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FUZZY SYSTEM RELIABILITY WITH MINIMIZATION OF COEFFICIENT OF VARIANCE FOR MULTISTAGE SERIES-PARALLEL SYSTEMS

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Abstract

System reliability optimization models containing uncertainty are expressed and a new formulation is proposed to maximize the reliability subject to the available cost of the system reliability estimate. The redundancy allocation problem can be defined as the selection of the system configuration and the type of the components to optimize some objective functions while satisfying some system related constraints. This paper proposes a solution for multistage series-parallel system with nonlinear function to evaluate the maximum reliability subject to available cost using triangular intuitionistic fuzzy number. In addition to that, we propose the

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coefficient of variance of the system reliability estimate for each membership function and non-membership function. These methods are utilized to identify minimum CV with high reliability estimate.

1. Introduction

The reliability optimization is one of the important research tasks in engineering and operation research. A systematic theory of reliability is based on probability theory. The probabilistic method is used in the reliability analysis. It is well known that the conventional reliability analysis using the probabilities has been found to be inadequate to handle uncertainty in failure data and modeling. To overcome this problem, the concept of fuzzy approach has been used in the evaluation of the reliability of a system. The concept of fuzzy reliability has been introduced and formulated in Cai [3] due to uncertainties and imprecision of data. Some researchers applied the fuzzy set theory to reliability analysis. Using the fuzzy set theoretic approach, Cai et al. [4-6] introduced the fuzzy state assumption and possibility assumption to replace binary state assumption and probability assumption.

Chen [7] presented a method for fuzzy system reliability analysis using simplified fuzzy arithmetic operations of fuzzy numbers, in which the reliability of each component is represented by a triangular fuzzy number. Hong and Do [9] presented fuzzy system reliability by the use of *t*-norm based convolution of fuzzy arithmetic operation, in which the reliability of each component is represented by L-R type fuzzy numbers. Abdul Razak and Rajakumar [1] presented a new method for finding fuzzy system reliability using fuzzy product reliability theory in which transition fuzzy model is represented by trapezoidal fuzzy number.

In practical, the problem of series, parallel or series-parallel system reliability may be formed as a typical nonlinear programming with cost functions in fuzzy environment. Park [20] presented a two-component series system subject to a single constraint by fuzzy nonlinear programming technique. Ruan and Sun [21] presented an exact method for cost minimization problem in series reliability system with multiple component choices. Sung and Cho [24] presented a reliability optimization problem for a series system with multiple-choice to maximize the system reliability subject to the system budget.

Mahapatra and Roy [15] presented fuzzy reliability problem of series system model through fuzzy geometric programming using max-min and max additive operator. Lee et al. [12] analyzed the reliability of a parallel system using (λ, ρ) interval valued fuzzy numbers. Sardar Donighi and Khanmohammadi [22] presented a new approach in fuzzy reliability model for series-parallel system in which beta type distribution as its membership function.

Liu [13] presented redundancy-reliability allocation problems in multi-stage series-parallel system under uncertain environment. Mahapatra and Roy [16] presented optimal redundancy allocation problem in series-parallel system using generalized fuzzy number to find out to maximize the system reliability subject to available cost and weight. Many researchers have applied different techniques and solutions on series system, series-parallel system [10, 16] in different environments.

At present, intuitionistic fuzzy sets (IFSs) are being studied and used in various fields of science and engineering. Atanassov [2] introduced intuitionistic fuzzy sets (IFSs) which have been found to be very useful to deal with uncertain information or environment. In fuzzy sets, the degree of acceptance is alone considered, but IFS is characterized by a membership function and non-membership function so that the sum of both values is lesser than one [2]. Kumar et al. [11] also developed a new approach for analyzing the fuzzy system reliability of series system and parallel system using IFS theory.

Sharma et al. [23] analyze the approaches that are similar in the fuzzy system reliability using intuitionistic fuzzy sets. Mahapatra et al. [17, 18] presented a method to analyze the fuzzy reliability of the series system, parallel system in reliability evaluation using triangular intuitionistic fuzzy members arithmetic operation.

This paper presents a solution for multistage series-parallel system with nonlinear function to evaluate the maximum reliability subject to available cost using triangular intuitionistic fuzzy number. An approach is introduced in the coefficient of variance (CV) of the system reliability estimate for each membership function and non-membership function. Advantage of this approach is to find to minimum CV with high reliability estimate. The system reliability estimate of CV is obtained by Tekiner-Mogulkoc and Coit [8].

This paper is segmented into the mentioned below sections: Section 2 gives the mathematical model for multistage series-parallel systems, notations and fuzzy mathematics prerequisites. Section 3 gives the mathematical formulation in crisp model and fuzzy model. Section 4 describes the mathematical analysis and solution procedure for the series-parallel system models. Section 5 gives the coefficient of variance for series-parallel system models. Section 6 illustrates the constructed defined model for given system. Section 7 concludes the maximum reliability with CV.

2. Mathematical Model for Series-parallel Systems

For a series-parallel system, there are m subsystems connected in series and those subsystems consisting of n_j components in parallel for $j = 1, 2, ..., n_k$. Figure 1 shows the diagram for m-stages series-parallel systems.

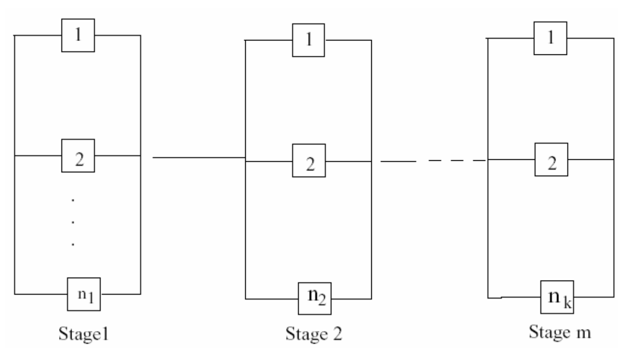

Figure 1. m-stages series-parallel system.

The series-parallel system model has been developed and worked out for the following notations:

2.1. Notations

In order to formulate the problem in series-parallel system model, following notations have been developed:

R_i - reliability of subsystem i, for $i = 1, 2, ..., m$;

r_{ij} - reliability of jth component in subsystem i, for $i = 1, 2, ..., m$, $j = 1, 2, ..., n_k$;

c_{ij} - cost of jth component in subsystem i, for $i = 1, 2, ..., m$, $j = 1, 2, ..., n_k$;

$R_{sp}(R_1, R_2, ..., R_m)$ - system reliability of m subsystem with each reliability R_i, for $i = 1, 2, ..., m$;

$C_{sp}(R_1, R_2, ..., R_m)$ - system cost of m subsystem with each reliability R_i, for $i = 1, 2, ..., m$.

The system reliability for ith stage parallel system is $R_{sp}(R_i) = 1 - \prod_{j=1}^{n_k}(1 - r_{ij})$ for $i = 1, 2, ..., m$. The series-parallel system of reliability has m subsystem with reliability R_i, $i = 1, 2, ..., m$ is

$$R_{sp}(R_1, R_2, ..., R_m) = \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n_k}(1 - r_{ij})\right].$$

The linear cost components for ith stage are $C_{sp}(R_i) = \sum_{j=1}^{n_k} c_{ij}r_{ij} \leq C_i$, $c_{ij} \geq 0$.

2.2. Fuzzy mathematics prerequisites

Fuzzy set theory was first introduced by Zadeh [25] in 1965. Let X be universe of discourse defined by $X = \{x_1, x_2, ..., x_n\}$. The grade membership of an element $x_i \in X$ in a fuzzy set is represented by real value between 0 and 1, but does not indicate the evidence against $x_i \in X$. Atanassov [2] presented the concept of an intuitionistic fuzzy set (IFS) theory in which $x_i \in X$ is characterized by a membership function $\mu_{\tilde{A}}(x)$ indicating evidence for $x \in X$ and non-membership function $\nu_{\tilde{A}}(x)$ indicating the evidence against $x \in X$, which is vital in real life situations.

Definition: intuitionistic fuzzy set

Let X be a fixed set. An intuitionistic fuzzy set $\tilde{A}$ in X is an object having the form $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : x \in X\}$, where $\mu_{\tilde{A}}(x) : X \to [0, 1]$ and $\nu_{\tilde{A}}(x) : X \to [0, 1]$ define the degree of membership and the degree of non-membership, respectively, of the element $x \in X$ to the set $\tilde{A}$ which is a subset of X, for every element of $x \in X$, $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$.

Definition: (α, β)-cuts for IFS

A set of (α, β)-cut generated by an IFS $\tilde{A}$ where $\alpha, \beta \in [0, 1]$ are fixed numbers such that $\alpha + \beta \leq 1$ defined as

$$\tilde{A}_{\alpha, \beta} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \geq \alpha, \nu_{\tilde{A}}(x) \leq \beta, \alpha, \beta \in [0, 1]\}.$$

(α, β)-cut is denoted by $\tilde{A}_{\alpha, \beta}$ and is defined as the crisp set of elements x which belongs to $\tilde{A}$ at least to the degree α and which belongs to $\tilde{A}$ almost to the degree β.

Definition: triangular intuitionistic fuzzy number

Let $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$. A triangular intuitionistic fuzzy number (TFN) $\tilde{A}$ in R written as $(a_1, a_2, a_3; a_1', a_2, a_3')$ has the membership function $\mu_{\tilde{A}}(x)$ and non-membership function $\nu_{\tilde{A}}(x)$ as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise,} \end{cases} \quad \nu_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a'_1} & \text{for } a'_1 \le x \le a_2 \\ \dfrac{x - a_2}{a'_3 - a_2} & \text{for } a_2 \le x \le a'_3 \\ 1 & \text{otherwise.} \end{cases}$$

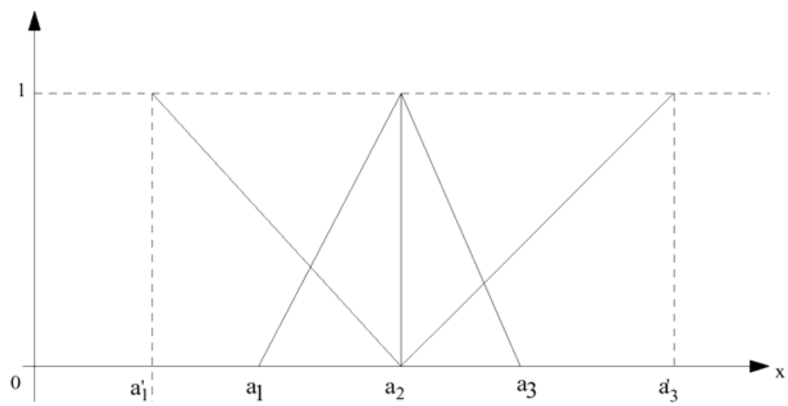

Figure 2. Membership and non-membership functions of TFN.

3. Mathematical Formulation

3.1. Crisp model

Consider the series-parallel model with m subsystem connected in series and those subsystems consisting of n_j, $j = 1, 2, ..., n_k$ components in parallel. The maximization of reliability is found to be $R_{sp}(R_1, R_2, ..., R_m)$ having subject to the limited available cost C_j, $j = 1, 2, ..., n_k$.

The mathematical forms of ith stage series-parallel system are as follows:

$$\text{Maximize } R_{sp}(R_i) = 1 - \sum_{j=1}^{n_k} (1 - r_{ij})$$

subject to $C_{sp}(R_i) = \sum_{j=1}^{n_k} c_{ij} r_{ij} \le C_i$, where $0 \le R_i < 1$, $0 \le r_{ij} \le 1$ and

$i = 1, 2, ..., m$, $j = 1, 2, ..., n_k$. (1)

3.2. Fuzzy model

Practice the system reliability of cost component and cost constraints can be involved in uncertain factors. So the constraint of system reliability becomes uncertain in a reliability optimization problem. Therefore, it can be represented as fuzzy nonlinear programming fuzzy number. The above problem can be modified as follows:

For *i*th stage of a given fuzzy model as

$$\text{Maximize } R_{sp}(R_i) = 1 - \sum_{j=1}^{n_k} (1 - r_{ij})$$

$$\text{subject to } C_{sp}(R_i) = \sum_{j=1}^{n_k} \tilde{c}_{ij} r_{ij} \le \tilde{C}_i,\ 0 < R_i \le 1 \text{ and}$$

$$0 < r_{ij} \le 1 \text{ for all } i, j. \tag{2}$$

4. Mathematical Analysis

Consider a nonlinear programming problem having one inequality constraint of the type:

$$\text{Maximize } Z = f(x_1, x_2, ..., x_n)$$

$$\text{subject to } g(x_1, x_2, ..., x_n) \le b,\ x_1, x_2, ..., x_n \ge 0.$$

The above problem can be expressed as

$$\text{Maximize } Z = f(x_1, x_2 \cdots x_n)$$

$$\text{subject to } h(x) \le 0, \text{ where } h(x) = g(x_1, x_1, ..., x_n) - b,\ h(x) \ge 0. \tag{3}$$

By Kuhn-Tucker condition, the necessary conditions for the maximization in nonlinear programming problem can be summarized as

$$\frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0,$$

$$\lambda h(x) = 0, \text{ where } h(x) \le 0 \text{ and } \lambda \ge 0. \tag{4}$$

The objective and constraint (9) into fuzzy nonlinear programming problem is as follows:

$$\text{Maximize } z = f(x_1, x_2, ..., x_n)$$

$$\text{subject to } \widetilde{h}(x) \le 0. \tag{5}$$

4.1. Solution procedure for series-parallel models

Step 1. Let the cost parameter be $\widetilde{c}_{ij} = (c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3})$ and the cost constraint be $\widetilde{C}_i = (C_{i1}, C_{i2}, C_{i3}; C'_{i1}, C_{i2}, C'_{i3})$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n_k$ are taken as triangular intuitionistics fuzzy numbers.

Step 2. Using α-cut membership function of cost parameters and cost constraints given by $\widetilde{c}_{ij} = (c_{ij1} + \alpha(c_{ij2} - c_{ij1}), c_{ij3} - \alpha(c_{ij3} - c_{ij2}))$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n_k$ and $\widetilde{C}_i = (C_{i1} + \alpha(C_{i2} - C_{i1}), C_{i3} - \alpha(C_{i3} - C_{i2}))$, $i = 1, 2, ..., m$, respectively.

Using β-cut membership function of cost parameters and cost constraints given by

$$\widetilde{c}'_{ij} = (c'_{ij1} + \beta(c_{ij2} - c'_{ij1}), c'_{ij3} - \beta(c'_{ij3} - c_{ij2})), i = 1, 2, ..., m, j = 1, 2, ..., n_k,$$

$$\widetilde{C}'_i = (C'_{i1} + \beta(C_{i2} - C'_{i1}), C'_{i3} - \beta(C'_{i3} - C_{i2})), i = 1, 2, ..., m.$$

Step 3. Applying the Kuhn-Tucker condition in a fuzzy nonlinear programming problem for given model with ith stage $i = 1, 2, ..., m$ of α-cut and β-cut is expressed as

$$\text{Maximize } R_{sp}(R_i) = 1 - \prod_{j=1}^{n_k} (1 - r_{ij})$$

$$\text{subject to } \sum_{j=1}^{n_k} \tilde{c}_{ij} r_{ij} - \tilde{C}_i \leq 0, \text{ where } 0 < r_{ij} \leq 1, \tilde{C}_i \geq 0,$$

$$\text{Maximize } R_{sp}(R_i') = 1 - \prod_{j=1}^{n_k} (1 - r_{ij})$$

$$\text{subject to } \sum_{j=1}^{n_k} \tilde{c}_{ij}' r_{ij} - C_i' \leq 0, \text{ where } 0 < r_{ij} \leq 1, C_i' \geq 0.$$

Step 4. To find the optimal solution R_i, $i = 1, 2, ..., m$ for each membership value of α and R_i', $i = 1, 2, ..., m$ for each non-membership value of β from Steps 2 and 3.

Step 5. To calculate the system reliability $R = \prod_{i=1}^{m} R_i$ and $R' = \prod_{i=1}^{m} R_i'$ with their corresponding CV for each value of α, β.

Step 6. To find out the maximum reliability and minimum CV from the values of α, β.

5. The Coefficient of Variance for Series-parallel System Models

The coefficient of variance (CV) is well-known statistical tool to characterize the estimation of uncertainty. CV is defined as the percentage of standard deviation divided by the mean or percentage of sample standard deviation divided by the average.

Let V be the variance of the system reliability estimate and R be the system reliability estimate. Then the coefficient of variance for series-parallel system reliability by Tekiner-Mogulkoc and Coit [8] is

$$(CV)^2 = \frac{V}{R^2} = \frac{\prod_{i=1}^{m}[R_i^2 + V_i] - \prod_{i=1}^{m} R_i}{\prod_{i=1}^{m} R^2} = \prod_{i=1}^{m}\left(1 + \frac{V_i}{R_i^2}\right) - 1.$$

6. Numerical Examples

In this section, series-parallel system reliability model has been presented numerically to find the optimal solution which can be maximized for system reliability subject to available cost in which considering there are two subsystems connected in series and those first subsystems consisting of 3 components and 2nd subsystem consisting of 2 components where cost coefficient and cost constraint are taken as ITFN.

For stage 1, the cost components of first subsystem are

$$C_1 = (6.5,\ 7,\ 7.5;\ 6.25,\ 7,\ 7.75),$$

$$C_2 = (12.25,\ 12.5,\ 13.0;\ 12.0,\ 12.5,\ 13.10),$$

$$C_3 = (9.75,\ 10.5,\ 11.0;\ 9.65,\ 10.5,\ 11.25).$$

Using α-cut membership function, the above cost coefficients are $\tilde{7} = 6.5 + 0.5\alpha$, $1\tilde{2}.5 = 12.25 + 0.25\alpha$, $1\tilde{0}.5 = 9.75 + 0.75\alpha$, respectively, and using β-cut non-membership function, the above cost coefficients are $\tilde{7} = 7 - 0.75\beta$, $1\tilde{2}.5 = 12.5 - 0.5\beta$, $1\tilde{0}.5 = 10.5 - 0.85\beta$, where $\alpha + \beta \leq 1$, i.e., $\beta \leq 1 - \alpha$.

The cost constraint of first subsystem is $C_1 = (25.5,\ 26,\ 26.5;\ 25.30,\ 26,\ 26.75)$ using α-cut membership function and β-cut non-membership functions as $2\tilde{6} = 25.5 + 0.5\alpha$ and $2\tilde{6} = 26.0 - 0.70\beta$, respectively.

For stage 2, the cost coefficients are $C_1 = (4.0,\ 4.5,\ 4.85;\ 3.75,\ 4.5,\ 5.0)$, $C_2 = (7.25,\ 7.75,\ 8.25;\ 7.0,\ 7.75,\ 8.50)$ using α-cut membership function, the above cost constraints are $\widetilde{4}.5 = 4 + 0.5\alpha,\ 7.\widetilde{7}5 = 7.25 + 0.5\alpha$ and using β-cut non-membership function, the above cost constraints are $4.\widetilde{5} = 4 - 0.75\beta,\ 7.\widetilde{7}5 = 7.75 - 0.75\beta,$ respectively.

The cost constraint is $C_2 = (9.75, 10.25, 10.75; 9.50, 10.25, 10.50)$ using α-cut membership function, β-cut non-membership functions as $10.\widetilde{2}5 = 9.75 + 0.5\alpha,\ 10.\widetilde{2}5 = 10.25 - 0.75\beta,$ respectively.

The following Table 1 and Table 2 show the optimal solution of fuzzy model through fuzzy parametric nonlinear programming for α, β with coefficient of variance for series-parallel system with value of fuzzy membership and fuzzy non-membership functions.

Table 1. Optimal solution of fuzzy membership value of α with CV

α	r_{11}	r_{12}	r_{13}	R_1	r_{21}	r_{22}	R_2	CV
0.0	0.84615385	0.91836735	0.89743590	0.99871191	0.81250000	0.89655172	0.98060345	0.052546282
0.1	0.84223919	0.91581806	0.89482612	0.99860323	0.80864198	0.89383562	0.97968459	0.053411474
0.2	0.83838384	0.91327913	0.89225589	0.99848991	0.80487805	0.89115640	0.97876222	0.054243959
0.3	0.83458647	0.91075051	0.88972431	0.99837199	0.80120482	0.88885135	0.97790418	0.055182988
0.4	0.83084577	0.90823212	0.88723051	0.99824949	0.79761905	0.88590604	0.97690956	0.055816226
0.5	0.82716049	0.90572391	0.88477366	0.99812243	0.79411765	0.88333333	0.97598039	0.056558422
0.6	0.82352941	0.90322581	0.88235294	0.99799085	0.79069767	0.88079470	0.97505005	0.057272884
0.7	0.81995134	0.90073776	0.87996756	0.99785478	0.78735632	0.87828947	0.97411903	0.057960695
0.8	0.81642512	0.89825971	0.87761675	0.99771425	0.78409091	0.87581699	0.97318776	0.058622902
0.9	0.81294964	0.89579158	0.87529976	0.99756932	0.78089888	0.87337662	0.97225668	0.05926049
1.0	0.80952381	0.89333333	0.87301587	0.99742000	0.77777778	0.87096774	0.97132616	0.059874414

Table 2. Optimal solution of fuzzy non-membership value of β with CV

β	r_{11}	r_{12}	r_{13}	R_1	r_{21}	r_{22}	R_2	CV
1.0	0.86133333	0.92777778	0.91018998	0.99910057	0.83333333	0.91071429	0.98511905	0.048324880
0.9	0.85559947	0.92420470	0.90618045	0.99897316	0.82679739	0.90636042	0.98378138	0.049815939
0.8	0.85000000	0.92066116	0.90224033	0.99883658	0.82051282	0.90209790	0.98242783	0.051219483
0.7	0.84453024	0.91714678	0.89836783	0.99869086	0.81446541	0.89792388	0.98106135	0.052540984
0.6	0.83918575	0.91366120	0.89456123	0.99853603	0.80864198	0.89383562	0.97968459	0.053785492
0.5	0.83396226	0.91020408	0.89081886	0.99837216	0.80303030	0.88983051	0.97829995	0.054957691
0.4	0.82885572	0.90677507	0.88713991	0.99819933	0.79761905	0.88590604	0.97690956	0.056061946
0.3	0.82386224	0.90337382	0.88352042	0.99801757	0.79239766	0.88205980	0.97551534	0.057102078
0.2	0.81897810	0.90000000	0.87996128	0.99782704	0.78735632	0.87828947	0.97411903	0.058081976
0.1	0.81419976	0.89665328	0.87646023	0.99762781	0.78248588	0.87459283	0.97272217	0.059005018
0.0	0.80952341	0.89333333	0.87301587	0.99742000	0.77777778	0.87096774	0.97132616	0.059874524

7. Conclusion

Considering various situations of uncertainty, it is important that the decision makers specify the reliability evaluations are of risk averse for which they want to have higher reliability and lower estimations. In general, the series-parallel system is a nonlinear programming problem with system available cost. But cost component and cost constraint are fuzzy numbers in nature. This paper attempts to provide a definition of IFN in accordance to the approach of fuzzy number is proposed, and the cost component and cost constraint of each subsystem are taken as triangular IFN. Kuhn-Tucker conditions are taken into consideration to solve the nonlinear programming problem with fuzzy coefficient to find out the optimal solutions for membership and non-membership values of α, β, respectively. This can be maximized for the system reliability subject to the available cost. Table 1 shows the optimal solution of fuzzy membership value in which the system reliability and CV decreasing with α, which identifies that the maximum reliability and minimum CV for α is 0.9806. Table 2 shows the optimal solution of fuzzy non-membership value in which the system reliability and CV increasing with β which also identifies that the maximum reliability and minimum CV for β is 0.9851. Decision maker can make use of the strategy of system reliability where optimization is involved.

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