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GENERALLY WEIGHTED MOVING AVERAGE CONTROL CHART FOR ZERO-INFLATED POISSON

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Abstract

The objective of this research is to propose an approximation of average run length (ARL) by Markov chain approach (MCA) for generally weighted moving average (GWMA) control chart when observations are from zero-inflated Poisson (ZIP) distribution. The main characteristics of a control chart are the average run length (ARL_0) which is mean of false alarm times and the average delay time (ARL_1) which is mean delay of true alarm times. The ARL_0 should be sufficiently large while the process is still in-control and the ARL_1 should be small when the process goes out-of-control. The results obtained from MCA are compared with Monte Carlo simulation (MC). The results found that the numerical results obtained from MCA are as good as from MC; however, MCA is very time saving. Furthermore, the performance of GWMA chart is superior to EWMA chart for small to moderate changes.

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1. Introduction

Control charts are often used to monitor processes for the purpose of detecting, monitoring and improving for a change in a process. A variety of statistical methods have been developed in many areas of interest including epidemiology and health care, industrial, engineering and others. Attribute control charts are an important technique in SPC to monitor the discrete data. When the quality characteristic cannot be measured on a continuous scale, for instance, in counting the number of defective products or the number of nonconformities in a production process, an attribute control chart must be used, for example; *p*, *np*, *c* and *u* charts. Additionally, exponentially weighted moving average (EWMA) control chart and cumulative sum chart (CUSUM) for attribute data have also been applied to discrete processes (see, e.g., Page [11] and Montgomery [9]). The EWMA was first suggested by Roberts [12]. Borror et al. [1] presented EWMA chart for monitoring Poisson observations showing that the performance of EWMA chart is superior to the Shewhart *c* chart. Later, Zhang et al. [17] presented double exponentially weighted moving average (DEWMA) control chart for Poisson observations and showed that this chart is more sensitive to small process changes than the EWMA chart. Recently, Sheu and Lin [13] developed generally weighted moving average (GWMA) control chart for monitoring process changes. This chart is better than other control charts especially sensitive for detecting a small shift. Sheu and Yang [14] studied GWMA chart when observations are Poisson observations and found that GWMA chart performs better than *c* and EWMA charts for large process changes.

Some quality characteristics which the researchers are recently interested in to monitor, are, for example, infection rates, rates of patient falls, number of congenital malformations in a society. However, an unusually large number of zeros are presented in the samples, for instance, in order to detect and monitor an increase in the incidence rate of a rare health event issue or in industrial as a high yield process (Noorossana et al. [10]). Traditional control charts by using Poisson distribution tend to result in underestimated mean and variance; subsequently leading to a higher false alarm rate in detecting out-of-control signals. Most of the statistical methods are developed based on the type of interested quality characteristic observations; whether they are attribute or variable. By improvement, statistical methods have been developed for monitoring such situations as attribute-type quality characteristics. To account this problem, zero-inflated Poisson (ZIP) distribution can be applied. The ZIP model was first suggested in Cohen [3]. One relatively novel approach to problem is to consider an attribute quality characteristic as zero inflation in a probability distribution. Through this approach, we can model overdispersion in the data in an effective manner. Xie et al. [16] proposed ZIP model instead of conventional Poisson model in statistical process control. They studied the efficiency of this chart for detecting a shift in process mean of nonconformities process. Later, Sim and Lim [15] studied control charts for zero-inflated samples in both binomial and Poisson distributions. Recently, He et al. [7] studied a control chart procedure using a combination of 2 cumulative sums (CUSUMs) for monitoring a zero-inflated Poisson process by using simulation method. The exponentially weighted moving average (EWMA) control chart for ZIP random variable to monitoring needle-stick rare occurrences in a hospital was also presented by Fatahi et al. [5].

Common characteristics of control charts which have been used to evaluate and compare the performance of different control charts are average run length (ARL_0) and average delay time (ARL_1). The ARL_0 is the expectation of the time or observations before the control chart gives a false alarm that an in-control process has gone out-of-control. A second important characteristic is the ARL_1 which is the expectation of the time or observations between a process going out-of-control and the control chart giving the alarm that the process has gone out-of-control. The ARL_0 of an acceptable chart should be large enough and the ARL_1 should be small.

Many methods for evaluating the ARL_0 and ARL_1 for control charts have been studied in the literature. A simple approach that is often used to test accuracy with other methods is Monte Carlo (MC) simulation. Roberts [12] studied the ARL for EWMA charts by using simulations for processes following a normal distribution and derived nomograms that can be used to find the ARL for a variety of parameter values. Brook and Evans [2] obtained an approximate formula for the ARL of an EWMA chart by using a finite-state Markov chain approach (MCA). Crowder [4] used numerical quadrature methods to solve the exact integral equations (IE) for the ARL for the normal distribution. The ARL for EWMA control chart for the exponential distribution by using differential equations was studied by Gan [6].

In this paper, we proposed Markov chain approach (MCA) for evaluating average run length (ARL) of generally weighted moving average (GWMA) control chart for a zero-inflated Poisson (ZIP) distribution. Moreover, the performances of GWMA and EWMA charts are compared.

2. Control Charts and their Properties

Let observations $X_1, X_2, ..., X_m$ be identical independently distributed random variables with zero-inflated Poisson distribution, where X_i number of nonconforming is items in sample i of m samples of size n. A simple way to model zero-inflated is to include a proportion π of extra-zeros and proportion $(1-\pi)\times e^{-c}$ follow from a Poisson distribution. The zero-inflated Poisson density function can be written as

$$f_X(x; c, \pi) = \begin{cases} \pi + (1-\pi)\times e^{-c}; & x = 0, \\ (1-\pi)\times \dfrac{c^x e^{-c}}{x!}; & x = 1, 2, 3, ..., \end{cases}$$

where π is the probability that the observation is zero by a binomial process and c is the mean of the Poisson. For the above distribution, mean and variance of the number of nonconforming can be calculated by

$$E(X) = \mu = c(1-\pi),$$

$$V(X) = \mu + \frac{\pi}{(1-\pi)}\mu^2 = c(1-\pi)(1+\pi c).$$

It is assumed that $c = c_0$ while the process is in-control and $c = c_1 > c_0$

when the process goes out-of-control. It is assumed that there is a change-point time $\theta \leq \infty$ at which the parameter changes from $c = c_0$ to $c = c_1$. Note that $\theta = \infty$ means that the process always remains in the in-control state.

Let $E_\theta(\cdot)$ denote the expectation that the change-point from $c = c_0$ to $c = c_1$ for a distribution function $F(x; c, \pi)$ occurs at time θ, where $\theta \leq \infty$. In the literature on quality control, the quantity $E_\infty(\tau)$ is called the *average run length* (ARL_0) of the chart for the given process.

A typical condition imposed on an ARL_0 is that

$$ARL_0 = E_\infty(\tau) = T, \tag{1}$$

where T is given (usually large). For given distribution function and chart, this condition then determines choices for the UCL and LCL.

A typical definition of the ARL_1 is that

$$ARL_1 = E_1(\tau \mid \tau \geq 1), \tag{2}$$

for the change-point occurs at $\theta = 1$.

2.1. Exponentially weighted moving average (EWMA) control chart

Robert [12] first introduced EWMA chart which is a weighted moving average of sequential historical observations same as GWMA chart but the weighted is less than GWMA chart. The statistic of EWMA chart is as follows:

$$Z_t = \lambda X_t + (1 - \lambda) Z_{t-1}, \tag{3}$$

where

Z_t is the EWMA statistic at time tth, where the initial statistic value $Z_0 = c_0$,

X_t is the Poisson observation at the tth time; $t = 1, 2, \ldots,$

λ is a weighted parameter $(0 \leq \lambda \leq 1)$.

Mean and variance of EWMA statistic are $E(Z_t) = c_0$ and

$$Var(Z_t) = \sigma^2_{Z_t} = \sigma^2\left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}],$$

respectively. Therefore, the upper and lower control limits of EWMA chart are

$$UCL = c_0 + H\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]} = h_U, \tag{4}$$

$$LCL = c_0 - H\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]} = h_L = 0. \tag{5}$$

The alarm time for the EWMA procedure is given by

$$\tau = \inf\{i > 0 : Z_i > UCL \text{ or } Z_i < LCL\},$$

where H is the width of control limit and let $LCL = h_L = 0$ as we considered EWMA chart for monitoring the case of increasing of mean and the number of nonconformities cannot be less than 0.

2.2. Generally weighted moving average (GWMA) control chart

The GWMA chart was first presented by Sheu and Lin [13]. This chart is developed and implemented method from EWMA chart by adding an adjustment smoothing constant (w). If the weighted historical observation constant is equal to $q = 1-\lambda$ and $w = 1$, then the GWMA chart coincides the EWMA chart.

The statistic of GWMA chart is as following:

$$Y_t = \sum_{i=1}^{t}(q^{(i-1)^w} - q^{i^w})X_{t-i+1} + q^{t^w}Y_0. \tag{6}$$

Using geometric series can be rewritten as

$$Y_t = \frac{(1-q)(q-1)-(q-1)q(q-1)}{(q-1)(1-q)}X_{t-i+1} + q^w Y_0, \tag{7}$$

where

Y_t is the GWMA statistic at time tth, where the initial statistic value $Y_0 = c_0$,

X_{t-i+1} is the Poisson observation at the $t - i + 1$th; $t = 2, 3, \ldots,$

q is a weighted parameter $(0 \leq q \leq 1)$,

w is an adjustment smoothing constant $(w > 0)$.

Mean and variance of GWMA statistic are $E(Y_t) = \alpha_0$ and $Var(Y_t) = \sigma_{Y_t}^2 = Q_t \sigma^2$, respectively.

Therefore, the upper and lower control limits of GWMA chart are

$$UCL = c_0 + L\sigma\sqrt{Q_t} = h_U, \tag{8}$$

$$LCL = c_0 - L\sigma\sqrt{Q_t} = h_L = 0, \tag{9}$$

respectively, where $Q_t = \sum_{i=1}^{t} (q^{(i-1)^w} - q^{i^w})^2$ and L is the width of control limit.

The alarm time for the GWMA procedure is given by

$$\tau = \inf\{i > 0 : Y_i > UCL \text{ or } Y_i < LCL\}.$$

3. Approximation of ARL Using Markov Chain Approach

Lucas and Saccucci [8] introduced Markov chain approach for approximate ARL t state is in-control process where they assumed that observation x_j; $j = 1, 2, \ldots, n$ is in-control state and $j = n + 1$ is out-of-control state. The transition probability, P_{ij}, is the probability of moving from state i to state j in one step and is given by

$$P_{ij} = (X_{ij} = x_j \mid X_t = x_i). \tag{10}$$

We can replace to the transition matrix $(\mathbf{P})$ and element of matrix (P_{ij}) is

$$\mathbf{P} = \begin{bmatrix} P_{11} & \cdots & P_{1n} & | & P_{1,n+1} \\ \vdots & \ddots & \vdots & | & \vdots \\ P_{n1} & \cdots & P_{nn} & | & P_{n,n+1} \\ --- & --- & --- & | & --- \\ 0 & \cdots & 0 & | & 1 \end{bmatrix} \text{ or}$$

$$\mathbf{P} = \begin{bmatrix} P_{11} & \cdots & P_{1(n+1)} \\ \vdots & \ddots & \vdots \\ P_{(n+1)1} & \cdots & P_{(n+1)(n+1)} \end{bmatrix} \quad \text{or} \quad \mathbf{P} = \begin{bmatrix} \mathbf{R} & (\mathbf{I}-\mathbf{R})\mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix}, \qquad (11)$$

where

$\mathbf{R}$ is the $n \times n$ transition probability matrix among the in-control states,

$\mathbf{I}$ is the $n \times n$ identity matrix,

$\mathbf{1}$ is the $n \times 1$ column vector of ones,

$\mathbf{0}$ is the $1 \times n$ row vector of zeros,

1 is the scalar of one.

An approximation of ARL by using MCA for detecting mean changes of process is in interval of lower control limit and upper control limit. The region of in-control state is divided into *n* subintervals.

The *j*th subinterval of upper control limit (U_j), *j*th subinterval of lower control limit (L_j) and the *i*th subinterval of midpoint (m_i) are given by

$$U_j = h_L + \frac{j(h_U - h_L)}{n},$$

$$L_j = h_L + \frac{(j-1)(h_U - h_L)}{n},$$

$$m_i = h_L + \frac{(2i-1)(h_U - h_L)}{2n}.$$

Consequently, the transition probability equation (P_{ij}) can be rewritten as

$$P_{ij} = P(L_j \le Z_t \le U_j \mid Z_{t-1} = m_i) \tag{12}$$

and substitute GWMA statistic (Y_t), L_j, U_j and m_i into equation (12). This transition probability equation is

$$P_{ij} = P\left(L_j < \frac{(1-q)(q-1)-(q-1)q(1-q)}{(q-1)(1-q)} X_{t-i+1} + q^w Y_{t-1} < U_j \mid Y_{t-1} = m_i\right)$$

$$= P\left(L_j < \frac{(1-q)(q-1)-(q-1)q(1-q)}{(q-1)(1-q)} X_{t-i+1} + q^w m_i < U_j\right)$$

$$= P\left(\begin{array}{l} \dfrac{[2nh_L + 2(j-1)(h_U - h_L) - 2nq^w h_L - q^w(2i-1)(h_U - h_L)](q-1)(1-q)}{2n[(1-q)(q-1)-(q-1)q(1-q)]} < X_{t-i+1} \\ \\ < \dfrac{[2nh_L + 2j(h_U - h_L) - 2nq^w h_L - q^w(2i-1)(h_U - h_L)](q-1)(1-q)}{2n[(1-q)(q-1)-(q-1)q(1-q)]} \end{array}\right). \tag{13}$$

We define the transition probability matrix from state i to state j in ith order as

$$\mathbf{P}^i = \begin{bmatrix} \mathbf{R}^i & (\mathbf{I} - \mathbf{R}^i)\mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix}, \tag{14}$$

where

$(\mathbf{I} - \mathbf{R}^i)\mathbf{1}$ is the $n \times n$ transition probability vector state $i \le n+1$ in ith order,

$\mathbf{R}^i$ is the $n \times n$ transition probability matrix among the in-control states in ith order,

$\mathbf{0}$ is the $n \times 1$ column vector of ones,

1 is the scalar of one.

The approximation of ARL is given by

$$ARL(t) = \sum_{i=1}^{\infty} iP(RL = i) \tag{15}$$

and then substitute $P(RL = i) = \mathbf{p}^{(i)T}(\mathbf{R}^{i-1} - \mathbf{R}^{i})\mathbf{1}$ in equation (15). The ARL can be rewritten as

$$\begin{aligned} ARL(t) &= \sum_{i=1}^{\infty} i\mathbf{P}^{(i)T}(\mathbf{R}^{i-1} - \mathbf{R}^{i})\mathbf{1} \\ &= \sum_{i=1}^{\infty} \mathbf{P}^{(i)T}\mathbf{R}^{i-1}\mathbf{1} \\ &= \mathbf{P}^{(i)T}(\mathbf{I} - \mathbf{R})^{-1}\mathbf{1}, \end{aligned} \tag{16}$$

where $\mathbf{P}^{(i)T}$ is the initial probability vector $[0, ..., 0, 1, 0, ..., 0]_{1\times n}$.

4. Numerical Results

In this section, we show an approximation of ARL of GWMA chart using MCA and MC approaches and comparison of performance between GWMA and EWMA charts for ZIP is presented. Tables 1-4 show the accuracy of the numerical results of ARL for GWMA chart obtained from MCA and MC when observations are from ZIP. We assumed that the ARL_0 value is 370, the mean of process $c_0 = 1$, $\pi = 0.3, 0.5$ and the magnitudes of change in the process mean $\delta = 0.00, 0.01, 0.05, 0.07, 0.1$ and 0.2, respectively. The results found that the numerical results obtained from MCA are in good agreement with the results obtained from MC. We also compare the performance of GWMA and EWMA charts by ARL_1. The results found that the GWMA control chart performs better than EWMA for small to moderate values of change and that the performance of EWMA chart is superior to GWMA chart for large changes. Note that the calculations with MCA obviously take the computational time much less than MC.

Table 1. Comparison of ARL by MCA and MC between GWMA and EWMA charts when $q = 0.9$, $\pi = 0.3$ and $ARL_0 = 370$

$\alpha_0 = 1$	GWMA				EWMA
$\pi = 0.3$	$W = 0.1$	$W = 0.3$	$W = 0.5$	$W = 0.7$	$W = 1$
$q = 0.9$	$UCL = 7.056$	$UCL = 2.994$	$UCL = 2.0321$	$UCL = 1.6434$	$UCL = 1.3135$
0.00 MCA	370.291	370.305	370.904	370.817	370.0833
	(104.56)	(103.63)	(102.80)	(104.52)	(101.88)
MC	370.542 ± 1.86	370.457 ± 1.80	370.529 ± 1.79	370.554 ± 1.68	370.979 ± 1.92
	(704.31)	(705.94)	(710.29)	(708.62)	(705.06)
0.01 MCA	349.897	339.764*	339.929	340.349	340.4703
	(105.71)	(103.61)	(100.13)	(105.75)	(103.29)
MC	349.615 ± 1.36	339.671 ± 1.32	339.714 ± 1.42	340.034 ± 1.38	340.576 ± 1.47
	(683.82)	(667.28)	(655.89)	(652.23)	(654.38)
0.05 MCA	287.626	249.637	246.667*	247.173	248.436
	(105.63)	(104.37)	(103.77)	(102.40)	(104.71)
MC	287.476 ± 1.18	249.602 ± 1.12	246.725 ± 1.10	247.417 ± 1.10	248.839 ± 1.11
	(563.69)	(490.16)	(465.93)	(470.04)	(474.22)
0.07 MCA	264.638	218.114	213.405	213.376*	214.450
	(105.83)	(101.56)	(102.67)	(102.10)	(105.00)
MC	264.332 ± 0.90	218.411 ± 0.96	213.264 ± 1.08	213.334 ± 1.03	214.869 ± 1.01
	(511.58)	(427.98)	(413.59)	(412.84)	(411.75)
0.10 MCA	236.832	181.786	174.726	173.685*	174.087
	(105.32)	(102.03)	(101.03)	(102.65)	(102.65)
MC	236.332 ± 0.49	181.167 ± 0.76	174.459 ± 0.89	173.081 ± 0.82	174.645 ± 0.76
	(465.86)	(352.84)	(332.51)	(329.49)	(332.81)
0.20 MCA	177.596	113.292	101.918	97.576	95.344*
	(104.36)	(103.98)	(103.71)	(104.46)	(103.50)
MC	177.756 ± 0.31	113.233 ± 0.41	101.459 ± 0.47	97.125 ± 0.45	95.857 ± 0.39
	(341.90)	(219.17)	(181.21)	(182.49)	(183.59)

Table 2. Comparison of ARL by MCA and MC between GWMA and EWMA charts when $q = 0.95$, $\pi = 0.3$ and $ARL_0 = 370$

$\alpha_0 = 1$	GWMA				EWMA
$\pi = 0.3$ $q = 0.95$	$W = 0.1$ $UCL = 6.02$	$W = 0.3$ $UCL = 2.636$	$W = 0.5$ $UCL = 1.7695$	$W = 0.7$ $UCL = 1.3768$	$W = 1$ $UCL = 1.087$
0.00 MCA	370.941 (104.74)	370.89 (102.40)	370.185 (105.43)	370.952 (104.13)	370.368 (102.52)
MC	370.717 ± 1.91 (713.22)	370.121 ± 1.86 (717.50)	370.912 ± 1.82 (725.66)	370.215 ± 1.95 (714.62)	370.794 ± 1.95 (708.35)
0.01 MCA	359.463 (103.41)	343.211 (98.94)	337.581* (104.29)	335.999 (103.74)	338.41 (102.59)
MC	359.557 ± 1.40 (672.46)	343.415 ± 1.32 (666.66)	337.556 ± 1.65 (658.99)	335.136 ± 1.58 (627.46)	338.688 ± 1.48 (628.64)
0.05 MCA	320.661 (103.21)	262.197 (103.64)	242.918 (103.85)	234.164* (104.68)	238.92 (103.12)
MC	320.356 ± 0.89 (529.46)	262.857 ± 0.91 (506.95)	242.187 ± 0.92 (498.22)	234.999 ± 0.90 (486.43)	238.064 ± 0.85 (478.69)
0.07 MCA	304.606 (103.31)	233.884 (104.79)	210.418 (102.49)	189.226* (104.58)	192.949 (103.15)
MC	304.896 ± 0.64 (462.18)	233.054 ± 0.76 (441.53)	210.604 ± 0.62 (436.85)	199.334 ± 0.72 (429.18)	192.643 ± 0.62 (431.55)
0.10 MCA	283.654 (102.13)	201.012 (104.44)	173.404 (102.26)	159.638 (102.63)	138.630* (103.04)
MC	283.925 ± 0.53 (386.44)	201.775 ± 0.61 (379.74)	173.597 ± 0.52 (359.66)	159.081 ± 0.48 (340.27)	138.864 ± 0.52 (325.65)
0.20 MCA	232.155 (101.38)	136.855 (104.59)	105.072 (102.35)	88.259 (103.68)	65.934* (102.66)
MC	232.667 ± 0.28 (264.38)	136.869 ± 0.35 (259.74)	105.485 ± 0.34 (248.66)	88.964 ± 0.31 (223.42)	65.319 ± 0.45 (236.53)

Table 3. Comparison of ARL by MCA and MC between GWMA and EWMA charts when $q = 0.9$, $\pi = 0.5$ and $ARL_0 = 370$

$\alpha_0 = 1$	GWMA				EWMA
$\pi = 0.5$ $q = 0.9$	$W = 0.1$ $UCL = 5.19$	$W = 0.3$ $UCL = 2.3108$	$W = 0.5$ $UCL = 1.6394$	$W = 0.7$ $UCL = 1.334$	$W = 1$ $UCL = 1.0021$
0.00 MCA	370.271	370.83	370.03	370.383	370.199
	(102.55)	(104.69)	(105.21)	(105.14)	(103.49)
MC	370.586 ± 1.88	370.133 ± 1.90	370.296 ± 1.85	370.705 ± 1.86	370.794 ± 1.85
	(726.38)	(731.30)	(724.65)	(725.63)	(722.56)
0.01 MCA	351.607	344.455	343.244*	343.573	345.891
	(100.25)	(105.28)	(104.99)	(105.63)	(98.58)
MC	351.693 ± 1.72	344.284 ± 1.75	343.621 ± 1.66	343.485 ± 1.67	345.684 ± 1.62
	(678.58)	(670.38)	(659.46)	(660.78)	(663.52)
0.05 MCA	292.733	262.968	259.274	258.717*	261.811
	(103.20)	(101.62)	(106.02)	(99.87)	(100.24)
MC	292.566 ± 1.16	261.378 ± 1.28	259.445 ± 1.18	258.425 ± 1.14	261.574 ± 1.21
	(534.46)	(503.31)	(501.24)	(498.63)	(499.51)
0.07 MCA	270.293	232.925	227.867	226.654*	228.653
	(104.19)	(104.22)	(105.07)	(107.72)	(102.5)
MC	270.897 ± 0.96	232.828 ± 1.13	227.421 ± 1.11	226.828 ± 0.93	228.254 ± 0.97
	(482.16)	(447.70)	(439.28)	(425.18)	(427.67)
0.10 MCA	242.601	197.091	190.132	187.908*	188.399
	(104.37)	(104.15)	(104.63)	(106.33)	(102.22)
MC	242.206 ± 0.84	197.073 ± 0.92	190.875 ± 0.84	187.057 ± 0.91	188.405 ± 0.84
	(396.37)	(376.57)	(365.47)	(359.44)	(360.17)
0.20 MCA	182.071	125.305	114.157	109.347	106.383*
	(100.52)	(104.83)	(103.43)	(104.71)	(103.74)
MC	182.417 ± 0.56	125.981 ± 0.54	114.955 ± 0.47	109.647 ± 0.52	106.754 ± 0.46
	(273.41)	(242.97)	(236.59)	(228.12)	(224.16)

Table 4. Comparison of ARL by MCA and MC between GWMA and EWMA charts when $q = 0.95$, $\pi = 0.5$ and $ARL_0 = 370$

$\alpha_0 = 1$	GWMA				EWMA
$\pi = 0.5$ $q = 0.95$	$W = 0.1$ $UCL = 4.38$	$W = 0.3$ $UCL = 1.988$	$W = 0.5$ $UCL = 1.3652$	$W = 0.7$ $UCL = 1.1015$	$W = 1$ $UCL = 1.0012$
0.00 MCA	370.921 (104.64)	370.803 (104.24)	370.022 (104.16)	370.089 (104.77)	370.793 (105.27)
MC	370.755 ± 1.98 (719.58)	370.877 ± 1.95 (723.84)	370.704 ± 1.97 (728.11)	370.559 ± 0.87 (721.64)	370.194 ± 1.95 (733.12)
0.01 MCA	359.322 (104.27)	345.324 (104.01)	339.769* (103.96)	340.64 (104.44)	342.754 (103.95)
MC	359.463 ± 1.76 (656.94)	345.319 ± 1.75 (649.22)	339.385 ± 1.70 (646.77)	340.948 ± 1.72 (648.67)	342.688 ± 1.73 (624.95)
0.05 MCA	319.924 (104.55)	267.799 (104.06)	248.62 (104.25)	233.203* (104.22)	235.308 (102.48)
MC	319.693 ± 1.23 (550.96)	267.743 ± 1.28 (536.29)	248.178 ± 1.15 (521.72)	233.412 ± 1.04 (514.62)	235.276 ± 1.29 (486.47)
0.07 MCA	303.563 (104.62)	239.581 (104.14)	216.031 (103.79)	197.307* (103.42)	199.437 (102.93)
MC	303.874 ± 1.01 (464.29)	239.826 ± 1.03 (429.69)	216.366 ± 0.97 (418.56)	197.473 ± 0.86 (415.22)	199.388 ± 1.05 (419.28)
0.10 MCA	282.184 (104.24)	206.059 (104.16)	178.012 (100.31)	156.185 (105.73)	152.446* (103.45)
MC	282.749 ± 0.80 (375.69)	206.673 ± 0.78 (340.54)	178.526 ± 0.73 (312.53)	156.954 ± 0.74 (308.64)	152.195 ± 0.75 (302.18)
0.20 MCA	229.652 (104.10)	138.521 (103.95)	105.319 (100.23)	81.302 (104.04)	78.643* (105.46)
MC	229.676 ± 0.45 (268.49)	138.375 ± 0.51 (213.64)	105.953 ± 0.53 (204.18)	81.497 ± 0.53 (197.44)	78.442 ± 0.41 (185.84)

5. Conclusion

An approximation of average run length (ARL) by Markov chain approach (MCA) for generally weighted moving average (GWMA) control chart when observations are from zero-inflated Poisson (ZIP) distribution is presented. The numerical results of ARL for GWMA chart obtained from MCA and MC approaches for ZIP are compared. The results found that the numerical results obtained from those methods are in good agreement, however, MCA is very time saving with CPU times about 1 minute whereas MC consumes CPU times between 10 minutes per case study. Additionally, the performance of GWMA chart is superior to EWMA chart for small to moderate shifts.

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