Far East Journal of Applied Mathematics
Published Online: March 2015
http://dx.doi.org/10.17654/FJAMFeb2015_115_147
Volume 90, Number 2, 2015, Pages 115-147 **ISSN: 0972-0960**

STOKES FLOW OF AN INCOMPRESSIBLE MICROPOLAR FLUID PAST A POROUS SPHEROID

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Abstract

In this paper, we study the Stokes flow of an incompressible micropolar fluid past and within an isolated porous spheroid directed along its axis of symmetry. We assume that the flow outside the spheroid is governed by the micropolar fluid flow equations under the Stokesian approximation and that within the porous spheroid by Darcy's equation. We determine the velocity field $\vec{q}$, microrotation field $\vec{\nu}$ and the pressure distribution p outside the porous spheroid and also the velocity components and the pressure distribution within the porous region. The expression for the drag on the spheroid is obtained and its variation is studied numerically with respect to the geometric parameter, micropolarity parameter and the permeability constant. It is observed that as the permeability is increasing, the drag on the spheroid is also increasing. For diverse values of the parameters under consideration, the variations in the drag and the stream line pattern are presented through graphs.

Received: November 20, 2014; Accepted: January 31, 2015

2010 Mathematics Subject Classification: 76S05.

Keywords and phrases: porous prolate spheroid, micropolar fluid, permeability constant, prolate angular and radial spheroidal wave functions, stream lines, drag.

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Communicated by K. K. Azad

1. Introduction

Payne and Pell in their classic paper [1] have discussed the Stokes flow of a viscous fluid past a class of axisymmetric bodies with a uniform velocity at infinity which is parallel to the axis of symmetry. Though Stokes flows are somewhat rare, their mathematical analysis has received considerable attention in view of their occurrence in the important field of small particle dynamics. Motivated by this study, the Stokes flow past a sphere has been extensively studied for a wide variety of fluids over years. Another geometry that has attracted the attention of researchers is that of a spheroid. However, all these researches are concerned with solid bodies which are impervious. In nature, as well as in diverse chemical processes, the particles that occur are porous in character. In view of this, in the past few decades, significant contributions have been made mainly dealing with viscous fluid flows past axisymmetric porous bodies. Many of the contributions deal with sphere geometry. Of course, a few of the works also deal with a porous spheroid or a porous approximate sphere. In this context, the works of Joseph and Tao [2], Sutherland and Tan [3], Ooms et al. [4], Neale et al. [5], Jones [6], Adler [7, 8], Vainshtein et al. [9] and Srinivasacharya [10] need special mention. Nanda Kumar and Masliyah [11] studied numerically the flow field inside and around an isolated porous sphere. In all these problems, the flow field variables outside the porous body are governed by the Stokesian approximation of the Navier-Stokes equations and the flow inside the porous body is assumed to follow either Darcian model or Brinkman model.

To explain the behavior of real fluids in certain contexts, Eringen has proposed the theory of micropolar fluids [12] in 1966. In the words of Lukaszewicz, this is a well founded and significant generalization of the classical Navier-Stokes model covering both in theory and applications, many more phenomena than the classical one can [13]. Ever since this theory appeared, several Stokes flow past axisymmetric bodies dealing with micropolar fluid flows have been studied by Lakshmana Rao and Bhujanga Rao [14], Lakshmana Rao and Iyengar [15] and Iyengar and Srinivasacharya

[16]. Significant contributions are also made by Ramkissoon and Majumdar [17] and Ramkissoon [18].

All these problems deal with axisymmetric impervious solid objects like sphere, spheroid and approximate sphere. The problems dealing with flows of micropolar fluids past porous bodies, it seems, have not been given the attention which they richly deserve. This is possibly due to the reason that the micropolar fluid flow equations constitute a coupled system of partial differential equations and in the case of porous bodies determining the arbitrary constants in the solution using the appropriate boundary conditions more complicated than in the counterpart situations in viscous fluids. A micropolar fluid flow is characterized by a coupled system of vector differential equations involving the fluid velocity $\vec{q}$ and the microrotation vector $\vec{\text{v}}$ (independent of $\vec{q}$) in addition to the usual equation of continuity. The equations are to be solved employing the usual no slip velocity on the boundary and the hyperstick boundary condition for the microrotation vector $\vec{\text{v}}$ which stipulates that the microrotation on an impervious boundary coincides with the angular velocity of the boundary. In the case of porous boundaries, however, these are to be suitably modified. In the present problem, we have employed the following boundary conditions: At the boundary of the porous body (i) the normal velocity component is continuous, (ii) the tangential velocity vanishes, (iii) the microrotation component vanishes and (iv) the pressure is continuous. The problems, of course, can also be investigated with other possible boundary conditions.

In this paper, we study the flow of an incompressible micropolar fluid past a porous prolate spheroid kept in an infinite expanse of the fluid with uniform streaming at infinity along the direction of the axis of the spheroid. We assume that the flow outside the spheroid is governed by the micropolar fluid flow equations under the Stokesian approximation and that within the porous spheroid by Darcy's equation. We determine the velocity field $\vec{q}$, microrotation field $\vec{\text{v}}$ and the pressure distribution p outside the porous spheroid and also the velocity components and the pressure distribution within the porous region. The expressions for the velocity and microrotation components are obtained in terms of Legendre functions, associated Legendre functions, radial prolate spheroidal wave functions and angular prolate spheroidal wave functions [20]. The stresses acting on the outer surface are estimated and the drag experienced by the spheroid is obtained. The variation of the drag on the spheroid is studied numerically with respect to the geometric parameter, micropolarity parameter and the permeability constant. The results are displayed through graphs. The stream line pattern for various values of parameters under consideration is also plotted.

The analysis for the case of the oblate spheroid can also be studied on similar lines and hence is not attempted here.

2. Basic Equations

The field equations governing an incompressible micropolar fluid flow are [12]

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{q}) = 0, \tag{1}$$

$$\rho \frac{d\vec{q}}{dt} = \rho \vec{f} - grad\ p + k\ curl\ \vec{\nu} - (\mu + k) curl\ curl\ \vec{q}$$

$$+ (\lambda_1 + 2\mu + k) grad\ div\ \vec{q}, \tag{2}$$

$$\rho j \frac{d\vec{\nu}}{dt} = \rho \vec{l} - 2k\vec{\nu} + k\ curl\ \vec{q} - \gamma\ curl\ curl\ \vec{\nu} + (\alpha + \beta + \gamma) grad\ div\ \vec{\nu} \tag{3}$$

in which $\vec{q}$, $\vec{\nu}$ are velocity and microrotation vectors, $\vec{f}$, $\vec{l}$ are body force per unit mass, body couple per unit mass, respectively, and p is the fluid pressure at any point. ρ and j are density of the fluid and gyration parameters, respectively, and are assumed to be constants. The material constants (λ_1, μ, k) are viscosity coefficients and (α, β, γ) gyro viscosity coefficients. These conform to the inequalities

$$k \geq 0; \quad 2\mu + k \geq 0; \quad 3\lambda_1 + 2\mu + k \geq 0;$$

$$\gamma \geq 0; \quad |\beta| \leq \gamma; \quad 3\alpha + \beta + \gamma \geq 0 \tag{4}$$

and are assumed to be constants.

The stress tensor t_{ij} and the couple stress tensor m_{ij} are given by

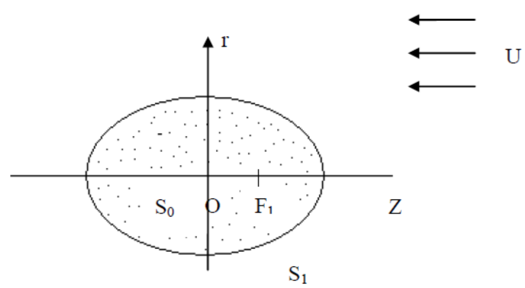
$$t_{ij} = (-p + \lambda_1 + div\,\overline{q})\delta_{ij} + (2\mu + k)e_{ij} + k\varepsilon_{ljm}(w_m - v_m), \tag{5}$$

$$m_{ij} = \alpha(div\,\overline{v})\delta_{ij} + \beta v_{i,j} + \gamma v_{j,i} \tag{6}$$

in which the symbols δ_{ij}, e_{ij}, $2w_m$ and v_m, respectively, denote Kronecker symbol, components of rate of strain, vorticity vector components and microrotation vector components. Comma denotes covariant differentiation.

3. Mathematical Formulation of the Problem

Consider a prolate spheroid S. Let O be the center of the spheroid and F_1 its focus. Introduce the cylindrical polar coordinate system (r, θ, z) with respect to O as origin and OF_1 extended on either side as z-axis.



Let us consider the slow stationary flow of an incompressible micropolar fluid past the spheroid S with a uniform flow with velocity U in the direction of the z-axis far away from the body. Let the region inside S be porous. The fluid flow generated is assumed to be axially symmetric and is the same in any meridian plane, and thus the flow variables will be independent of the azimuth angle.

We shall introduce the prolate spheroidal coordinates (ξ, η, φ) with $(\overline{e}_\xi, \overline{e}_\eta, \overline{e}_\phi)$ as base vectors and (h_1, h_2, h_3) as the corresponding scale factors through the definition

$$z + ir = c\cosh(\xi + i\eta). \tag{7}$$

We assume that the flow is Stokesian as in the classical investigation of the problem by Payne and Pell [1] and Lakshmana Rao and Iyengar [15] and this enables us to drop the inertial terms in the momentum equation and bilinear terms in the balance of first stress moments.

Let $(\overline{q}^{(1)}, \overline{\nu}^{(1)}, p^{(1)})$ denote the velocity, microrotation and pressure in the region S_1 and let $(\overline{q}^{(0)}, p^{(0)})$ be the velocity and pressure in the porous region S_0.

In view of the symmetry of the flow, we take

$$\overline{q}^{(1)} = u^{(1)}(\xi, \eta)\overline{e}_\xi + v^{(1)}(\xi, \eta)\overline{e}_\eta, \tag{8}$$

$$\overline{\nu}^{(1)} = C^{(1)}(\xi, \eta)\overline{e}_\phi, \tag{9}$$

$$p^{(1)} = p^{(1)}(\xi, \eta). \tag{10}$$

Ignoring the body force and body couple $\vec{f}$ and $\vec{l}$, respectively, in the field equations, the basic equations governing the Stokesian flow in region S_1 can be written in the form

$$div(\vec{q}^{(1)}) = 0, \tag{11}$$

$$-grad\ p^{(1)} + k\ curl\ \vec{\nu}^{(1)} - (\mu + k) curl\ curl\ \vec{q}^{(1)} = 0, \tag{12}$$

$$-2k\vec{\nu}^{(1)} + k\ curl\ \vec{q}^{(1)} - \gamma\ curl\ curl\ \vec{\nu}^{(1)} + (\alpha + \beta + \gamma) grad\ div\ \vec{\nu}^{(1)} = 0. \tag{13}$$

In view of the continuity equation, we introduce the stream function $\psi^{(1)}$ through

$$h_2 h_3 u^{(1)} = -\frac{\partial \psi^{(1)}}{\partial \eta}; \quad h_1 h_3 v^{(1)} = \frac{\partial \psi^{(1)}}{\partial \xi}. \tag{14}$$

Using (8) and (14),

$$curl\ \vec{q}^{(1)} = \left(\frac{1}{h_3} E^2 \psi^{(1)}\right)\vec{e}_\phi \tag{15}$$

in which the Stokes stream function operator E^2 is given by

$$E^2 = \frac{h_3}{h_1 h_2}\left(\frac{\partial}{\partial \xi}\left(\frac{h_2}{h_1 h_3}\frac{\partial}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{h_1}{h_2 h_3}\frac{\partial}{\partial \eta}\right)\right). \tag{16}$$

Evaluating the expressions for $curl\, curl\, \vec{q}^{(1)}$, $div\, \vec{\text{v}}^{(1)}$ (which is equal to zero), $curl\, \vec{\text{v}}^{(1)}$, $curl\, curl\, \vec{\text{v}}^{(1)}$, the basic equations describing the flow in region S_1 are

$$-\frac{1}{h_1}\frac{\partial p^{(1)}}{\partial \xi} + \frac{k}{h_2 h_3}\frac{\partial}{\partial \eta}(h_3 C^{(1)}) - \frac{\mu + k}{h_2 h_3}\frac{\partial}{\partial \eta}(E^2 \psi^{(1)}) = 0, \tag{17}$$

$$-\frac{1}{h_2}\frac{\partial p^{(1)}}{\partial \eta} - \frac{k}{h_1 h_3}\frac{\partial}{\partial \xi}(h_3 C^{(1)}) + \frac{\mu + k}{h_1 h_3}\frac{\partial}{\partial \xi}(E^2 \psi^{(1)}) = 0, \tag{18}$$

$$-2kC^{(1)} + \frac{k}{h_3}E^2 \psi^{(1)} + \gamma\left(\nabla^2 - \frac{1}{h_3^2}\right)C^{(1)} = 0, \tag{19}$$

where ∇^2 is the Laplacian operator given by

$$\nabla^2 = \frac{1}{h_1 h_2 h_3}\left\{\frac{\partial}{\partial \xi}\left(\frac{h_2 h_3}{h_1}\frac{\partial}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{h_3 h_1}{h_2}\frac{\partial}{\partial \eta}\right)\right\}. \tag{20}$$

Using the identity

$$h_3\left(\nabla^2 - \frac{1}{h_3^2}\right)C^{(1)} = E^2(h_3 C^{(1)}), \tag{21}$$

equation (19) can be recasted in the form

$$2kh_3 C^{(1)} = kE^2\psi^{(1)} + \gamma E^2(h_3 C^{(1)}). \tag{22}$$

Eliminating $p^{(1)}$ from (17) and (18), we have

$$(\mu + k)E^4\psi^{(1)} - kE^2(h_3 C^{(1)}) = 0. \tag{23}$$

From (22) and (23), we get

$$2h_3 C^{(1)} = E^2\psi^{(1)} + \frac{\gamma(\mu + k)}{k^2}E^4\psi^{(1)}. \tag{24}$$

Operating E^2 on equation (24) and using equation (23), we obtain

$$\left(E^6 - \frac{\lambda^2}{c^2}E^4\right)\psi^{(1)} = 0 \tag{25}$$

which can be written as

$$E^4\left(E^2 - \frac{\lambda^2}{c^2}\right)\psi^{(1)} = 0, \tag{26}$$

where

$$\frac{\lambda^2}{c^2} = \frac{k(2\mu + k)}{\gamma(\mu + k)}. \tag{27}$$

Thus, the flow variables in the region S_1 are completely determinable from the system of partial differential equations (26) and (24) using the appropriate boundary and regularity conditions.

As mentioned earlier, the flow in the porous region S_0 is assumed to be Darcian. In view of this, the equations governing the flow in the region S_0 are given by

$$div(\bar{q}^{(0)}) = 0, \tag{28}$$

$$\bar{q}^{(0)} = -k^{(1)} grad\ p^{(0)} \tag{29}$$

which implies that the pressure $p^{(0)}$ is a harmonic function given by the equation

$$\nabla^2 p^{(0)} = 0. \tag{30}$$

Boundary conditions

The determination of the relevant flow field variables $\psi^{(i)}$, $C^{(i)}$ and $p^{(i)}$ is subjected to the following boundary and regularity conditions:

(i) Continuity of the normal velocity component on the interface:

$$u^{(1)} = u^{(0)} \text{ on } S. \tag{31}$$

(ii) Vanishing of the tangential velocity components on the interface:

$$v^{(1)} = 0 \text{ on } S. \tag{32}$$

(iii)

$$C^{(1)}(s, t) = 0 \text{ on } S. \tag{33}$$

(iv) Continuity of pressure on the interface:

$$p^{(1)} = p^{(0)} \text{ on } S. \tag{34}$$

In addition to the above boundary conditions, it is natural to have regularity of the flow field variables on the axis of symmetry. Further, as the flow is a uniform stream at infinity, we have

$$\psi = -\frac{1}{2}Ur^2 \text{ far away from the body.} \tag{35}$$

4. Solution for the Flow in the Region S_1

Since we are dealing with a prolate spheroidal coordinate system, we have

$$h_1 = h_2 = c\sqrt{(s^2 - t^2)},\ h_3 = c\sqrt{(s^2 - 1)(1 - t^2)}, \tag{36}$$

$$E^2 = \frac{1}{c^2(s^2 - t^2)}\left((s^2 - 1)\frac{\partial^2}{\partial s^2} + (1 - t^2)\frac{\partial}{\partial t^2}\right), \tag{37}$$

$$\nabla^2 = \frac{1}{c^2(s^2 - t^2)}\left((s^2 - 1)\frac{\partial^2}{\partial s^2} + (1 - t^2)\frac{\partial}{\partial t^2} + 2s\frac{\partial}{\partial s} - 2t\frac{\partial}{\partial t}\right), \tag{38}$$

where

$$s = \cosh \xi; \quad t = \cos \eta. \tag{39}$$

We assume that the boundary of the spheroid is given by $s = s_1$.

The solution of equation (26) can be obtained by superposing the solutions of the equations

$$E^4\psi = 0 \tag{40}$$

and

$$\left(E^2 - \frac{\lambda^2}{c^2}\right)\psi = 0. \tag{41}$$

Solution of equation (40)

The solution of (40) can be written in the form

$$\psi = \psi_0 + \psi_1, \tag{42}$$

where

$$\psi_0 = -\frac{1}{2}Uc^2(s^2 - 1)(1 - t^2) \tag{43}$$

and

$$\psi_1 = c^2(s^2 - 1)(1 - t^2)\sum_{n=0}^{\infty} G_{n+1}(s)P'_{n+1}(t), \tag{44}$$

where $P'_{n+1}(t)$ is the derivative of $P_{n+1}(t)$ with respect to *t*. The function ψ_0 in (43) represents the stream function due to a uniform stream of magnitude *U* parallel to the axis of symmetry far away from the spheroid. We notice that $E^2\psi_0 = 0$ and hence $E^4\psi_0 = 0.$ In view of this, ψ_1 must satisfy

$$E^4\psi_1 = 0. \tag{45}$$

It can be verified that the expression

$$f = c^2(s^2 - 1)(1 - t^2)\sum_{n=0}^{\infty} A_{n+1}Q'_{n+1}(s)P'_{n+1}(t), \tag{46}$$

where $Q'_{n+1}(s)$ is the derivative of Legendre function of second kind $Q_{n+1}(s)$ with respect to s, satisfies $E^2 f = 0$. In view of this, we shall impose the restriction on the functions $G_{n+1}(s)$ through

$$E^2\psi_1 = c^2(s^2-1)(1-t^2)\sum_{n=0}^{\infty} A_{n+1}Q'_{n+1}(s)P'_{n+1}(t) \tag{47}$$

so that $E^4\psi_1 = 0$.

Now operating E^2 on equation (44) and equating the result with the right hand side of (47), we get

$$\sum_{n=0}^{\infty}[\{(s^2-1)G_{n+1}(s)\}'' - (n+1)(n+2)G_{n+1}(s)]P'_{n+1}(t)$$

$$= \sum_{n=0}^{\infty} A_{n+1}c^2(s^2-t^2)Q'_{n+1}(s)P'_{n+1}(t). \tag{48}$$

Following (Lakshmana Rao and Iyengar [15]), we note that $G_{n+1}(s)$ is governed by the differential equation

$$(s^2-1)G''_{n+1}(s) + 4sG'_{n+1}(s) - n(n+3)G_{n+1}(s) = g_{n+1}(s), \tag{49}$$

where

$$g_{n+1}(s) = c^2\left[\frac{(n+1)(n+2)}{(2n+3)(2n+5)}A_{n+1} - \frac{(n+3)(n+4)}{(2n+5)(2n+7)}A_{n+3}\right]Q'_{n+3}(s)$$

$$-c^2\left[\frac{(n-1)(n)}{(2n-1)(2n+1)}A_{n-1} - \frac{(n+1)(n+2)}{(2n+1)(2n+3)}A_{n+1}\right]Q'_{n-1}(s). \tag{50}$$

Equations (49) and (50) are valid for $n = 0, 1, 2, 3, \ldots$ with an understanding that the term involving

$$Q'_{-1}(s) \text{ is } -\frac{s}{s^2-1} \text{ and } A_{-1} = 0. \tag{51}$$

Using the method of variation of parameters, we note that

$$G_{n+1}(s) = \alpha_{n+1}P'_{n+1}(s) + B_{n+1}Q'_{n+1}(s)$$

$$- \frac{P'_{n+1}(s)}{(n+1)(n+2)}\int_{s_1}^{s}(s^2-1)Q'_{n+1}(s)g_{n+1}(s)ds$$

$$+ \frac{Q'_{n+1}(s)}{(n+1)(n+2)}\int_{s_1}^{s}(s^2-1)P'_{n+1}(s)g_{n+1}(s)ds \text{ for } n = 0, 1, 2, ..., \quad (52)$$

where $s = s_1$ represents the value specifying the spheroid past in which the flow is being studied. Thus, the flow region S_1 is given by $s > s_1$. As $s \to \infty$, $\psi^{(1)}$ must tend to 0. In view of this, we have to take $\alpha_{n+1} = 0$. Hence the appropriate expression for $G_{n+1}(s)$ is given by

$$G_{n+1}(s) = B_{n+1}Q'_{n+1}(s) - \frac{P'_{n+1}(s)}{(n+1)(n+2)}\int_{s_1}^{s}(s^2-1)Q'_{n+1}(s)g_{n+1}(s)ds$$

$$+ \frac{Q'_{n+1}(s)}{(n+1)(n+2)}\int_{s_1}^{s}(s^2-1)P'_{n+1}(s)g_{n+1}(s)ds$$

$$\text{for } n = 0, 1, 2, \ldots. \quad (53)$$

As $g_{n+1}(s)$ involves one set $\{A_{n+1}\}$ of arbitrary constants, the functions $G_{n+1}(s)$ involve two sets of arbitrary constants $\{A_{n+1}\}$ and $\{B_{n+1}\}$. Using this in equation (44), we get ψ_1.

Solution of equation (41)

To solve equation (41) (viz.) $\left(E^2 - \frac{\lambda^2}{c^2}\right)\psi = 0,$ we take the solution in the form

$$\psi = c\sqrt{(s^2-1)(1-t^2)}\,R(s)S(t). \quad (54)$$

Substituting this in equation (41), we notice that $R(s)$ and $S(t)$, respectively, satisfy the differential equations

$$(s^2-1)R''(s)+2sR'(s)-\left(\Lambda+\lambda^2 s^2+\frac{1}{s^2-1}\right)R(s)=0 \tag{55}$$

and

$$(1-t^2)S''(t)-2t\,S'(t)+\left(\Lambda+\lambda^2 t^2-\frac{1}{1-t^2}\right)S(t)=0, \tag{56}$$

where Λ is a separation constant [20]. These are spheroidal wave differential equations of radial and angular type, respectively. To ensure regularity of solution at infinity and in the flow region, we have to choose the solutions of equations (55) and (56) in the form

$$R_{1n}^{(3)}(i\lambda,\,s)=\left[i^{n+2}\sum_{r=0,1}^{\infty}{}'(r+1)(r+2)d_r^{1n}(i\lambda)\right]^{-1}\left(\frac{s^2-1}{s^3}\right)^{1/2}$$

$$\times\left(\frac{2}{\pi\lambda}\right)^{1/2}\sum_{r=0,1}^{\infty}{}'(r+1)(r+2)d_r^{1n}(i\lambda)K_{r+3/2}(\lambda s) \tag{57}$$

and

$$S_{1n}^{(1)}(i\lambda,\,t)=\sum_{r=0,1}^{\infty}{}'d_r^{1n}(i\lambda)P_{r+1}^{(1)}(t), \tag{58}$$

where

$$P_{r+1}^{(1)}(t)=\sqrt{1-t^2}\,\frac{d}{dt}P_{r+1}(t) \tag{59}$$

denotes the associated Legendre function of the first kind.

The coefficients $d_r^{1n}(i\lambda)$ in the above expansions are constants depending on the parameter $i\lambda$ and the suffix r has the value 1, 3, 5, ... or 0, 2, 4, 6, ... depending upon the odd or even values of $n+1$. We have, therefore, the solution

$$\psi_2 = c\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda,\, s) S_{1n}^{(1)}(i\lambda,\, t), \tag{60}$$

where C_n's are constants.

Hence, the stream function for the region S_1 is given by

$$\psi^{(1)}(s,\, t) = -\frac{1}{2}Uc^2(s^2-1)(1-t^2)$$

$$+\, c^2(s^2-1)(1-t^2)\sum_{n=0}^{\infty} G_{n+1}(s) P'_{n+1}(t)$$

$$+\, c\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda,\, s) S_{1n}^{(1)}(i\lambda,\, t). \tag{61}$$

We can see that

$$E^2\psi^{(1)} = c^2(s^2-1)(1-t^2)\sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(s) P'_{n+1}(t)$$

$$+\, \frac{\lambda^2}{c}\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda,\, s) S_{1n}^{(1)}(i\lambda,\, t) \tag{62}$$

and

$$E^4\psi^{(1)} = \frac{\lambda^4}{c^3}\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda,\, s) S_{1n}^{(1)}(i\lambda,\, t). \tag{63}$$

Using equations (62) and (63) in equation (24), we have

$$C^{(1)}(s,\, t) = \frac{c}{2}\sqrt{(s^2-1)(1-t^2)}\sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(s) P'_{n+1}(t)$$

$$+\, \frac{\lambda^2}{c^2}\,\frac{\mu+k}{k}\sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda,\, s) S_{1n}^{(1)}(i\lambda,\, t). \tag{64}$$

Pressure distribution in S_1

Equations (17) and (18) using equation (36) lead to

$$\frac{\partial p^{(1)}}{\partial s} = \frac{(2\mu + k)}{2c(s^2 - 1)} \frac{\partial}{\partial t} (E^2 \psi^{(1)}) - \frac{\gamma(\mu + k)}{2kc(s^2 - 1)} \frac{\partial}{\partial t} (E^4 \psi^{(1)}) \tag{65}$$

and

$$\frac{\partial p^{(1)}}{\partial t} = -\frac{(2\mu + k)}{2c(1 - t^2)} \frac{\partial}{\partial s} (E^2 \psi^{(1)}) + \frac{\gamma(\mu + k)}{2kc(1 - t^2)} \frac{\partial}{\partial s} (E^4 \psi^{(1)}). \tag{66}$$

Using the expressions in equations (62) and (63) in (65) and (66) and integrating the resulting equations, we get

$$p^{(1)}(s, t) = -\frac{(2\mu + k)c}{2} \sum_{n=0}^{\infty} A_{n+1} Q_{n+1}(s)(n + 1)(n + 2) P_{n+1}(t). \tag{67}$$

Thus, $\psi^{(1)}(s, t)$, $C^{(1)}(s, t)$ and $p^{(1)}(s, t)$ given in equations (61), (64) and (67) give, respectively, the stream function, microrotation and pressure distribution for the region S_1. These involve the three sets of constants $\{A_n\}$, $\{B_n\}$, $\{C_n\}$ as can be seen from equations (61), (53) and (64).

5. Solution for the Flow in the Region S_0

We have seen earlier that the flow in the porous region S_0 is governed by equations (28) and (29) which lead to equation (30). Equation (30) implies that the pressure distribution $p^{(0)}(s, t)$ in S_0 is harmonic and hence it is given by

$$p^{(0)}(s, t) = \sum_{n=0}^{\infty} (\alpha_n P_n(s) + \beta_n Q_n(s)) P_n(t), \tag{68}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ constitute two sets of arbitrary constants to be determined. This $p^{(0)}(s, t)$ has to be regular within the spheroid and hence

$\beta_n = 0$ for all n. The velocity components $u^0(s, t)$ and $v^{(0)}(s, t)$ can be determined from equations (29) and (68).

In view of the continuity equation in the region S_0, we introduce the stream function $\psi^{(0)}$ through

$$h_2 h_3 u^{(0)} = -\frac{\partial \psi^{(0)}}{\partial \eta}; \quad h_1 h_3 v^{(0)} = \frac{\partial \psi^{(0)}}{\partial \xi} \tag{69}$$

as in equation (14). Using (68) and (29), the stream function $\psi^{(0)}$ takes the form

$$\psi^{(0)}(s, t) = -k^{(1)} c(s^2 - 1) \sum_{n=0}^{\infty} \alpha_{2n+1} P'_{2n+1}(s) \int_{-1}^{t} P_{2n+1}(t) dt. \tag{70}$$

Thus, in all, we have four sets of unknown constants $\{A_n\}$, $\{B_n\}$, $\{C_n\}$, $\{\alpha_n\}$ and these can be determined by using the boundary conditions given by equations (31), (32), (33) and (34).

6. Velocity and Microrotation Components in the Regions S_0, S_1

The expressions for the velocity components $u^{(1)}(s, t)$ and $v^{(1)}(s, t)$ are as

$$u^{(1)}(s, t) = \frac{1}{c^2 \sqrt{(s^2 - t^2)(s^2 - 1)}} \frac{\partial \psi^{(1)}}{\partial t},$$

$$v^{(1)}(s, t) = \frac{1}{c^2 \sqrt{(s^2 - t^2)(1 - t^2)}} \frac{\partial \psi^{(1)}}{\partial s}. \tag{71}$$

Further,

$$u^{(0)}(s, t) = -\frac{k^{(1)} \sqrt{s^2 - 1}}{c\sqrt{(s^2 - t^2)}} \frac{\partial p^{(0)}}{\partial s},$$

$$v^{(0)}(s, t) = \frac{k^{(1)}\sqrt{1-t^2}}{c\sqrt{(s^2-t^2)}}\frac{\partial p^{(0)}}{\partial t}. \tag{72}$$

These can be obtained by using the expressions for $\psi^{(1)}$ given in equation (61) and $p^{(0)}$ given in equation (68). Thus, the expressions for the velocity components $u^{(1)}, v^{(1)}; u^{(0)}, v^{(0)};$ the microrotation component $C^{(1)}$ can all be written explicitly. Using these expressions and those of $p^{(0)}$ and $p^{(1)}$ in the boundary conditions given by equations (31), (32), (33) and (34), we can write the equations that lead to the determination of the arbitrary constants.

7. Determination of Arbitrary Constants

In view of the continuity of the normal velocity components on the interface $s = s_1$ given by equation (31), we have

$$Uc^2(s_1^2-1) - c^2(s_1^2-1)\sum_{n=0}^{\infty} G_{n+1}(s_1)(n+1)(n+2)P_{n+1}(t)$$

$$-c\sqrt{s_1^2-1}\sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda, s_1)\frac{d}{dt}(\sqrt{1-t^2}\,S_{1n}^{(1)}(i\lambda, t))$$

$$= -k^{(1)}c(s_1^2-1)\sum_{n=0}^{\infty}\alpha_{n+1}P'_{n+1}(s_1)P_{n+1}(t). \tag{73}$$

As the tangential velocity components are to vanish on the boundaries, equation (32) leads to

$$-Uc^2 s_1(1-t^2)P_1'(t) + c^2\sum_{n=0}^{\infty}\frac{d}{ds}((s^2-1)G_{n+1}(s))_{s=s_1}(1-t^2)P'_{n+1}(t)$$

$$+c\sum_{n=1}^{\infty} C_n\frac{d}{ds}[\sqrt{s^2-1}R_{1n}^{(3)}(i\lambda, s)]_{\text{on } s=s_1}$$

$$\times \sum_{r=0,1}^{\infty} {}' d_r^{1n}(i\lambda)(1-t^2)P'_{r+1}(t) = 0. \tag{74}$$

The condition (33) on microrotation gives rise to the equation

$$\frac{c}{2}\sqrt{(s_1^2-1)}\sum_{n=0}^{\infty} A_{n+1}Q'_{n+1}(s_1)\sqrt{1-t^2}\,P'_{n+1}(t)$$

$$+\frac{(\mu+k)}{k}\frac{\lambda^2}{c^2}\sum_{n=1}^{\infty} C_n R_{1m}^{(3)}(i\lambda, s_1)\sum_{r=0,1}^{\infty} {}' d_r^{1n}(i\lambda)\sqrt{1-t^2}P'_{r+1}(t) = 0. \tag{75}$$

The continuity of pressure on the interfaces given by equation (34) yields

$$-\frac{(2\mu+k)c}{2}\sum_{n=0}^{\infty} A_{n+1}Q_{n+1}(s_1)(n+1)(n+2)P_{n+1}(t)$$

$$= \sum_{n=0}^{\infty}(\alpha_{n+1}P_{n+1}(s_1))P_{n+1}(t). \tag{76}$$

Using the orthogonality property of Legendre functions and the associated Legendre functions, equations (73) to (76) give rise to the following equations adopting some simple algebraic manipulation:

$$Uc^2(s_1^2-1)\delta_{n0} - c^2(s_1^2-1)B_{n+1}Q'_{n+1}(s_1)(n+1)(n+2)$$

$$- c(n+1)(n+2)\sum_{m=1}^{\infty} C_m\sqrt{s_1^2-1}R_{1n}^{(3)}(i\lambda, s_1)d_n^{1m}(i\lambda)$$

$$= -k^{(1)}c(s_1^2-1)(\alpha_{n+1}P'_{n+1}(s_1)), \tag{77}$$

$$- Uc^2 s_1\delta_{0n} + c^2 B_{n+1}(n+1)(n+2)Q_{n+1}(s_1)$$

$$+ c\sum_{m=1}^{\infty} C_m \frac{d}{ds}[\sqrt{s^2-1}R_{1n}^{(3)}(i\lambda, s)]_{\text{on } s=s_1} d_n^{1m}(i\lambda) = 0, \tag{78}$$

$$\frac{c}{2}\sqrt{(s_1^2-1)}\,A_{n+1}Q'_{n+1}(s_1)+\frac{(\mu+k)}{k}\frac{\lambda^2}{c^2}\sum_{m=1}^{\infty}C_m R_{1m}^{(3)}(i\lambda,\, s_1)\,d_n^{1m}(i\lambda)=0, \quad (79)$$

$$-\frac{(2\mu+k)c}{2}A_{n+1}Q_{n+1}(s_1)(n+1)(n+2)=\alpha_{n+1}P_{n+1}(s_1). \quad (80)$$

From equations (77) and (78), the coefficient B_{n+1} can be eliminated and using (79) and (80), we get a non-homogeneous linear system of algebraic equations for the determination of constants $\{C_n\}$. This system is seen to be

$$\sum_{m=1}^{\infty}D_{nm}C_m = -Uc\delta_{0n}, \quad n = 0, 1, 2, ..., \quad (81)$$

where

$$\begin{aligned} D_{nm} &= d_{2n}^{1m}(i\lambda)\Bigg[\Bigg\{\sqrt{s_1^2-1}\,\frac{d}{ds}R_{1m}^{(3)}(i\lambda,\, s_1)+\frac{s_1}{\sqrt{s_1^2-1}}R_{1m}^{(3)}(i\lambda,\, s_1)\Bigg\}(s_1^2-1)Q'_{2n+1}(s_1) \\ &\quad - 2(n+1)(2n+1)\sqrt{s_1^2-1}\,Q_{2n+1}(s_1)R_{1m}^{(3)}(i\lambda,\, s_1) \\ &\quad + (2\mu+k)(n+1)(2n+1)\frac{\mu+k}{k}\frac{\lambda^2}{c^2}2k^{(1)}(s_1^2-1)\frac{P'_{2n+1}(s_1)}{P_{2n+1}(s_1)} \\ &\quad \cdot\frac{Q_{2n+1}(s_1)}{Q'_{2n+1}(s_1)}\frac{Q_{2n+1}(s_1)}{\sqrt{s_1^2-1}}R_{1m}^{(3)}(i\lambda,\, s_1)\Bigg]. \end{aligned} \quad (82)$$

The above linear system splits into two complementary subsystems where n is even and n is odd. The subsystem when n is odd reduces to the homogeneous set of equations

$$\sum_{m=1}^{\infty}D_{2n+1,\,2m}C_{2m} = 0 \quad (83)$$

and we, therefore, have $C_2 = C_4 = C_6 \cdots = 0$. Hence A_n, B_n are all zero when n is even. The analytical determination of the odd suffixed constants is not possible. In view of this, we propose to determine them numerically. Here we truncate the system (81) to fifth order and numerically evaluate the coefficients C_1, C_3, C_5, C_7 and C_9. This is the maximum extent to which the order of truncation can be extended, since the coefficients of spheroidal wave functions needed for a higher order truncation are not explicitly available in the standard literature [20].

After determining these, it is possible to evaluate numerically the other constants. The details of the manipulations are omitted in view of the lengthiness of the expressions and the final system only is reported here.

8. Determination of Drag

To evaluate the drag on the body, we need the stress components and the couple stress components. The stress tensor is given by equation (5) and we need to evaluate the rate of strain components e_{ij} and the spin component ω_ϕ.

The velocity vector $\vec{q}$ can be written in the form

$$\vec{q} = u\vec{e}_\xi + v\vec{e}_\eta, \tag{84}$$

where

$$u = \frac{1}{c^2\sqrt{(s^2 - t^2)(s^2 - 1)}}\frac{\partial\psi}{\partial t},$$

$$v = \frac{1}{c^2\sqrt{(s^2 - t^2)(1 - t^2)}}\frac{\partial\psi}{\partial s}. \tag{85}$$

The rates of strain components are given by

$$e_{\xi\xi} = \frac{1}{c^3(s^2 - t^2)}\left(\psi_{st} + \frac{t}{s^2 - t^2}\psi_s - \frac{s(2s^2 - 1 - t^2)}{(s^2 - t^2)(s^2 - 1)}\psi_t\right),$$

$$e_{\xi\eta} = e_{\eta\xi} = \frac{(s^2-1)\psi_{ss} - (1-t^2)\psi_{tt}}{2c^3(s^2-t^2)\sqrt{(s^2-1)(1-t^2)}} - \frac{s\sqrt{s^2-1}}{c^3(s^2-t^2)^2\sqrt{1-t^2}}\psi_s$$

$$- \frac{t\sqrt{1-t^2}}{c^3(s^2-t^2)^2\sqrt{s^2-1}}\psi_t,$$

$$e_{\eta\eta} = \frac{1}{c^3(s^2-t^2)}\left(-\psi_{st} + \frac{s}{s^2-t^2}\psi_t + \frac{t(2t^2-1-s^2)}{(s^2-t^2)(1-t^2)}\psi_s\right),$$

$$e_{\phi\phi} = \frac{1}{c^3(s^2-t^2)}\left(\frac{s}{s^2-1}\psi_t + \frac{t}{(1-t^2)}\psi_{st}\right), \tag{86}$$

$e_{\xi\phi} = e_{\phi\xi} = e_{\eta\phi} = e_{\phi\eta} = 0$. The spin $= \frac{1}{2} curl\, \vec{q}$ has only one non-zero component ω_ϕ in the direction of the vector $\vec{e}_\phi$ and this is given by

$$\omega_\phi = \frac{1}{2c\sqrt{(s^2-1)(1-t^2)}}E^2\psi. \tag{87}$$

The surface stress t_{ij} for the micropolar fluid is given by equation (5) and we find that the only non-vanishing components of t_{ij} are $t_{\xi\xi}$, $t_{\eta\eta}$, $t_{\phi\phi}$, $t_{\xi\eta}$ and $t_{\eta\xi}$. These are given by

$$t_{\xi\xi} = -p + (2\mu + k)e_{\xi\xi},$$

$$t_{\eta\eta} = -p + (2\mu + k)e_{\eta\eta},$$

$$t_{\phi\phi} = -p + (2\mu + k)e_{\phi\phi},$$

$$t_{\xi\eta} = (2\mu + k)e_{\xi\eta} + k\omega_\phi,$$

$$t_{\eta\xi} = (2\mu + k)e_{\eta\xi} - k\omega_\phi. \tag{88}$$

The stress vector $\vec{t}$ on the boundary of the body is given by

$$\vec{t} = t_{\xi\xi}\vec{e}_\xi + t_{\xi\eta}\vec{e}_\eta. \tag{89}$$

We find that

$$(t_{\xi\xi})_{s=s_1} = -p^{(1)}(s_1,\, t) + \frac{k^{(1)} s_1 (2s_1^2 - 1 - t^2)(2\mu + k)}{c^2 (s_1^2 - t^2)^2} \times \sum_{n=0}^{\infty} (\alpha_{n+1} P'_{n+1}(s_1)) P_{n+1}(t) \tag{90}$$

and

$$\begin{aligned}(t_{\xi\eta})_{s=s_1} &= \frac{(2\mu + k)c}{2} \sqrt{(s_1^2 - 1)(1 - t^2)} \sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(s_1) P'_{n+1}(t) \\ &+ \frac{(2\mu + k)}{2} \frac{\lambda^2}{c^2} \sum_{n=0}^{\infty} C_n R_{1n}^{(3)}(i\lambda,\, s_1) S_{1n}^{(1)}(i\lambda,\, t) \\ &+ \frac{(2\mu + k)}{c^2 (s_1^2 - t^2)} \sqrt{(s_1^2 - 1)(1 - t^2)} \sum_{n=0}^{\infty} (\alpha_{n+1} P'_{n+1}(s_1) + \beta_{n+1} Q'_{n+1}(s_1)) P'_{n+1}(t) \\ &- \frac{(2\mu + k) s_1 k^{(1)}}{c^2 (s_1^2 - t^2)^2} \sqrt{(s_1^2 - 1)(1 - t^2)} \sum_{n=0}^{\infty} (\alpha_{n+1} P'_{n+1}(s_1) + \beta_{n+1} Q'_{n+1}(s_1)) P'_{n+1}(t) \\ &+ \frac{k}{2c\sqrt{(s_1^2 - 1)(1 - t^2)}} \left(c^2 (s_1^2 - 1)(1 - t^2) \sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(s_1) P'_{n+1}(t) \right. \\ &\left. + \frac{\lambda^2}{c} \sqrt{(s_1^2 - 1)(1 - t^2)} \sum_{n=0}^{\infty} C_n R_{1n}^{(3)}(i\lambda,\, s) S_{1n}^{(1)}(i\lambda,\, t) \right).\end{aligned} \tag{91}$$

The stress vector has the component

$$(stress)_{axial} = \frac{1}{\sqrt{s_1^2 - t^2}} (t\sqrt{s^2 - 1}\, t_{\xi\xi} - s\sqrt{1 - t^2}\, t_{\xi\eta})_{s=s_1} \tag{92}$$

in the direction of the axis of symmetry and

$$(stress)_{radial} = \frac{1}{\sqrt{s_1^2 - t^2}} \left(s\sqrt{1 - t^2}\, t_{\xi\xi} + t\sqrt{s^2 - 1}\, t_{\xi\eta}\right)_{s=s_1} \tag{93}$$

in the radial direction of the meridian plane. The resultants of these two vector components over the entire surface of the body are obtained by integration and it is seen that the radial component integrates to zero. Thus, the resultant of the stress vector on the body is the force in the direction of the axis of symmetry and this gives the drag on the body. The drag D can be written in the form

$$D = 2\pi c^2 \sqrt{s_1^2 - 1} \int_{-1}^{1} \left(t\sqrt{s^2 - 1}\, t_{\xi\xi} - s\sqrt{1 - t^2}\, t_{\xi\eta}\right)_{s=s_1} dt \tag{94}$$

and this simplifies to

$$\begin{aligned}
&2\pi c^2 \sqrt{s_1^2 - 1} \Bigg(\int_{-1}^{1} t\sqrt{s_1^2 - 1}\, p^{(1)}(s_1, t)\, dt \\
&+ \frac{2(2\mu + k) s_1 (s_1^2 - 1)^{3/2} k^{(1)}}{c^2} \sum_{n=0}^{\infty} (\alpha_{n+1} P'_{n+1}(s)) \int_{-1}^{1} \frac{t P_{n+1}(t)}{(s_1^2 - t^2)^2}\, dt \\
&- \frac{(2\mu + k)}{c^2} s_1 \sqrt{s_1^2 - 1}\, k^{(1)} \sum_{n=0}^{\infty} (\alpha_{n+1} P'_{n+1}(s)) \int_{-1}^{1} \frac{(1 - t^2) P'_{n+1}(t)}{(s_1^2 - t^2)}\, dt \\
&- (\mu + k) c s_1 \sqrt{s_1^2 - 1} \sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(s_1) \int_{-1}^{1} (1 - t^2) P'_{n+1}(t)\, dt \\
&- (\mu + k) s_1 \frac{\lambda^2}{c^2} \sum_{n=0}^{\infty} C_n R_{1n}^{(3)}(i\lambda, s_1) \int_{-1}^{1} \sqrt{(1 - t^2)}\, S_{1n}^{(1)}(i\lambda, t)\, dt \Bigg).
\end{aligned} \tag{95}$$

Using the relations

$$\int_{-1}^{1} \frac{(1 - t^2) P'_n(t)}{(s_1^2 - t^2)}\, dt = -\frac{2}{s_1} (s_1^2 - 1) Q'_n(s_1) \tag{96}$$

and

$$\int_{-1}^{1} \frac{tP_n(t)}{(s_1^2 - t^2)^2}\, dt = -\frac{1}{s_1} Q_n'(s_1), \tag{97}$$

drawn from "The Theory of Spherical and Ellipsoidal Harmonics" due to Hobson [19], the drag simplifies to

$$D = 2\pi c\sqrt{s_1^2 - 1}\left(cA_1 \frac{2}{3}\sqrt{s_1^2 - 1}((2\mu + k)Q_1(s_1) - 2(\mu + k)s_1 Q_1'(s_1)) \right.$$

$$\left. - \frac{4}{3}(\mu + k)\frac{\lambda^2}{c^2} s_1 \sum_{n=0}^{\infty}{}' C_n R_{1n}^{(3)}(i\lambda,\, s_1) d_0^{(1n)}(i\lambda) \right). \tag{98}$$

Using equation (79), we may eliminate the series involving the constants C_n in the above expression for the drag and after further simplification we see that the drag due to the surface stress is given by the simple formula

$$D = \frac{4}{3}(2\mu + k)\pi c^3 A_1. \tag{99}$$

Introducing the non-dimensionalization scheme given by

$$A_{n+1} = \frac{U}{c^2}\tilde{A}_{n+1}; \quad C_n = Uc\tilde{C}_n; \quad D = 4\pi(2\mu + k)Uc\tilde{D}. \tag{100}$$

It is seen, after dropping the tildes, that the non-dimensional drag D is given by

$$D = \frac{1}{3} A_1, \tag{101}$$

where

$$A_1 = -\frac{2(\mu + k)}{k}\lambda^2 \frac{\sum_{m=1}^{\infty}{}' C_m R_{1m}^{(3)}(i\lambda,\, s_1) d_0^{1m}(i\lambda)}{\sqrt{s_1^2 - 1}\, Q_1'(s_1)}. \tag{102}$$

This depends upon the eccentricity of the spheroid, the material constant

λ, the micropolarity parameter $pl = \frac{k}{\mu + k}$ and the non-dimensional permeability parameter kp defined through $kp = k^{(1)} \frac{\mu + k}{c^2}$.

9. Special Case: (Impervious Spheroid)

When $k^{(1)} = 0$, then the problem reduces to the case of flow past an impervious spheroid. In this case, the system is

$$\sum_{m=1}^{\infty} D_{nm} C_m = -Uc\delta_{0n}, \quad n = 0, 1, 2..., \tag{103}$$

where

$$\begin{aligned} D_{nm} = d_{2n}^{1m}(i\lambda) \Bigg[& \left\{ \sqrt{s_1^2 - 1} \frac{d}{ds} R_{1m}^{(3)}(i\lambda, s_1) + \frac{s_1}{\sqrt{s_1^2 - 1}} R_{1m}^{(3)}(i\lambda, s_1) \right\} \\ & \times (s_1^2 - 1) Q'_{2n+1}(s_1) \\ & - 2(n+1)(2n+1)\sqrt{s_1^2 - 1}\, Q_{2n+1}(s_1) R_{1m}^{(3)}(i\lambda, s_1) \Bigg]. \end{aligned} \tag{104}$$

The drag experienced is given by

$$D = \frac{1}{3} A_1, \tag{105}$$

where

$$A_1 = -\frac{2(\mu + k)}{k} \lambda^2 \frac{\sum_{m=1}^{\infty}{}' C_m R_{1m}^{(3)}(i\lambda, s_1) d_0^{1m}(i\lambda)}{\sqrt{s_1^2 - 1}\, Q'_1(s_1)}. \tag{106}$$

The result agrees with that obtained by Lakshmana Rao and Iyengar [15].

10. Numerical Discussion

The non-homogeneous linear system given in equation (81) is truncated to a 5×5 system as mentioned already. The system is numerically solved and the needed constant A_1 is determined using equation (102). The numerical work is carried out for the micropolarity parameter $pl = 0.2, 0.4, 0.6$ and 0.8; the material constant $\lambda = 1.0, 1.2, 1.5$ and 1.8 and permeability parameter $kp = 0.001$ and 0.02. The variation of the drag is displayed through Figures 1 to 6.

It can be noticed from Figure 1 that for a fixed permeability parameter $kp = 0.001,$ the drag on the spheroid increases steadily as λ increases for a fixed value of the micropolarity parameter *pl*. As *pl* increases the drag decreases for each of the λ's. The same trend is observed for other values of the permeability parameter $kp = 0.02$ (Figure 2). As the micropolarity viscosity *k* increases, *pl* increases and thus a part of energy will be spent in opposing the rotational nature of the fluid particle and this naturally results in a reduction in drag. Thus, an increase in *pl* for a fixed λ leads to a reduction in drag as can be seen in Figures 1 and 2.

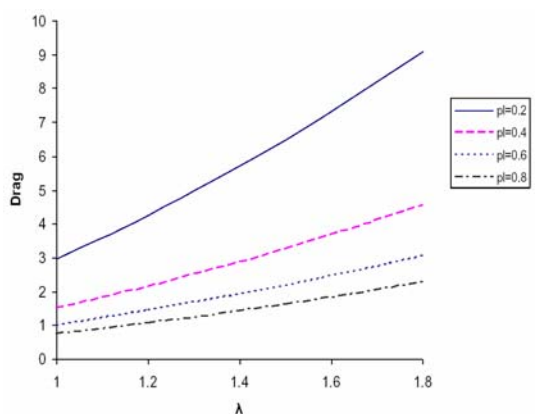


Figure 1. Variation of drag with respect to λ for different values of the polarity parameter *pl* when $s_1 = 1.2$ and the permeability parameter $kp =$ 0.001.

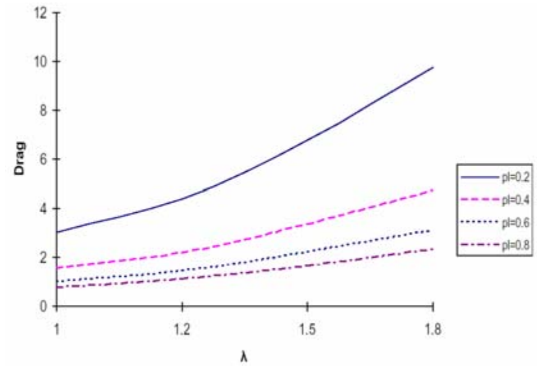


Figure 2. Variation of drag with respect to λ for different values of the polarity parameter *pl* when $s_1 = 1.2$ and the permeability parameter $kp =$ 0.02.

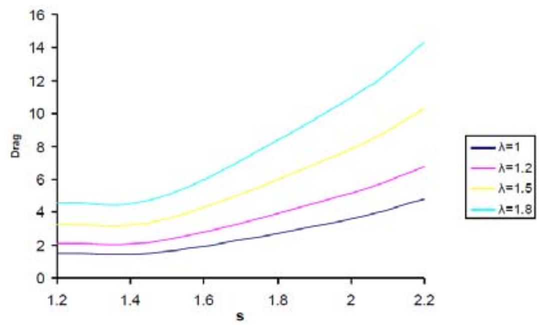


Figure 3. Variation of drag with respect to *s* for different values of λ when the polarity parameter $pl = 0.4$ and the permeability parameter $kp = 0.001$.

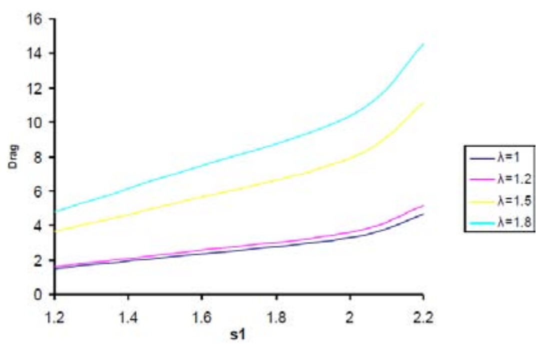


Figure 4. Variation of drag with respect to *s* for different values of λ when the polarity parameter $pl = 0.4$ and the permeability parameter $kp = 0.02$.

Figure 3 shows the variation of drag on the spheroid with the eccentricity s_1 for polarity parameter $p1 = 0.4$ and permeability parameter $kp = 0.001$. As s_1 is increasing for different values of the polarity parameter *pl* with fixed permeability constant $kp = 0.001$ and $\lambda = 1.5$, the drag is increasing. An increase in s_1 leads to the increase in the volume of the porous region in the spheroid. This naturally leads to greater absorption of the fluid in the vicinity of the outer boundary $s = s_1$. As the volume of the body itself is increasing with increase in s_1, we can expect that the body experiences a larger drag. Figure 4 also indicates the same trend for the drag with polarity parameter $pl = 0.4$ and permeability parameter $kp = 0.02$.

Figures 5 and 6 indicate the variation of the drag with respect to the geometric parameter s_1 for different values of the polarity parameter *pl* when *kp* and λ are fixed as (0.001, 1.5) and (0.02, 1.5). The drag is not changing much for an increase of s_1 from 1.2 to 1.5 but therefrom, as s_1 increases it is showing an increasing trend.

In the present problem, plotting the stream lines is a difficult task in view of the complicated nature of the stream functions $\psi^{(0)}$ and $\psi^{(1)}$. Using the expressions for the radial and angular prolate spheroidal wave functions and the related coefficients, we have attempted to compute these by a program developed and computed the values of the stream functions for values of (s, t), within and outside a spheroid $s_1 = 2$.

The stream line pattern for diverse values of λ, micropolarity parameter *pl* and permeability parameter *kp* is presented. It is observed that for smaller permeability parameter, the flow is almost uniform as in Figures 7, 8 and 9. As the permeability is increased, the flow field is seen to be disturbed as is observed by Raja Sekhar and Osamu Sano in their study of viscous flow past a circular/spherical void in porous media [21]. This phenomenon is seen in Figure 10.

As the material constant λ is further increased, with a large permeability, the micropolar nature of the fluid must be further interacting and dividing stream line patterns which are generated and we find the stream line patterns as in Figures 11 and 12. Suitable experimental studies concerning micropolar fluid flow past spherical or spheroidal objects may throw further light on this aspect.

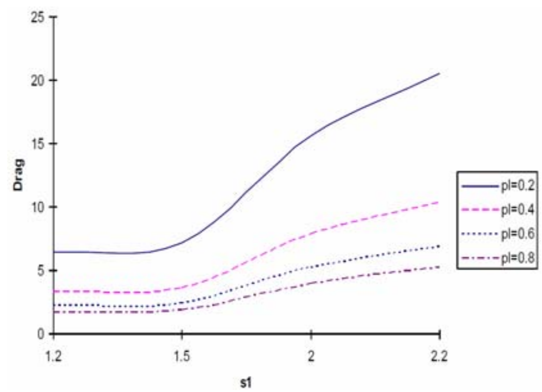


Figure 5. Variation of drag with respect to s for different values of the polarity parameter pl when the permeability parameter $kp = 0.001$ and $\lambda = 1.5$.

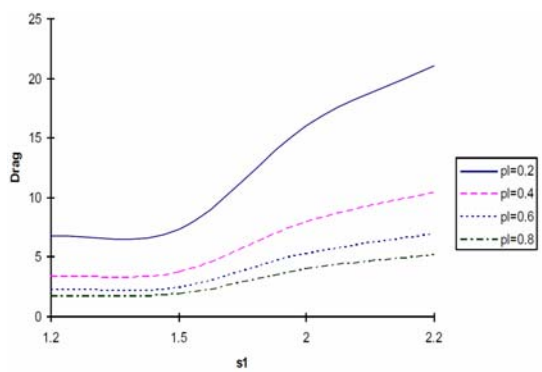


Figure 6. Variation of drag with respect to s for different values of the polarity parameter pl when the permeability parameter $kp = 0.02$ and $\lambda = 1.5$.

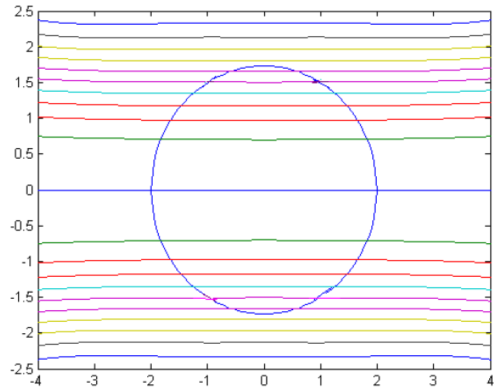


Figure 7. Stream lines for $\lambda = 1.2,\ pl = 0.4,\ kp = 0.01$.

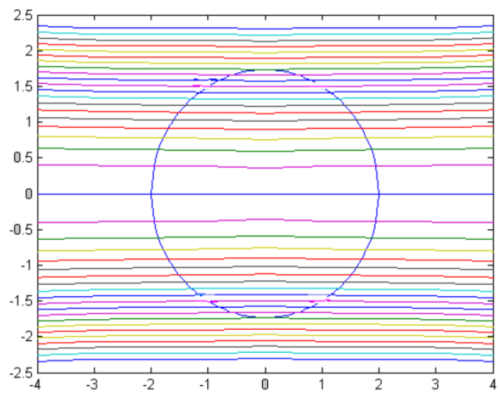


Figure 8. Stream lines for $\lambda = 1.2,\ pl = 0.6,\ kp = 0.01$.

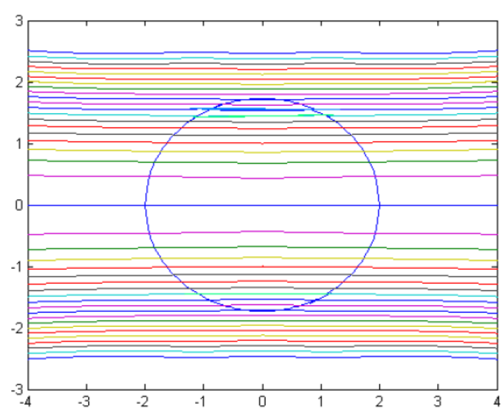


Figure 9. Stream lines for $\lambda = 1.5,\ pl = 0.6,\ kp = 0.01$.

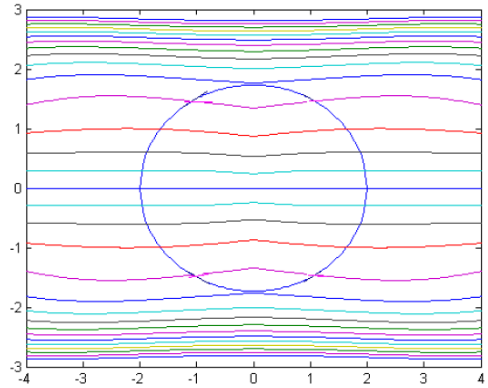


Figure 10. Stream lines for $\lambda = 1.2,\ pl = 0.4,\ kp = 0.02.$

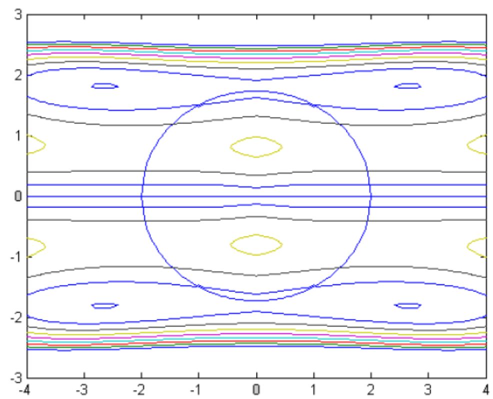


Figure 11. Stream lines for $\lambda = 2.0,\ pl = 0.6,\ kp = 0.1.$

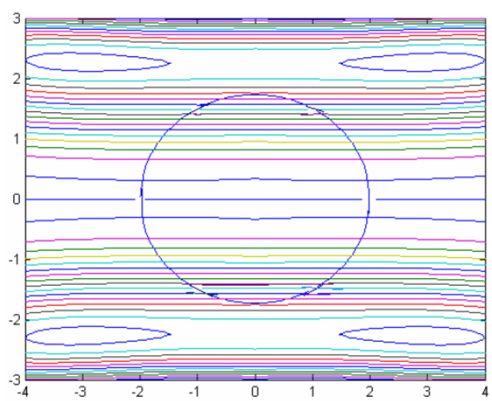


Figure 12. Stream lines for $\lambda = 3.0,\ pl = 0.6,\ kp = 0.1.$

Acknowledgement

The authors thank Dr. J. V. Ramana Murty, Department of Mathematics, NITW for useful discussions.

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