accretion shock velocity and the corresponding mass flow rate are shown in Figure 5. The accretion shock moves outward at first and propagates inwardly at 35ms after the implosion. The absolute magnitude of the accretion mass flow rate which is certainly dependent on the medium density has its maximum at 595ms after the implosion and reduces to 1.8×10^{23} kg/s at 1200ms after the implosion. The time dependent velocity of the accretion shock is found to be always greater than the collapse velocity of the iron core as can be confirmed from Figures 2 and 5, which indicates that the accretion shock just follows the movement of the collapsing iron core.

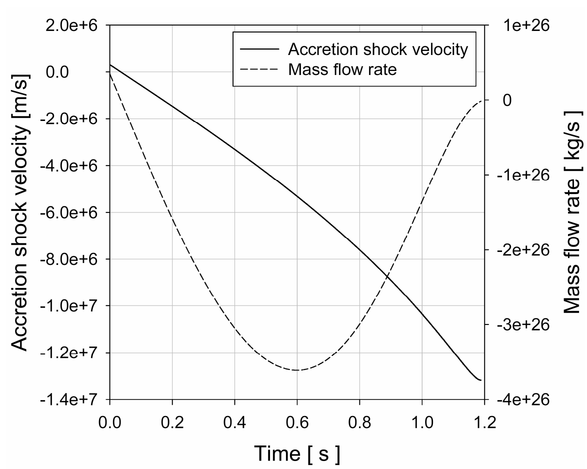

Figure 5. Accretion shock velocity and the corresponding mass flow rate.

With a specific heat value of $7/6$ and the minimum radius of protoneutron star, 108km, the asymptotic explosion velocity calculated from equation (38) is about 60400km/s provided that the core-mass of $1.5M_{\odot}$ during infall is fully involved in explosion process. The corresponding energy needed for the explosion is calculated to be about 1.1×10^{53}ergs. Equation (38) indicates that the magnitude of the explosion velocity depends on the core-mass involved in the explosion process. In fact, a larger trapped lepton fraction during infall results in a larger inner core-mass, which, in turn, produces a stronger bounce shock [21].

Time dependent radius and the explosion velocity of the protoneutron star are given in Figure 6. The velocity reaches its asymptotic limit of 60400km/s after 400ms and the star expands continuously because the heat flow into the star during expansion as can be confirmed from equation (39). The shock speed after the post explosion, which is slightly less than the explosion velocity as can be seen in Figure 6 is comparable to the shock speed of 30000km/s obtained by three-dimensional numerical simulation [8] if only 25% $(0.37M_{\odot})$ of the core-mass during infall $(1.5M_{\odot})$ is involved in the explosion process. It is noted that a mass less than 38% $(0.58M_{\odot})$ of the infalling mass should be involved in the explosion to satisfy the condition given in equation (33) if one uses the center properties given in Table 1.

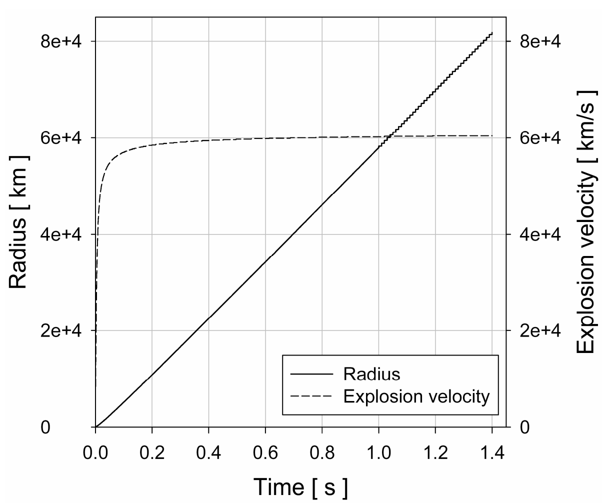

Figure 6. Time dependent radius and explosion velocity of the protoneutron star.

In Figure 7, the outward mass flow rates accompanying the shock after the explosion are also shown. It takes about 200~300ms for the outward mass flow rate to compete with the mass accretion rate of $1.8 \times 10^{23}\,\mathrm{kg/s}$ at the end of the collapse. The outgoing shock may stall during this stage [2]. Recent numerical simulations indicate that three-dimensional core explodes approximately at 250ms after the bounce [8]. The interaction between the outgoing shock and the accretion shock needs another extensive study.

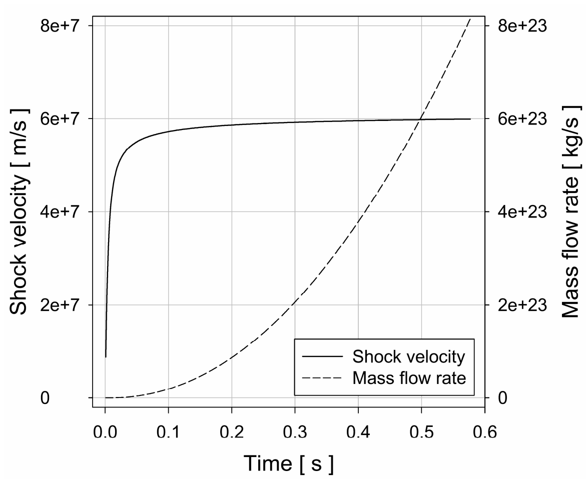

Figure 7. Outgoing shock velocity due to the explosion of the protoneutron star and mass flow rate accompanying the shock.

8. Conclusions

A simple hydrodynamic theory on the core-collapse supernovae by taking the star as an ideal gas system was present in this study. The core-collapsing velocity and the subsequent explosion velocity of the protoneutron star were also obtained. Explicit form of the heat transport from the collapsing star and the expanding star was obtained hydrodynamically. The heat flow from the iron core of $10^{51}\,\mathrm{ergs}$ during the collapse, which is related to the neutrinos issued from the collapse, and the heat flow of $10^{52}\,\mathrm{ergs}$ into the expanding star, which is related to the electron-type neutrinos emerged during the expansion phase, was obtained. These results suggest that some results obtained from the analytical hydrodynamics study for the core-collapse supernova may be interpretable by micro physics. *Our results indicate that hydrodynamic equations could reveal the core-collapse supernova explosion remarkably without any detailed ingredients*. *Also*, *our study reveals that the heat transfer across the system boundary determines the evolution of the ideal gas system*. *On the other hand*, *one can calculate the heat transfer rate through the system boundary if one knows the evolution of the system* [14, 23].

References

[1] A. Burrows, Supernova explosions in the universe, Nature 403 (2000), 727-733.

[2] S. A. Colgate and R. H. White, The hydrodynamics behavior of supernova explosions, Astrophys. J. 143 (1966), 626-681.

[3] A. Burrows, J. Hays and B. A. Fryxell, On the nature of core-collapse supernova explosions, Astrophys. J. 450 (1995), 830-850.

[4] E. Livne, A. Burrows, R. Walder, I. Lichtenstadt and T. A. Thompson, Two-dimensional, time-dependent, multigroup, multiangle radiation hydrodynamics test simulation in the core-collapse supernova context, Astrophys. J. 609 (2004), 277-287.

[5] M. Liebendoerfer, M. Rampp, H.-Th. Janka and A. Mezzacappa, Supernova simulations with Boltzmann neutrino transport: a comparison of methods, Astrophys. J. 620 (2005), 840-860.

[6] R. Buras, M. Rampp, H.-Th. Janka and K. Kifonidis, Two-dimensional hydrodynamics core-collapse supernova simulations with spectral neutrino transport, I. Numerical method and results for a $15M_{\odot}$ star, Astron. Asrophys. 447 (2006), 1049-1092.

[7] J. W. Murphy and A. Burrows, Criteria for core-collapse supernova explosions by the neutrino mechanism, Astrophys. J. 688 (2008), 1159-1175.

[8] J. Nordhaus, A. Burrows, A. Almgren and J. Bell, Dimension as a key to the neutrino mechanism of core-collapse supernova explosions, Astrophys. J. 720 (2010), 694-703.

[9] M. Rampp and H.-Th. Janka, Radiation hydrodynamics with neutrinos: variable Eddington factor method for core-collapse supernova simulations, Astron. Astrophys. 396 (2002), 361-392.

[10] H. Shen, H. Toki, K. Oyamatsu and K. Sumiyoshi, Relativistic equation of nuclear matter for supernova and neutron star, Nucl. Phys. A 637 (1998), 435-450.

[11] H. Kwak, I. Ko, K. Kang and J. Kang, Fire-ball expansion and subsequent shock wave propagation from explosives detonation, Int. J. Therm. Sci. 95 (2012), 9-16.

[12] J. Jun and H. Kwak, Gravitational collapse of Newtonian stars, Internat. J. Modern Phys. D 9 (2000), 35-42.

[13] H. Kwak and H. Yang, An aspect of sonoluminescence from hydrodynamic theory, J. Phys. Soc. Japan 64 (1995), 1980-1992.

[14] H. Kwak and J. Na, Hydrodynamic solutions for a sonoluminescing gas bubble, Phys. Rev. Lett. 77 (1996), 4454-4457.

[15] H. Kwak, S.-D. Oh and C.-H. Park, Bubble dynamics on the evolving bubble formed from the droplet at the superheat limit, Int. J. Heat Mass Trans. 38 (1995), 1709-1718.

[16] H. Kwak and J.-H. Jun, Hydrodynamics and thermodynamics of Newtonian stars, Geophys. Astrophys. Fluid Dyn. 97 (2003), 45-58.

[17] J. R. Oppenheimer and H. Snyder, On continued gravitational contraction, Phys. Rev. 56 (1939), 455-459.

[18] Y. Lee, S. W. Karng, J. Jeon and H. Kwak, Shock pulse from a sonoluminescing gas bubble, J. Phys. Soc. Japan 66 (1997), 2537-2540.

[19] R. T. Knapp, J. W. Daily and F. G. Hammitt, Cavitation, McGraw-Hill, New York, 1970.

[20] A. Marek and H.-Th. Janka, Delayed neutrino-driven supernova explosions aided by the standing accretion-shock instability, Astrophys. J. 694 (2009), 664-696.

[21] S. W. Bruenn, The prompt-shock supernova mechanism, I. The effect of the free-proton mass fraction and the neutrino transport algorithm, Astrophys. J. 340 (1989), 955-965.

[22] 3D supernova explosion animation by R. A. Fesen, Dartmouth College, USA and J. Long, ESA/Hubble.

[23] K. Y. Kim, K.-T. Byun and H. Kwak, Characteristics of sonoluminescing bubbles in aqueous solutions of sulfuric acid, J. Phys. Soc. Japan 75 (2006), 114705-1-114705-7.