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FLEXURAL MODES IN A FLUID-LOADED TRANSVERSELY ISOTROPIC PLATE

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Abstract

Propagation of flexural guided waves in a fluid loaded transversely isotropic plate of infinite length is studied. Numerical results are presented for highly dense transversely isotropic material cobalt, when immersed in water. The phase velocities are not significantly affected except for several modes in which energy leakage occurs into water over certain frequency ranges. Corresponding attenuation spectra for the leaky modes are also plotted.

1. Introduction

The study of the interaction of elastic waves with fluid-loaded solids is considered as tool for the nondestructive evaluation of solid structures. The reflected acoustic field from a fluid-solid interface has assets of information which if exploited, reveals details of many characteristics of the solid, for example, solid properties, existence of internal defects, quality of interface etc.

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The problem of wave propagation in fluid-loaded plates was studied by many scientists. Rayleigh [15] and Lamb [5] were the first who investigated the Lamb modes in an infinite elastic plate. Schoch [17], Osborne and Hart [13], Merkulov [8], Viktorov [20], Pitts et al. [14] and Selezov et al. [19] studied the effects of water loading on plates under variety of conditions. Their results showed that most modes are strongly attenuated by leaking into fluid, but the phase velocity is unaffected. More recent research by Nayfeh and Chimenti [11], and Rokhlin et al. [16] showed that when the ratio of densities of the fluid and the elastic material, (ρ_0/ρ_1) is small, the mode spectrum of the loaded plate is only slightly different from that of a free plate. However, an increase in this ratio leads to an interaction between various modes until in the limiting case $\rho_0/\rho_1 \to \infty$, the spectrum transform into that of plate clamped on its surfaces with slip boundary conditions. Ahmad et al. [2] have generalized Rayleigh-Lamb wave equation and studied the propagation of longitudinal guided waves in a transversely isotropic plate loaded by an inviscid fluid. In this paper we will extend this to the case of propagation of flexural guided waves in inviscid fluid loaded transversely isotropic plate. Zhu and Wu [21] extended their approach to Leaky Lamb waves propagating along a solid plate in contact with a viscous fluid.

Dayal [3], Scott [18] and Nagy [10] have studied the same problem for cylindrical geometry. Meeker and Meitzler [7] have reviewed the wave propagation in free plates and cylinders.

2. Dispersion Curves

Ahmad et al. [2, Section 2] derived the Rayleigh-Lamb dispersion relations for the antisymmetric modes in a free plate in the generalized form of [1] as

$$\frac{\tan(p_1 h)}{\tan(p_2 h)} = \frac{p_1(1 + q_1)(C_{33} p_2^2 q_2 + C_{13} k^2)}{p_2(1 + q_2)(C_{33} p_1^2 q_1 + C_{13} k^2)}. \tag{1}$$

For the case of antisymmetric modes we take the potential functions φ and ψ [2] in terms of sine as

$$\varphi(x_1, x_3, t) = A\sin(px_3)\exp i(kx_1 - \omega t),$$

$$\psi(x_1, x_3, t) = B\sin(px_3)\exp i(kx_1 - \omega t). \tag{2}$$

Doing all the calculations parallel to [2] gives the determinant for the dispersion relation of flexural modes for the fluid-loaded plate as

$$\begin{vmatrix} p_1(1+q_1)\cos(p_1h) & p_2(1+q_2)\cos(p_2h) & 0 \\ (C_{33}p_1^2q_1 + C_{13}k^2)\sin(p_1h) & (C_{33}p_2^2q_2 + C_{13}k^2)\sin(p_2h) & 1 \\ p_1q_1\cos(p_1h) & p_2q_2\cos(p_2h) & -\dfrac{ip_3}{\omega^2\rho_0} \end{vmatrix} = 0. \tag{3}$$

Define the dimensionless parameters

$$p_1h = s_1,\ \ p_2h = s_2,\ \ p_3h = s_3,\ \ kh = s,\ \frac{C_{33}}{C_{13}} = a.$$

Equation (3) can be re-written as

$$D_1 - \frac{i(h\omega)^2\rho_0}{C_{13}s_3}D_2 = 0, \tag{4}$$

where

$$D_1 = s_1(1+q_1)(as_2^2q_2 + s^2)\cos s_1 \sin s_2$$

$$- s_2(1+q_2)(as_1^2q_1 + s^2)\cos s_2 \sin s_1, \tag{5}$$

$$D_2 = s_1s_2(q_2 - q_1)\cos s_1 \cos s_2. \tag{6}$$

Equation (3) is the dispersion relation for the fluid loaded plate, which generalizes the results of [16] for the isotropic plate. $D_1 = 0$ is the case for the free plate wave propagation modes and second term is the correction term introduced by the fluid loaded. In the limit, $\rho_0 \to \infty$, $D_2 = 0$ gives the modes for the fluid loaded plate.

The sketching of wave propagation modes in a plate as well as cylinder was made relatively easy by the work of Mindlin and his coworkers [9, 12], who developed the method of bounds to plot dispersion relations for axially symmetric waves. The wave numbers of any of the guided modes propagating in free or fluid-loaded plate can be calculated at any frequency by numerically searching for the zeroes of Eqs. (1) and (3), respectively. We shall represent the dispersion relations in terms of frequency dependent phase velocity $\omega/\mathrm{Re}(k)$. In the limit of the high frequency, the left hand side of Eq. (1) approaches unity and the phase velocity is obtained from the following relation:

$$\frac{p_2(1+q_2)}{p_1(1+q_1)} = \frac{C_{33}p_2^2q_2 + C_{13}k^2}{C_{33}p_1^2q_1 + C_{13}k^2}. \tag{7}$$

Equation (7) is the generalization of the Rayleigh equation for a transversely isotropic material. Its solution for the cobalt, gives $C_R = 2674.03$ m/sec.

To draw the dispersion curves for cobalt plate, we need the material constants which are taken from Landolt and Börenstein [6] and produced in Table 1.

Table 1. Material constant for cobalt

	Stiffness	10^11	N/m^2		Density
C_{11}	C_{12}	C_{13}	C_{33}	C_{44}	ρ_1 Kg/m^3
2.95	1.59	1.11	3.35	0.71	8900

The velocity of sound is taken as 1475 m/s and the density of water is taken as $\rho_0 = 1000$ Kg/m^3 [4].

In Figure 1 we have plotted first four flexural modes as f1, f2, f3, and f4. The normalized velocity is plotted as a function of normalized frequency. The phase velocity has been normalized with respect to Rayleigh wave velocity C_R. Fluid loading significantly perturbs the spectrum of guided waves only when the density ratio is $(\rho_0/\rho_1 \approx 1)$. In our present case this ratio is 0.112, so the dispersion curves of the fluid loaded plate cannot be seen as they overlap to the curves of the free plate and are indistinguishable from them.

The lowest mode f1 exhibits the expected behavior in that the normalized velocity approaches unity at high frequencies, i.e., approaches to Rayleigh velocity. The wave number of higher modes f2, f3, f4 have small imaginary part giving rise to the leakage of energy from solid to fluid. They suffer from attenuation for all frequencies and they appear to be asymptotically approaching the transverse velocity, C_T at high frequencies. In our present case the transverse velocity is

$$C_T = \sqrt{C_{44}/\rho_1} = 2824.45 \text{ m/sec}.$$

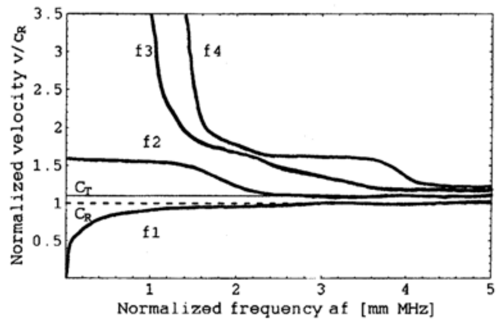

Figure 1. Dispersion curves for the first four flexural modes

3. Attenuation Spectrum

Figures 2 and 3 show the attenuation spectra of the mode when cobalt plate immersed in water. The normalized attenuation is plotted as a function of the normalized frequency. We have plotted the attenuation curves of the f1, f3 and f4 flexural modes as a1, a3, and a4, respectively. For the lowest mode f1, the attenuation a1 is an increasing function of normalized frequency.

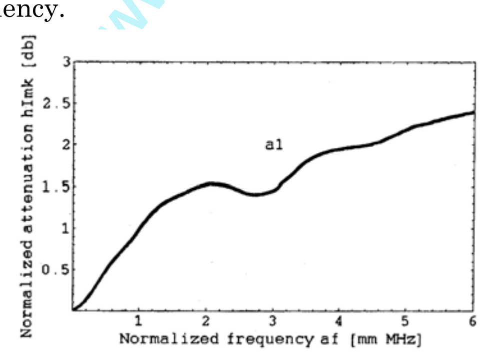

Figure 2. Leaky attenuation spectrum for the first flexural mode a1

In Figure 3 the solid line shows the attenuation a3 of f3 mode and dashed line shows the attenuation a4 of f4 mode. For a3, the attenuation increases sharply when normalized frequency exceeds 2.8 and the curve rises towards the maximum and then slows down. The a4 mode is undamped in the beginning and attenuation becomes small when *af* exceeds 4.6.

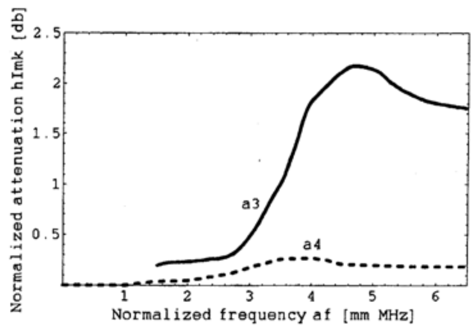

Figure 3. Leaky attenuation spectra of a3 and a4

The overall attenuation spectrum shows that for high frequencies, the leaky modes approach the leaky Rayleigh mode and in this limit, the attenuation is proportional to normalized frequency *af*.

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