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INTRODUCTION TO MULTI-FUZZY SOFT TOPOLOGICAL SPACES

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Abstract

The aim of this paper is to introduce and study the concept of multi-fuzzy soft topological space and in the meantime, some of its structural properties such as neighbourhood of a multi-fuzzy soft set, interior multi-fuzzy soft set, multi-fuzzy soft basis, multi-fuzzy soft subspace topology are studied. Also, we introduced multi-fuzzy soft cover, multi-fuzzy soft open cover, multi-fuzzy soft compactness which are defined and presented several results on it.

1. Introduction

Algebraic structures play a prominent role in the mathematics with wide range of applications in many disciplines such as theoretical physics, computer science, coding theory, topological spaces, etc. This provides

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sufficient motivation to the researchers to review various concepts and results from the area of abstract algebra in the broader framework of fuzzy setting. One of the structures which is most extensively discussed in the mathematics and its applications is lattice theory.

In real life situation, the problems in economics, engineering, social sciences, medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e., which we can use as mathematical tools for dealings with uncertainties. But all these theories have their inherent difficulties. The reason for these difficulties Molodtsov initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties.

Maji et al. [13] gave first practical application of soft sets in decision making problems. They have also introduced the concept of fuzzy soft set, a more generalized concept, which is a combination of fuzzy set and soft set and also studied some of its properties.

A new type of fuzzy set (multi-fuzzy set) was introduced in a paper of Sebastian and Ramakrishnan [17] by using the ordered sequences of membership function. The notion of multi-fuzzy sets provides a new method to represent some problems which are difficult to explain in other extensions of fuzzy set theory, such as color of pixels.

The notion of multi-fuzzy complex numbers and multi-fuzzy complex sets are introduced first time in the work of Dey and Pal [6]. Using these concepts Dey and Pal [7] introduced multi-fuzzy complex nilpotent matrices over a distributive lattice. Recently, Yong et al. [20] proposed the concept of the multi-fuzzy soft set which is a more general fuzzy soft set and gave its application in decision making.

The purpose of our paper is to introduce the multi-fuzzy soft topology, from which we can obtain a new notion for modern analysis. To facilitate our discussion, we first review some background on soft set, fuzzy soft set, multi-fuzzy set and multi-fuzzy soft set in Section 2.

In Section 3, the concept of multi-fuzzy soft topological space is introduced and some of its structural properties such as neighbourhood of a multi-fuzzy soft set, interior multi-fuzzy soft set, multi-fuzzy soft basis, multi-fuzzy soft subspace topology are studied. In Section 4, we introduced multi-fuzzy soft cover, multi-fuzzy soft open cover, multi-fuzzy soft compactness. Finally, some conclusions are pointed out in Section 5.

2. Preliminaries

Throughout this paper, U refers to an initial universal set, E is a set of parameters, $P(U)$ is the power set of U and $A \subseteq E$.

Definition 1 [15] (Soft sets)**.** A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a mapping from parameters to $P(U)$, and it is not a set, but a parameterized family of subsets of U.

Example 1. Let $U = \{b_1, b_2, b_3, b_4, b_5\}$ be a set of bikes under consideration. Let $A = \{e_1, e_2, e_3\}$ be a set of parameters, where $e_1 =$ expensive, $e_2 =$ beautiful and $e_3 =$ good milage. Suppose that $F(e_1) = \{b_2, b_4\}$, $F(e_2) = \{b_1, b_4, b_5\}$, $F(e_3) = \{b_1, b_3\}$. The soft set (F, A) describes the "attractiveness of the bikes". $F(e_1)$ means "bikes (expensive)" whose function value is the set $\{b_2, b_4\}$, $F(e_2)$ means "bikes (beautiful)" whose function value is the set $\{b_1, b_4, b_5\}$ and $F(e_3)$ means "bikes (good milage)" whose function value is the set $\{b_1, b_3\}$.

Definition 2 [12] (Fuzzy soft sets)**.** Let $\widetilde{P}(U)$ be all fuzzy subsets of U. A pair $(\widetilde{F}, A)$ is called *fuzzy soft set* over U, where $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \to \widetilde{P}(U)$.

Example 2. Consider Example 1.

The fuzzy soft set $(\tilde{F}, A)$ can describe the "attractiveness of the bikes" under the fuzzy circumstances.

$$\tilde{F}(e_1) = \{b_1/0.3, b_2/0.8, b_3/0.4, b_4/0.7, b_5/0.5\},$$

$$\tilde{F}(e_2) = \{b_1/0.7, b_2/0.2, b_3/0.4, b_4/0.8, b_5/0.9\},$$

$$\tilde{F}(e_3) = \{b_1/0.6, b_2/0.4, b_3/0.7, b_4/0.2, b_5/0.1\}.$$

Definition 3 [17] (Multi-fuzzy sets)**.** Let k be a positive integer. A multi-fuzzy set $\tilde{A}$ in U is a set of ordered sequences $\tilde{A} = \{u/(\mu_1(u), \mu_2(u), ..., \mu_k(u)) : u \in U\}$, where $\mu_i \in \tilde{P}(U)$, $i = 1, 2, ..., k$.

The function $\mu_{\tilde{A}} = (\mu_1(u), \mu_2(u), ..., \mu_k(u))$ is called the *multi-membership function* of multi-fuzzy set $\tilde{A}$, k is called *dimension* of $\tilde{A}$. The set of all multi-fuzzy sets of dimension k in U is denoted by $M^k FS(U)$.

Note 1. A multi-fuzzy set of dimension 1 is a Zadeh's fuzzy set, and a multi-fuzzy set of dimension 2 with $\mu_1(u) + \mu_2(u) \leq 1$ is an Atanassov's intuitionistic fuzzy set.

Note 2. If $\sum_{i=1}^{k} \mu_i(u) \leq 1$, for all $u \in U$, then the multi-fuzzy set of dimension k is called *normalized multi-fuzzy set*. If $\sum_{i=1}^{k} \mu_i(u) = l > 1$ for some $u \in U$, we redefine the multi-membership degree $(\mu_1(u), \mu_2(u), ..., \mu_k(u))$ as $\frac{1}{l}(\mu_1(u), \mu_2(u), ..., \mu_k(u))$, then the non-normalized multi-fuzzy set can be changed into a normalized multi-fuzzy set.

Definition 4 [17]**.** Let $\tilde{A} \in M^k FS(U)$. If $\tilde{A} = \{u/(0, 0, ..., 0) : u \in U\}$, then $\tilde{A}$ is called the *null multi-fuzzy set* of dimension k, denoted by $\tilde{\Phi}_k$. If

$\widetilde{A} = \{u/(1, 1, ..., 1) : u \in U\}$, then $\widetilde{A}$ is called the *absolute multi-fuzzy set* of dimension k, denoted by $\widetilde{1}_k$.

Example 3. Suppose a color image is approximated by an $m \times n$ matrix of pixels. Let U be the set of all pixels of the color image. For any pixel u in U, the membership values $\mu_r(u), \mu_g(u), \mu_b(u)$ being the normalized red value, green value and blue value of the pixel u respectively. So the color image can be approximated by the collection of pixels with the multi-membership function $(\mu_r(u), \mu_g(u), \mu_b(u))$ and it can be represented as a multi-fuzzy set $\widetilde{A} = \{u/(\mu_r(u), \mu_g(u), \mu_b(u)) : u \in U\}$. In a two dimensional image, color of pixels cannot be characterized by a membership function of an ordinary fuzzy set, but it can be characterized by a three dimensional membership function $(\mu_r(u), \mu_g(u), \mu_b(u))$. In fact, a multi-fuzzy set can be understood to be a more general fuzzy set using ordinary fuzzy sets as its building blocks.

Definition 5 [17]**.** Let $\widetilde{A} = \{u/(\mu_1(u), \mu_2(u), ..., \mu_k(u)) : u \in U\}$ and $\widetilde{B} = \{u/(\nu_1(u), \nu_2(u), ..., \nu_k(u)) : u \in U\}$ be two multi-fuzzy sets of dimension k in U. We define the following relations and operations:

(1) $\widetilde{A} \sqsubseteq \widetilde{B}$ if and only if $\mu_i(u) \leq \nu_i(u), \forall u \in U$ and $1 \leq i \leq k$.

(2) $\widetilde{A} = \widetilde{B}$ if and only if $\mu_i(u) = \nu_i(u), \forall u \in U$ and $1 \leq i \leq k$.

(3) $\widetilde{A} \sqcup \widetilde{B} = \{u/(\mu_1(u) \vee \nu_1(u), \mu_2(u) \vee \nu_2(u), ..., \mu_k(u) \vee \nu_k(u)) : u \in U\}$.

(4) $\widetilde{A} \sqcap \widetilde{B} = \{u/(\mu_1(u) \wedge \nu_1(u), \mu_2(u) \wedge \nu_2(u), ..., \mu_k(u) \wedge \nu_k(u)) : u \in U\}$.

(5) $\widetilde{A}^c = \{u/(\mu_1^c(u), \mu_2^c(u), ..., \mu_k^c(u)) : u \in U\}$.

Definition 6 [20] (Multi-fuzzy soft sets)**.** A pair $(\widetilde{F}, A)$ is called a *multi-fuzzy soft set* of dimension k over U, where $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \to M^k FS(U)$.

A multi-fuzzy soft set is a mapping from parameters to $M^k FS(U)$. It is a parameterized family of multi-fuzzy subsets of U. For $e \in A$, $\tilde{F}(e)$ may be considered as the set of e-approximate elements of the multi-fuzzy soft set $(\tilde{F}, A)$.

Example 4. Suppose that $U = \{c_1, c_2, c_3, c_4, c_5\}$ is the set of cell phones under consideration, $A = \{e_1, e_2, e_3\}$ is the set of parameters, where e_1 stands for the parameter color which consists of red, green and blue, e_2 stands for the parameter ingredient which is made from plastic, liquid crystal and metal, and e_3 stands for the parameter price which can be various: high, medium and low. We define a multi-fuzzy soft set of dimension 3 as follows:

$$\tilde{F}(e_1) = \{c_1/(0.4, 0.2, 0.3), c_2/(0.2, 0.1, 0.6), \\ c_3/(0.1, 0.3, 0.4), c_4/(0.3, 0.1, 0.3), c_5/(0.7, 0.1, 0.2)\},$$

$$\tilde{F}(e_2) = \{c_1/(0.1, 0.2, 0.6), c_2/(0.3, 0.2, 0.4), \\ c_3/(0.5, 0.3, 0.1), c_4/(0.6, 0.1, 0.3), c_5/(0.6, 0.2, 0.1)\},$$

$$\tilde{F}(e_3) = \{c_1/(0.3, 0.4, 0.1), c_2/(0.4, 0.1, 0.2), \\ c_3/(0.2, 0.2, 0.5), c_4/(0.7, 0.1, 0.2), c_5/(0.5, 0.2, 0.3)\}.$$

Definition 7 [20]**.** Let $A, B \subseteq E$. Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two multi-fuzzy soft sets of dimension k over U. $(\tilde{F}, A)$ is said to be a *multi-fuzzy soft subset* of $(\tilde{G}, B)$ if

(1) $A \subseteq B$, and

(2) $\tilde{F}(e) \sqsubseteq \tilde{G}(e)$ for all $e \in A$.

In this case, we write $(\tilde{F}, A) \tilde{\sqsubseteq} (\tilde{G}, B)$.

Definition 8 [20]**.** Let $A, B \subseteq E$. Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two multi-fuzzy soft sets of dimension k over U. $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are said to be a *multi-fuzzy soft equal* if $(\tilde{F}, A)$ is a multi-fuzzy soft subset of $(\tilde{G}, B)$ and $(\tilde{G}, B)$ is a multi-fuzzy soft subset of $(\tilde{F}, A)$.

In this case, we write $(\tilde{F}, A) \cong (\tilde{G}, B)$.

Definition 9 [20] (Null multi-fuzzy soft sets)**.** A multi-fuzzy soft set $(\tilde{F}, A)$ of dimension k over U is said to be *null multi-fuzzy soft set*, denoted by $\tilde{\Phi}_A^k$ if $\tilde{F}(e) = \tilde{\Phi}_k$ for all $e \in A$.

Definition 10 [20] (Absolute multi-fuzzy soft sets)**.** A multi-fuzzy soft set $(\tilde{F}, A)$ of dimension k over U is said to be *absolute multi-fuzzy soft set*, denoted by $\tilde{U}_A^k$ if $\tilde{F}(e) = \tilde{1}_k$ for all $e \in A$.

Definition 11 [20]**.** The complement of a multi-fuzzy soft set $(\tilde{F}, A)$ of dimension k over U is denoted by $(\tilde{F}, A)^c$ and is defined by $(\tilde{F}, A)^c = (\tilde{F}^c, A)$, where $\tilde{F}^c : A \to M^k FS(U)$ is a mapping given by $\tilde{F}^c(e) = (\tilde{F}(e))^c$ for all $e \in A$. Clearly, $(\tilde{F}^c)^c$ is the same as $\tilde{F}$ and $((\tilde{F}, A)^c)^c = (\tilde{F}, A)$. Suppose $\tilde{\Phi}_A^k$ and $\tilde{U}_A^k$ are multi-fuzzy soft sets of dimension k over U, then $(\tilde{\Phi}_A^k)^c = \tilde{U}_A^k$ and $(\tilde{U}_A^k)^c = \tilde{\Phi}_A^k$.

Example 5. Consider Example 4, we have $(\tilde{F}, A)^c$ as follows:

$$\tilde{F}^c(e_1) = \{c_1/(0.6, 0.8, 0.7), c_2/(0.8, 0.9, 0.4),$$
$$c_3/(0.9, 0.7, 0.6), c_4/(0.7, 0.9, 0.7), c_5/(0.3, 0.9, 0.8)\},$$

$$\tilde{F}^c(e_2) = \{c_1/(0.9, 0.8, 0.4), c_2/(0.7, 0.8, 0.6),$$
$$c_3/(0.5, 0.7, 0.9), c_4/(0.4, 0.9, 0.7), c_5/(0.4, 0.8, 0.9)\},$$

$$\tilde{F}^c(e_3) = \{c_1/(0.7, 0.6, 0.9), c_2/(0.6, 0.9, 0.8),$$
$$c_3/(0.8, 0.8, 0.5), c_4/(0.3, 0.9, 0.8), c_5/(0.5, 0.8, 0.7)\}.$$

By the suggestions given by Molodtsov in [15], we present the notion of AND and OR operations on two multi-fuzzy soft sets as follows.

Definition 12 [20]**.** If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two multi-fuzzy soft sets of dimension *k* over *U*, the $(\tilde{F}, A)$ AND $(\tilde{G}, B)$, denoted by $(\tilde{F}, A) \wedge (\tilde{G}, B)$ is defined by $(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B)$, where $\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \sqcap \tilde{G}(\beta)$, for all $\alpha, \beta \in A \times B$.

Definition 13 [20]**.** If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two multi-fuzzy soft sets of dimension *k* over *U*, the $(\tilde{F}, A)$ OR $(\tilde{G}, B)$, denoted by $(\tilde{F}, A) \vee (\tilde{G}, B)$ is defined by $(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{O}, A \times B)$, where $\tilde{O}(\alpha, \beta) = \tilde{F}(\alpha) \sqcup \tilde{G}(\beta)$, for all $\alpha, \beta \in A \times B$.

Theorem 1 [20]**.** *Let* $(\tilde{F}, A)$ *and* $(\tilde{G}, B)$ *be two multi-fuzzy soft sets of dimension k over U. Then*

(1) $((\tilde{F}, A) \wedge (\tilde{G}, B))^c = ((\tilde{F}, A))^c \vee ((\tilde{G}, B))^c$.

(2) $((\tilde{F}, A) \vee (\tilde{G}, B))^c = ((\tilde{F}, A))^c \wedge ((\tilde{G}, B))^c$.

Definition 14 [20]**.** The union of two multi-fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ of dimension *k* over *U* is the multi-fuzzy soft set $(\tilde{H}, C)$, where $C = A \bigcup B$, and $\forall e \in C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e), & \text{if } e \in A - B, \\ \tilde{G}(e), & \text{if } e \in B - A, \\ \tilde{F}(e) \sqcup \tilde{G}(e), & \text{if } e \in A \bigcap B. \end{cases}$$

Definition 15 [20]**.** The intersection of two multi-fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ of dimension *k* over *U* with $A \bigcap B \neq \varnothing$ is the multi-fuzzy soft set $(\tilde{H}, C)$, where $C = A \bigcap B$, and $\forall e \in C$, $\tilde{H}(e) = \tilde{F}(e) \sqcap \tilde{G}(e)$.

We write $(\tilde{F}, A) \sqcap (\tilde{G}, B) = (\tilde{H}, C)$.

Theorem 2 [20]**.** *Let* $(\tilde{F}, A)$ *and* $(\tilde{G}, B)$ *be two multi-fuzzy soft sets of dimension k over U. Then*

(1) $(\tilde{F}, A)\tilde{\sqcup}(\tilde{F}, A) = (\tilde{F}, A)$,

(2) $(\tilde{F}, A)\tilde{\sqcap}(\tilde{F}, A) = (\tilde{F}, A)$,

(3) $(\tilde{F}, A)\tilde{\sqcup}\tilde{\Phi}_A^k = (\tilde{F}, A)$,

(4) $(\tilde{F}, A)\tilde{\sqcap}\tilde{\Phi}_A^k = \tilde{\Phi}_A^k$,

(5) $(\tilde{F}, A)\tilde{\sqcup}\tilde{U}_A^k = \tilde{U}_A^k$,

(6) $(\tilde{F}, A)\tilde{\sqcap}\tilde{U}_A^k = (\tilde{F}, A)$,

(7) $(\tilde{F}, A)\tilde{\sqcup}(\tilde{G}, B) = (\tilde{G}, B)\tilde{\sqcup}(\tilde{F}, A)$,

(8) $(\tilde{F}, A)\tilde{\sqcap}(\tilde{G}, B) = (\tilde{G}, B)\tilde{\sqcap}(\tilde{F}, A)$.

3. Multi-fuzzy Soft Topology

Now we are ready to define the concept of multi-fuzzy soft topology. Let U be an initial universe, E be the set of parameters, $\mathcal{P}(U)$ be the set of all subsets of U and $\tilde{\mathcal{F}}^k(U; E)$ be the family of all multi-fuzzy soft sets over U via parameters in E.

Definition 16. Let $(\tilde{F}, X)$ be an element of $\tilde{\mathcal{F}}^k(U; E)$, $\mathcal{P}(\tilde{F}, X)$ be the set of all multi-fuzzy soft subsets of $(\tilde{F}, X)$ and $\tilde{\tau}$ be a subfamily of $\mathcal{P}(\tilde{F}, X)$. Then $\tilde{\tau}$ is called *multi-fuzzy soft topology* on $(\tilde{F}, X)$ if the following conditions are satisfied:

(1) $\tilde{\Phi}_A^k, (\tilde{F}, X) \in \tilde{\tau}$,

(2) $(\tilde{G}_1, A), (\tilde{G}_2, B) \in \tilde{\tau} \Rightarrow (\tilde{G}_1, A)\tilde{\sqcap}(\tilde{G}_2, B) \in \tilde{\tau}$,

(3) $\{(\tilde{G}_k, A_k) : k \in K\} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigsqcup}_{k \in K}(\tilde{G}_k, A_k) \in \tilde{\tau}$.

The pair $(X_{\tilde{F}}, \tilde{\tau})$ is called a *multi-fuzzy soft topological space*. Each member of $\tilde{\tau}$ is called $\tilde{\tau}$*-open multi-fuzzy soft set*. A multi-fuzzy soft set is called $\tilde{\tau}$*-closed* iff its complement is $\tilde{\tau}$-open. We shall call a $\tilde{\tau}$*-open* $(\tilde{\tau}$*-closed*$)$ *multi-fuzzy soft set simply open* (*closed*) *set*.

Here, $\{\tilde{\Phi}_A^k, (\tilde{F}, X)\}$ and $\mathcal{P}(\tilde{F}, X)$ are two examples for the multi-fuzzy soft topology on $(\tilde{F}, X)$ and shall call indiscrete multi-fuzzy soft topology and discrete multi-fuzzy soft topology, respectively as called in point set topology.

Example 6. Let $(\tilde{F}, X)$ be as $(\tilde{F}, A)$ in Example 4. Then the subfamily

$$\tau = \{\tilde{\Phi}_A^k, (\tilde{F}, X),$$

$$\{e_1 = \{c_1/(0.4, 0.1, 0.1), c_2/(0.1, 0.0, 0.5),$$

$$c_3/(0.1, 0.3, 0.2), c_4/(0.2, 0.1, 0.3), c_5/(0.4, 0.1, 0.1)\},$$

$$e_2 = \{c_1/(0.1, 0.1, 0.5), c_2/(0.2, 0.2, 0.4),$$

$$c_3/(0.4, 0.2, 0.1), c_4/(0.6, 0.1, 0.3), c_5/(0.6, 0.2, 0.1)\},$$

$$e_3 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$$c_3/(0.0, 0.0, 0.0), c_4/(0.0, 0.0, 0.0), c_5/(0.0, 0.0, 0.0)\}\},$$

$$\{e_3 = \{c_1/(0.3, 0.4, 0.1), c_2/(0.4, 0.1, 0.2),$$

$$c_3/(0.2, 0.2, 0.4), c_4/(0.6, 0.1, 0.2), c_5/(0.4, 0.1, 0.2)\}\},$$

$$\{e_1 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$$c_3/(0.0, 0.0, 0.0), c_4/(0.0, 0.0, 0.0), c_5/(0.0, 0.0, 0.0)\},$$

$$e_2 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$$c_3/(0.0, 0.0, 0.0), c_4/(0.0, 0.0, 0.0), c_5/(0.0, 0.0, 0.0)\}\},$$

$$\{e_1 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$c_3/(0.0,\ 0.0,\ 0.0),\ c_4/(0.0,\ 0.0,\ 0.0),\ c_5/(0.0,\ 0.0,\ 0.0)\},$

$e_2 = \{c_1/(0.0,\ 0.0,\ 0.0),\ c_2/(0.0,\ 0.0,\ 0.0),$

$c_3/(0.0,\ 0.0,\ 0.0),\ c_4/(0.0,\ 0.0,\ 0.0),\ c_5/(0.0,\ 0.0,\ 0.0)\},$

$e_3 = \{c_1/(0.3,\ 0.4,\ 0.1),\ c_2/(0.4,\ 0.1,\ 0.2),$

$c_3/(0.2,\ 0.2,\ 0.4),\ c_4/(0.6,\ 0.1,\ 0.2),\ c_5/(0.4,\ 0.1,\ 0.2)\},$

$\{e_1 = \{c_1/(0.4,\ 0.1,\ 0.2),\ c_2/(0.1,\ 0.0,\ 0.5),$

$c_3/(0.1,\ 0.3,\ 0.2),\ c_4/(0.2,\ 0.1,\ 0.3),\ c_5/(0.4,\ 0.1,\ 0.1)\},$

$e_2 = \{c_1/(0.1,\ 0.1,\ 0.5),\ c_2/(0.2,\ 0.2,\ 0.4),$

$c_3/(0.4,\ 0.2,\ 0.1),\ c_4/(0.6,\ 0.1,\ 0.3),\ c_5/(0.6,\ 0.2,\ 0.1)\},$

$e_3 = \{c_1/(0.3,\ 0.4,\ 0.1),\ c_2/(0.4,\ 0.1,\ 0.2),$

$c_3/(0.2,\ 0.2,\ 0.4),\ c_4/(0.6,\ 0.1,\ 0.2),\ c_5/(0.4,\ 0.1,\ 0.2)\}$

$\{e_1 = \{c_1/(0.4,\ 0.1,\ 0.2),\ c_2/(0.1,\ 0.0,\ 0.5),$

$c_3/(0.1,\ 0.3,\ 0.2),\ c_4/(0.2,\ 0.1,\ 0.3),\ c_5/(0.4,\ 0.1,\ 0.1)\},$

$e_2 = \{c_1/(0.1,\ 0.1,\ 0.5),\ c_2/(0.2,\ 0.2,\ 0.4),$

$c_3/(0.4,\ 0.2,\ 0.1),\ c_4/(0.6,\ 0.1,\ 0.3),\ c_5/(0.6,\ 0.2,\ 0.1)\},$

$e_3 = \{c_1/(0.0,\ 0.0,\ 0.0),\ c_2/(0.0,\ 0.0,\ 0.0),$

$c_3/(0.0,\ 0.0,\ 0.0),\ c_4/(0.0,\ 0.0,\ 0.0),\ c_5/(0.0,\ 0.0,\ 0.0)\}\}$

of $\mathcal{P}(\tilde{F},\ X)$ is a multi-fuzzy soft topology on $(\tilde{F},\ X)$ and $(X_{\tilde{F}},\ \tilde{\tau})$ is a multi-fuzzy soft topological spaces.

Definition 17. Let $\tilde{\tau}$ be a multi-fuzzy soft topology on $(\tilde{F},\ X) \in \tilde{\mathcal{F}}^k(U;\ E)$ and $(\tilde{G}_2,\ B)$ be a multi-fuzzy soft set in $\mathcal{P}(\tilde{F},\ X)$. Then a multi-fuzzy soft set $(\tilde{G}_1, A) \in \mathcal{P}(\tilde{F}, X)$ is a neighbourhood of $(\tilde{G}_2, B)$ if and only if there exists an open multi-fuzzy soft set $(\tilde{G}_3, C) \in \tilde{\tau}$ such that $(\tilde{G}_2, B) \tilde{\sqsubseteq} (\tilde{G}_3, C) \tilde{\sqsubseteq} (\tilde{G}_1, A)$.

Example 7. In Example 6, taking

$$(\tilde{G}_1, A) = \{e_3 = \{c_1/(0.4, 0.4, 0.1), c_2/(0.5, 0.1, 0.2), c_3/(0.2, 0.4, 0.4), c_4/(0.7, 0.2, 0.2), c_5/(0.6, 0.4, 0.4)\}\},$$

$$(\tilde{G}_2, B) = \{e_3 = \{c_1/(0.2, 0.3, 0.1), c_2/(0.2, 0.1, 0.1), c_3/(0.1, 0.1, 0.2), c_4/(0.2, 0.1, 0.1), c_5/(0.1, 0.1, 0.1)\}\}$$

and

$$(\tilde{G}_3, C) = \{e_3 = \{c_1/(0.3, 0.4, 0.1), c_2/(0.4, 0.1, 0.2), c_3/(0.2, 0.2, 0.4), c_4/(0.6, 0.1, 0.2), c_5/(0.4, 0.1, 0.2)\}\}.$$

Then $(\tilde{G}_2, B) \tilde{\sqsubseteq} (\tilde{G}_3, C) \tilde{\sqsubseteq} (\tilde{G}_1, A)$. Therefore, $(\tilde{G}_1, A)$ is a neighbourhood of $(\tilde{G}_2, B)$.

Definition 18. Let $\tilde{\tau}$ be a multi-fuzzy soft topology on $(\tilde{F}, X)$ and $(\tilde{G}, B) \in \mathcal{P}(\tilde{F}, X)$. The collection of all neighbourhood of $(\tilde{G}, B)$ is denoted by $\tilde{\mathcal{N}}_{(\tilde{G}, B)}$ and it is called *neighbourhood system* of $(\tilde{G}, B)$.

Theorem 3. *A multi-fuzzy soft set $(\tilde{G}, B)$ in $\mathcal{P}(\tilde{F}, X)$ is an open multi-fuzzy soft set if and only if $(\tilde{G}, B)$ is a neighbourhood of each multi-fuzzy soft set $(\tilde{H}, C)$ contained in $(\tilde{G}, B)$.*

The proof is trivial and it is omitted.

Definition 19. Let $(X_{\tilde{F}}, \tilde{\tau})$ be a multi-fuzzy soft topological space and $(\tilde{G}_1, B), (\tilde{G}_2, B) \in \mathcal{P}(\tilde{F}, X)$ such that $(\tilde{G}_2, B) \tilde{\sqsubseteq} (\tilde{G}_1, A)$. Then $(\tilde{G}_2, B)$

is called an *interior multi-fuzzy soft set* of $(\tilde{G}_1, A)$ if $(\tilde{G}_1, A)$ is a neighbourhood of $(\tilde{G}_2, B)$.

The union of all interior multi-fuzzy soft set of $(\tilde{G}_1, A)$ is called the *interior* of $(\tilde{G}_1, A)$ and it is denoted by $(\tilde{G}_1, A)^o$.

Theorem 4. *Let* $\tilde{\tau}$ *be a multi-fuzzy soft topology on* $(\tilde{F}, X)$ *and* $(\tilde{G}_1, A) \in \mathcal{P}(\tilde{F}, X)$. *Then*

(i) $(\tilde{G}_1, A)^o$ *is the largest open multi-fuzzy soft set contained in* $(\tilde{G}_1, A)$.

(ii) *The multi-fuzzy soft set* $(\tilde{G}_1, A)$ *is open if and only if* $(\tilde{G}_1, A) = (\tilde{G}_1, A)^o$.

Proof. (i) $(\tilde{G}_1, A)^o = \tilde{\sqcup}\{(\tilde{G}_2, B) : (\tilde{G}_1, A)$ is a neighbourhood of $(\tilde{G}_2, B)\}$.

Thus, $(\tilde{G}_1, A)^o$ is itself an interior multi-fuzzy soft set of $(\tilde{G}_1, A)$. Therefore, there exists an open multi-fuzzy soft set $(\tilde{G}_3, C)$ such that $(\tilde{G}_1, A)^o \,\tilde{\sqsubseteq}\, (\tilde{G}_3, C) \,\tilde{\sqsubseteq}\, (\tilde{G}_1, A)$. But $(\tilde{G}_3, C)$ is an interior multi-fuzzy soft set of $(\tilde{G}_1, A)$.

Thus, $(\tilde{G}_3, C) \,\tilde{\sqsubseteq}\, (\tilde{G}_1, A)^o$.

Therefore, $(\tilde{G}_3, C) = (\tilde{G}_1, A)^o$.

This implies $(\tilde{G}_1, A)^o$ is open and $(\tilde{G}_1, A)^o$ is the largest open multi-fuzzy soft set contained in $(\tilde{G}_1, A)$.

(ii) The proof is obvious.

Definition 20. Let $(X_{\tilde{F}}, \tilde{\tau}_1), (X_{\tilde{F}}, \tilde{\tau}_2)$ be two multi-fuzzy soft topological spaces. If each $(\tilde{G}, A) \in \tilde{\tau}_1$ implies $(\tilde{G}, A) \in \tilde{\tau}_2$, then $\tilde{\tau}_2$ is called *multi soft* finer than $\tilde{\tau}_1$ or equivalently $\tilde{\tau}_1$ is multi-fuzzy soft coarser than $\tilde{\tau}_2$.

Example 8. Let $\tilde{\tau}_1$ be as $\tilde{\tau}$ in Example 6 and $\tilde{\tau}_2$ be as the following:

$$\tilde{\tau}_2 = \{\tilde{\Phi}_A^k, (\tilde{F}, X),$$

$$\{e_3 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$$c_3/(0.0, 0.0, 0.0), c_4/(0.0, 0.0, 0.0), c_5/(0.0, 0.0, 0.0)\}\},$$

$$\{e_3 = \{c_1/(0.3, 0.4, 0.1), c_2/(0.4, 0.1, 0.2),$$

$$c_3/(0.2, 0.2, 0.4), c_4/(0.6, 0.1, 0.2), c_5/(0.4, 0.1, 0.2)\}\},$$

$$\{e_1 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$$c_3/(0.0, 0.0, 0.0), c_4/(0.0, 0.0, 0.0), c_5/(0.0, 0.0, 0.0)\}\},$$

$$e_2 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$$c_3/(0.0, 0.0, 0.0), c_4/(0.0, 0.0, 0.0), c_5/(0.0, 0.0, 0.0)\},$$

$$e_3 = \{c_1/(0.3, 0.4, 0.1), c_2/(0.4, 0.1, 0.2),$$

$$c_3/(0.2, 0.2, 0.4), c_4/(0.6, 0.1, 0.2), c_5/(0.4, 0.1, 0.2)\}\}\}.$$

Then $\tilde{\tau}_1$ is multi soft finer than $\tilde{\tau}_2$.

Definition 21. Let $\tilde{\tau}$ be a multi-fuzzy soft topology on $(\tilde{F}, X)$ and $\tilde{\mathcal{B}}$ be a subfamily of $\tilde{\tau}$. If every element of $\tilde{\tau}$ be written as arbitrary multi-fuzzy soft union of some elements of $\tilde{\mathcal{B}}$, then $\tilde{\mathcal{B}}$ is called a *multi-fuzzy soft basis* for the multi-fuzzy soft topology $\tilde{\tau}$.

Example 9. Let us consider the multi-fuzzy soft topology $\tilde{\tau}_2$ as in Example 8. Then the subfamily

$$\tilde{\mathcal{B}} = \{\tilde{\Phi}_A^k, (\tilde{F}, X),$$

$$\{e_3 = \{c_1/(0.0, 0.0, 0.0), c_2/(0.0, 0.0, 0.0),$$

$$c_3/(0.0,\ 0.0,\ 0.0),\ c_4/(0.0,\ 0.0,\ 0.0),\ c_5/(0.0,\ 0.0,\ 0.0)\}\},$$
$$\{e_3 = \{c_1/(0.3,\ 0.4,\ 0.1),\ c_2/(0.4,\ 0.1,\ 0.2),$$
$$c_3/(0.2,\ 0.2,\ 0.4),\ c_4/(0.6,\ 0.1,\ 0.2),\ c_5/(0.4,\ 0.1,\ 0.2)\}\}\}$$

is a basis of $\tilde{\tau}_2$.

The next two results can be proved using the previous results.

Proposition 1. *Let* $\tilde{\tau}_1, \tilde{\tau}_2$ *be two multi-fuzzy soft topologies on* $(\tilde{F}, X)$ *and* $\tilde{\mathcal{B}}_1, \tilde{\mathcal{B}}_2$ *be multi-fuzzy soft bases for* $\tilde{\tau}_1, \tilde{\tau}_2$, *respectively. If* $\tilde{\mathcal{B}}_1 \tilde{\sqsubseteq} \tilde{\mathcal{B}}_2$, *then* $\tilde{\tau}_2$ *is multi-fuzzy soft finer than* $\tilde{\tau}_1$.

Proposition 2. *Let* $\tilde{\tau}$ *be a multi-fuzzy soft topology on* $(\tilde{F}, X)$ *and* $\tilde{\mathcal{B}}$ *be a multi-fuzzy soft basis for* $\tilde{\tau}$. *Then* $\tilde{\tau}$ *equals to the collection of multi-fuzzy soft unions of the elements of* $\tilde{\mathcal{B}}$.

Theorem 5. *Let* $(X_{\tilde{F}}, \tilde{\tau})$ *be a multi-fuzzy soft topological space and* $(\tilde{G}_1, A) \in \mathcal{P}(\tilde{F}, X)$. *Then the collection* $\tilde{\tau}_{(\tilde{G}_1, A)} = \{(\tilde{G}_1, A) \tilde{\sqcap} (\tilde{G}_2, B) : (\tilde{G}_2, B) \in \tilde{\tau}\}$ *is a multi-fuzzy soft topology on the multi-fuzzy soft subsets* $(\tilde{G}_1, A)$ *relative to the parameter set A.*

Proof. (i) Since $\tilde{\Phi}_A^k, (\tilde{F}, X) \in \tilde{\tau}$.

Thus, $\tilde{\Phi}_A^k = \tilde{\Phi}_A^k \tilde{\sqcap} (\tilde{G}_1, A) \in \tilde{\tau}_{(\tilde{G}_1, A)}$ and $(\tilde{G}_1, A) \tilde{\sqcap} (\tilde{F}, X) \in \tilde{\tau}_{(\tilde{G}_1, A)}$.

(ii) Let $(\tilde{G}_{11}, A_1), (\tilde{G}_{12}, A_2) \in \tilde{\tau}_{(\tilde{G}_1, A)}$.

Therefore, there exist $(\tilde{G}_{21}, B_1), (\tilde{G}_{22}, B_2) \in \tilde{\tau}$ such that $(\tilde{G}_{11}, A_1) = (\tilde{G}_1, A) \tilde{\sqcap} (\tilde{G}_{21}, B_1)$ and $(\tilde{G}_{12}, A_2) = (\tilde{G}_1, A) \tilde{\sqcap} (\tilde{G}_{22}, B_2)$.

Therefore,

$$\begin{aligned} & (\tilde{G}_{11}, A_1) \tilde{\sqcap} (\tilde{G}_{12}, A_2) \\ &= [(\tilde{G}_1, A) \tilde{\sqcap} (\tilde{G}_{21}, B_1)] \tilde{\sqcap} [(\tilde{G}_1, A) \tilde{\sqcap} (\tilde{G}_{22}, B_2)] \\ &= (\tilde{G}_1, A) \tilde{\sqcap} [(\tilde{G}_{21}, B_1) \tilde{\sqcap} (\tilde{G}_{22}, B_2)]. \end{aligned}$$

Since $(\tilde{G}_{21}, B_1) \tilde{\sqcap} (\tilde{G}_{22}, B_2) \in \tilde{\tau}$, therefore, $(\tilde{G}_{11}, A_1) \tilde{\sqcap} (\tilde{G}_{12}, A_2) \in \tilde{\tau}_{(\tilde{F}, X)}$.

(iii) Let $\{(\tilde{G}_k, B_k) : k \in K\}$ be a subfamily of $\tilde{\tau}_{(\tilde{G}_1, A)}$.

Thus, for each $k \in K$, there is a multi-fuzzy soft set $(\tilde{H}_k, C_k)$ of $\tilde{\tau}$ such that $(\tilde{G}_k, B_k) = (\tilde{G}_1, A) \tilde{\sqcap} (\tilde{H}_k, C_k)$.

Now,

$$\tilde{\sqcup}_{k \in K} (\tilde{G}_k, B_k) = \tilde{\sqcup}_{k \in K} [(\tilde{G}_1, A) \tilde{\sqcap} (\tilde{H}_k, C_k)] = (\tilde{G}_1, A) \tilde{\sqcap} (\tilde{\sqcup}_{k \in K} (\tilde{H}_k, C_k)).$$

Since $\tilde{\sqcup}_{k \in K} (\tilde{H}_k, C_k) \in \tilde{\tau}$, therefore, $\tilde{\sqcup}_{k \in K} (\tilde{G}_k, B_k) \in \tilde{\tau}_{(\tilde{G}_1, A)}$.

Definition 22. Let $(X_{\tilde{F}}, \tilde{\tau})$ be a multi-fuzzy soft topological space and $(\tilde{G}_1, A) \in \mathcal{P}(\tilde{F}, X)$. Then, the multi-fuzzy soft topology $\tilde{\tau}_{(\tilde{G}_1, A)}$ given in Theorem 5 is called *multi-fuzzy soft subspace topology* and it is called a *multi-fuzzy soft subspace* of $(X_{\tilde{F}}, \tilde{\tau})$.

The following result follows from the previous results.

Theorem 6. *Let $(X_{\tilde{F}}, \tilde{\tau})$ be a multi-fuzzy soft topological space on $(\tilde{F}, X)$, $\tilde{B}$ be a multi-fuzzy soft basis for $\tilde{\tau}$ and $(\tilde{G}_1, A) \in \mathcal{P}(\tilde{F}, X)$. Then the collection $\tilde{B}_{(\tilde{G}_1, A)} = \{(\tilde{G}_1, A) \tilde{\sqcap} (\tilde{G}_2, B) : (\tilde{G}_2, B) \in \tilde{B}\}$ is a multi-fuzzy soft basis for the multi-fuzzy soft subspace topology $\tilde{\tau}_{(\tilde{G}_1, A)}$.*

4. Compactness in Multi-fuzzy Soft Topological Spaces

In this section, we introduce multi-fuzzy soft cover, multi-fuzzy soft open cover, multi-fuzzy soft compactness and theorem of these concepts.

Let U be an initial universe, E be the set of parameters, $P(U)$ be the set of all subsets of U and $\tilde{\mathcal{F}}^k(U; E)$ be the family of all multi-fuzzy soft sets of dimension k over U via parameters in E.

Definition 23. Let $(\tilde{F}, X)$ be an element of $\tilde{\mathcal{F}}^k(U; E)$. A family $\{(\tilde{F}_k, A_k) : k \in K\} \tilde{\sqsubseteq} \tilde{\mathcal{F}}^k(U; E)$ of multi-fuzzy soft sets is a cover of $(\tilde{F}, X)$ if $(\tilde{F}, X) \tilde{\sqsubseteq} \tilde{\sqcup}_{k \in K} (\tilde{F}_k, A_k)$.

If each member of the family $\{(\tilde{F}_k, A_k) : k \in K\}$ is a multi-fuzzy soft open set then it is called a *multi-fuzzy soft open cover* of $(\tilde{F}, X)$.

If a subfamily of $\{(\tilde{F}_k, A_k) : k \in K\}$ which is also cover of $(\tilde{F}, X)$ is called a *subcover*.

Definition 24. Let $(\tilde{F}, X)$ be an element of $\tilde{\mathcal{F}}^k(U; E)$. Then $(\tilde{F}, X)$ is said to be a *multi-fuzzy soft compact* if each multi-fuzzy soft open cover of $(\tilde{F}, X)$ has a finite subcover.

Also, a multi-fuzzy soft topological space $(X_{\tilde{F}}, \tilde{\tau})$ is called *compact* if each multi-fuzzy soft open cover of $\tilde{U}_X^k$ has a finite subcover.

Note 3. Let $(X_{\tilde{F}}, \tilde{\tau}_1), (X_{\tilde{F}}, \tilde{\tau}_2)$ be two multi-fuzzy soft topological spaces and $\tilde{\tau}_2$ be multi-fuzzy soft finer than $\tilde{\tau}_1$. If $(X_{\tilde{F}}, \tilde{\tau}_2)$ is compact then $(X_{\tilde{F}}, \tilde{\tau}_1)$ is compact.

Theorem 7. *Let $(\tilde{G}, B)$ be a multi-fuzzy soft closed set in a multi-fuzzy soft compact topological space $(X_{\tilde{F}}, \tilde{\tau})$. Then $(\tilde{G}, B)$ is also compact.*

Proof. Let $\{(\tilde{F}_k, A_k) : k \in K\}$ be any multi-fuzzy soft open cover of $(\tilde{G}, B)$.

Then, $\tilde{U}_X^k \tilde{\sqsubseteq} \{\tilde{\sqcup}_{k \in K} (\tilde{F}_k, A_k)\} \tilde{\sqcup} (\tilde{G}, B)^c$.

Thus, $\{(\tilde{F}_k, A_k) : k \in K\}$ together with multi-fuzzy soft open set $(\tilde{G}, B)^c$ is a multi-fuzzy soft open cover of $\tilde{U}_X^k$.

Since $(X_{\tilde{F}}, \tilde{\tau})$ is compact, therefore, there exists a finite subcover.

Let $\{(\tilde{F}_{k_1}, A_{k_1}), (\tilde{F}_{k_2}, A_{k_2}), ..., (\tilde{F}_{k_n}, A_{k_n}), (\tilde{G}, B)^c\}$ be the finite subcover.

Therefore, $(\tilde{G}, B) \tilde{\sqsubseteq} \{\tilde{\sqcup}_{i=1}^n (\tilde{F}_{k_i}, A_{k_i})\} \tilde{\sqcup} (\tilde{G}, B)^c$.

This implies $(\tilde{G}, B) \tilde{\sqsubseteq} \tilde{\sqcup}_{i=1}^n (\tilde{F}_{k_i}, A_{k_i})$.

Hence, $(\tilde{G}, B)$ is compact.

Definition 25. Let $(X_{\tilde{F}}, \tilde{\tau})$ be a multi-fuzzy soft topological space and $e_1, e_2 \in X$ with $e_1 \neq e_2$. If there exist two multi-fuzzy soft open sets $(\tilde{F}, A), (\tilde{G}, B)$ such that $(\tilde{F}(e_1) \in (\tilde{F}, A)), (\tilde{G}(e_2) \in (\tilde{G}, B))$ and $(\tilde{F}, A) \tilde{\sqcap} (\tilde{G}, B) = \tilde{\Phi}_A^k$, then $(X_{\tilde{F}}, \tilde{\tau})$ is called a *multi-fuzzy soft Hausdroff space*.

Theorem 8. *Let $(\tilde{G}, B)$ be a multi-fuzzy soft compact set in a multi-fuzzy soft Hausdroff space $(X_{\tilde{F}}, \tilde{\tau})$. Then $(\tilde{G}, B)$ is multi-fuzzy soft closed set.*

Proof. Let $e_1 \in X$ with $\tilde{G}(e_1)$ does not belong to $(\tilde{G}, B)$, i.e., $\tilde{G}(e_1) \in (\tilde{G}, B)^c$.

Let $\tilde{G}(e_2) \in (\tilde{G}, B)$.

Therefore, $e_1 \neq e_2$ and $e_1, e_2 \in X$.

Thus, by Hausdroff property of $(X_{\tilde{F}}, \tilde{\tau})$ there exist two disjoint multi-fuzzy soft open sets $(\tilde{H}_{e_2}, A), (\tilde{G}_{e_2}, A)$ such that $\tilde{H}(e_1) \in (\tilde{H}_{e_2}, A)$ and $\tilde{G}(e_2) \in (\tilde{G}_{e_2}, A)$.

Thus, the collection

$$\{(\tilde{G}_{e_2}, A) : e_2 \in X, \tilde{G}(e_2) \in (\tilde{G}, B)\}$$

is a multi-fuzzy soft open cover of $(\tilde{G}, B)$.

Since $(\tilde{G}, B)$ is a multi-fuzzy soft compact set in $(X_{\tilde{F}}, \tilde{\tau})$, therefore, there exists a finite subcover for $(\tilde{G}, B)$.

Let $\{(\tilde{G}_{e_{r_1}}, A), (\tilde{G}_{e_{r_2}}, A), \ldots, (\tilde{G}_{e_{r_n}}, A)\}$ be the finite subcover.

Thus, $\tilde{\sqcap}_{i=1}^{n}(\tilde{H}_{e_{r_i}}, A)$ is a multi-fuzzy soft open set contained in $(\tilde{G}, B)^c$.

Therefore, $(\tilde{G}, B)^c$ is a multi-fuzzy soft open set.

Hence, $(\tilde{G}, B)$ is a multi-fuzzy soft closed set.

5. Conclusion

The purpose of this paper is to discuss some important properties of multi-fuzzy soft topological spaces. We introduce neighbourhood of a multi-fuzzy soft set and interior multi-fuzzy soft set. We also introduce multi-fuzzy soft basis and multi-fuzzy soft subspace topology and have established several interesting properties. To extend this work, one could study the connectedness and other interesting properties for multi-fuzzy soft topological spaces. We hope that this work would help enhancing this study on multi-fuzzy soft topological spaces for the researchers.

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