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A NOTE ON P_1- AND LIPSCHITZIAN MATRICES

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Abstract

In this note a counterexample is given to illustrate that a result in [SIAM J. Matrix Anal. Appl. 21 (1999), 636-641] is not true.

In [3], the authors asserted that if $A \in \mathbf{E}'$, then for every proper subset α of $\overline{n} = \{1, 2, \ldots, n\}$ such that $\det A_{\alpha\alpha} = 0$, $A_{\alpha\alpha} \notin \mathbf{P}_1$ (see Lemma 5 of [3]).

Now, we point out that this assertion is a mistake. First we give a lemma below.

Lemma 1. *Let* $A \in R^{n\times n} (n \geq 2)$ *and* $A \in \mathbf{C}_0$. *Then* $A \in \mathbf{E}'$ *if and only if A is an almost E-matrix.*

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Proof. Assume that $A \in \mathbf{E}'$. Then $A \notin \mathbf{E}$. From Theorem 2.2 in [2] it follows that every proper principal submatrix of A is an E-matrix. So A is an almost E-matrix. Conversely, if $A \in \mathbf{C}_0$, and A is an almost E-matrix, then $A \notin \mathbf{E}$. Now, we show that $A \in \mathbf{E}^*$.

Let $\forall 0 \neq q \geq 0$. Suppose that there exists a nonzero nonnegative vector z such that $z \in S(q, A)$, then

$$z \geq 0, \quad Az + q \geq 0, \quad z^T(Az + q) = 0.$$

Let $\alpha = supp\, z$, where $supp\, z = \{i : z_i \neq 0\}$. Then $\alpha \neq \emptyset$. If $\alpha = \overline{n}$, then $z > 0$. From the assumption that $A \in \mathbf{C}_0$, $z > 0$, $0 \neq q \geq 0$, we have $z^T Az \geq 0$ and $z^T q > 0$. Hence $z^T(Az + q) = z^T Az + z^T q > 0$. This contradicts to the assumption that $z \in S(q, A)$. Hence $\alpha \neq \overline{n}$, i.e., there exists i such that $z_i = 0$. Let $\beta = \bar{i}$. Then

$$A_{\beta\beta} z_\beta + q_\beta \geq 0, \quad 0 \neq z_\beta \geq 0, \quad z_\beta^T(A_{\beta\beta} z_\beta + q_\beta) = 0.$$

Hence we have $z_\beta \in S(q_\beta, A_{\beta\beta})$. From $q \geq 0$ we have $q_\beta \geq 0$, and thus $A_{\beta\beta} \notin \mathbf{E}$. This contradicts to the assumption that A is an almost E-matrix. This implies that for any nonzero nonnegative vector q, the zero vector is the only solution of $LCP(q, A)$. So $A \in \mathbf{E}^*$, hence $A \in \mathbf{E}'$.

A counterexample for Murthy et al. assertion is given below.

Let

$$A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -2 & 0 \\ -2 & -1 & 4 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}.$$

Then all principal minors of $A + A^T$ are nonnegative, which follows that A is positive semi definite and hence copositive. Obviously $A \in \mathbf{P}_0$. Let $x = (x_1, x_2, x_3, x_4)^T$ with $x_1 = x_2 = x_3 = x_4 > 0$. Then $x \in SOL(0, A)$, so $A \notin \mathbf{R}_0$, and thus $A \notin \mathbf{Q}$ (see [1]). Therefore $A \notin \mathbf{E}$. And we can easily prove that every proper principal submatrix of A is an E-matrix. So A is an almost E-matrix. It follows from Lemma 1 that $A \in \mathbf{E}'$. Let $\alpha = \{1, 2\}$. Then $\det A_{\alpha\alpha} = 0$, and $A_{\alpha\alpha} \in \mathbf{P}_1$, which implies that the above assertion is not true. Through Theorem 6 in [3] follows from the above assertion, this assertion still holds under the assumption of Theorem 6 [3].

Danao studied E^*-matrices and E'-matrices in [2], and presented the following conjecture: if $A \in \mathbf{P}_0$, then $A \in \mathbf{E}' \Leftrightarrow A \in \mathbf{P}_1^*$. This counterexample also illustrates that Danao's conjecture is not true (noting that this conjecture has been disproved in [4]).

References

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