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FUZZY PAIRWISE α-SEMI-IRRESOLUTE CONTINUOUS MAPPINGS

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Abstract

We define and characterize a fuzzy pairwise α-semi-irresolute continuous mapping and a fuzzy pairwise α-semiopen irresolute mapping on a fuzzy bitopological space.

1. Introduction

Kharal and Ahmad [3] defined a fuzzy α-semiopen set and studied a fuzzy α-semicontinuous mapping on a fuzzy topological space. Recently, Im et al. [2] characterized a fuzzy pairwise α-semicontinuous mapping and a

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fuzzy pairwise α-semiopen mapping (a fuzzy pairwise α-semiclosed mapping) on a fuzzy bitopological space as a natural generalization of a fuzzy topological space. One purpose of this paper is to find more stronger mapping than we studied in [2].

In this paper, we define a fuzzy pairwise α-semi-irresolute continuous mapping and a fuzzy pairwise α-semiopen irresolute mapping (a fuzzy pairwise α-semiclosed irresolute mapping) on a fuzzy bitopological space and study their properties. We also give an example, which is a fuzzy pairwise α-semicontinuous mapping but not a fuzzy pairwise α-semi-irresolute continuous mapping.

2. Preliminaries

Let X be a set and let τ_1 and τ_2 be fuzzy topologies on X. Then we call (X, τ_1, τ_2) a *fuzzy bitopological space* $[fbts]$.

Throughout this paper, we take an ordered pair (τ_i, τ_j) with $i, j \in \{1, 2\}$ and $i \neq j$. For simplicity, we abbreviate a τ_i-fuzzy open set μ and a τ_j-fuzzy closed set μ with a $\tau_i - fo$ set μ and a $\tau_j - fc$ set μ, respectively. Also, we denote the interior and the closure of μ for a fuzzy topology τ_i with $\tau_i - \mathrm{Int}\,\mu$ and $\tau_i - \mathrm{Cl}\,\mu$, respectively.

A fuzzy set μ on an $fbts$ X is called a (τ_i, τ_j)-*fuzzy semiopen* $[(\tau_i, \tau_j)$-*fso*$]$ set if $\mu \leq \tau_j - \mathrm{Cl}(\tau_i - \mathrm{Int}\,\mu)$, and μ is called a (τ_i, τ_j)-*fuzzy semiclosed* $[(\tau_i, \tau_j)$-*fsc*$]$ set if $\tau_j - \mathrm{Int}(\tau_i - \mathrm{Cl}\,\mu) \leq \mu$.

Let μ be a fuzzy set on an $fbts$ X. Then the (τ_i, τ_j)-*semi-interior of* μ, $(\tau_i, \tau_j) - \mathrm{sInt}\,\mu$, is $\bigvee\{\nu \mid \nu \leq \mu, \nu$ is a $(\tau_i, \tau_j) -$ *fso* set$\}$ and the (τ_i, τ_j)-*semi-closure of* μ, $(\tau_i, \tau_j) - \mathrm{sCl}\,\mu$, is $\bigwedge\{\nu \mid \nu \geq \mu, \nu$ is a $(\tau_i, \tau_j) -$ *fsc* set$\}$.

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is called a *fuzzy pairwise semicontinuous* $[fpsc]$ if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) -$ *fso* set on X for each

$\tau_i^* - fo$ set ν on Y and a mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is called a *fuzzy pairwise semiopen* $[fps\, open]$ if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - fso$ set on Y for each $\tau_i - fo$ set μ on X [6].

A fuzzy set μ on an $fbts$ X is called a (τ_i, τ_j)-*fuzzy* α-*open* $[(\tau_i, \tau_j)\text{-} f\alpha o]$ set if $\mu \le \tau_i - \operatorname{Int}(\tau_j - \operatorname{Cl}(\tau_i - \operatorname{Int}\mu))$, and μ is called a (τ_i, τ_j)-*fuzzy* α-*closed* $[(\tau_i, \tau_j)\text{-} f\alpha c]$ set if $\tau_i - \operatorname{Cl}(\tau_j - \operatorname{Int}(\tau_i - \operatorname{Cl}\mu)) \le \mu$.

Let μ be a fuzzy set on an $fbts$ X. Then the (τ_i, τ_j)-α-*interior of* μ, $(\tau_i, \tau_j) - \alpha\operatorname{Int}\mu$, is $\bigvee\{\nu \mid \nu \le \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\alpha o \text{ set}\}$ and the (τ_i, τ_j)-α-*closure of* μ, $(\tau_i, \tau_j) - \alpha\operatorname{Cl}\mu$, is $\bigwedge\{\nu \mid \nu \ge \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\alpha c \text{ set}\}$.

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is called a *fuzzy pairwise* α-*continuous* $[fp\alpha c]$ if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\alpha o$ set on X for each $\tau_i^* - fo$ set ν on Y and a mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is called a *fuzzy pairwise* α-*open* $[fp\alpha\ open]$ if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - f\alpha o$ set on Y for each $\tau_i - fo$ set μ on X [5].

A fuzzy μ on an $fbts$ X is called a (τ_i, τ_j)-*fuzzy* α-*semiopen* $[(\tau_i, \tau_j) - f\alpha so]$ set if $\mu \le \tau_i - \operatorname{Int}(\tau_j - \operatorname{Cl}((\tau_i, \tau_j) - \operatorname{sInt}\mu)))$, and μ is called a (τ_i, τ_j)-*fuzzy* α-*semiclosed* $[(\tau_i, \tau_j) - f\alpha sc]$ set on X if $\tau_i - \operatorname{Cl}(\tau_j - \operatorname{Int}((\tau_i, \tau_j) - \operatorname{sCl}\mu)) \le \mu$.

Let μ be a fuzzy set on an $fbts$ X. Then the (τ_i, τ_j)-α-*semi-interior of* μ, $(\tau_i, \tau_j) - \alpha\operatorname{sInt}\mu$, is $\bigvee\{\nu \mid \nu \le \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\alpha so \text{ set}\}$ and the (τ_i, τ_j)-α-*semi-closure of* μ, $(\tau_i, \tau_j) - \alpha\operatorname{sCl}\mu$, is $\bigwedge\{\nu \mid \nu \ge \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\alpha sc \text{ set}\}$.

Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called a *fuzzy pairwise* α*-semicontinuous* $[fp\alpha sc]$ if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\alpha so$ set on X for each $\tau_i^* - fo$ set ν on Y and f is called a *fuzzy pairwise* α*-semiopen* $[fp\alpha - semiopen]$ (*fuzzy pairwise* α*-semiclosed* $[fp\alpha - semiclosed]$) if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - f\alpha so$ $((\tau_i^*, \tau_j^*) - f\alpha sc)$ set on Y for each $\tau_i - fo$ $(\tau_i - fc)$ set μ on X.

Every $fp\alpha sc$ mapping is $fpsc$. And every $fp\alpha - semiopen$ $(fp\alpha - semiclosed)$ is fps open $(fps\ closed)$. But the converses are not true in general [2].

3. Fuzzy Pairwise α-semi-irresolute Continuous Mappings

In this section, we introduce a fuzzy pairwise α-semi-irresolute continuous mapping and a fuzzy pairwise α-semiopen irresolute mapping which are stronger than a fuzzy pairwise α-semicontinuous mapping and a fuzzy pairwise α-semiopen mapping, respectively. And we characterize a fuzzy pairwise α-semi-irresolute continuous mapping and a fuzzy pairwise α-semiopen irresolute mapping.

Definition 3.1. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called a *fuzzy pairwise* α*-semi-irresolute continuous* $[fp\alpha\text{-}semi\text{-}irresolute]$ *mapping* if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\alpha so$ set on X for each $(\tau_i^*, \tau_j^*) - f\alpha so$ set ν on Y.

It is clear that every *fp*α*-semi-irresolute* is *fp*α*sc* from the above definition. But the converse is not true in general as the following example shows.

Example 3.2. Let μ_1, μ_2 and μ_3 be fuzzy sets on $X = \{a, b, c\}$ with

$$\mu_1(a) = 0.2, \quad \mu_1(b) = 0.2, \quad \mu_1(c) = 0.2,$$

$$\mu_2(a) = 0.3, \quad \mu_2(b) = 0.3, \quad \mu_2(c) = 0.3 \text{ and}$$

$$\mu_3(a) = 0.4, \quad \mu_3(b) = 0.4, \quad \mu_3(c) = 0.4.$$

Let

$$\tau_1 = \{0_X, \mu_1, \mu_3, 1_X\}, \quad \tau_2 = \{0_X, \mu_2, \mu_3, 1_X\} \text{ and}$$

$$\tau_1^* = \{0_X, \mu_2, 1_X\}, \quad \tau_2^* = \{0_X, \mu_3, 1_X\}$$

be fuzzy topologies on X.

Then identity mapping $i_X : (X, \tau_1, \tau_2) \to (X, \tau_1^*, \tau_2^*)$ is $fp\alpha sc$. Since a $(\tau_i^*, \tau_j^*) - f\alpha so$ set μ_1^c is not $(\tau_i, \tau_j) - f\alpha so$, i_X is not *fpα-semi-irresolute*.

Theorem 3.1. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ *be a mapping. Then the following are equivalent*:

(1) *f is fpα-semi-irresolute*.

(2) *The inverse of each* $(\tau_i^*, \tau_j^*) - f\alpha sc$ *set on Y is a* $(\tau_i, \tau_j) - f\alpha sc$ *set on X*.

(3) $f((\tau_i, \tau_j) - \alpha sCl\,\mu) \le (\tau_i^*, \tau_j^*) - \alpha sCl(f(\mu))$ *for each fuzzy set* μ *on X*.

(4) $(\tau_i, \tau_j) - \alpha sCl(f^{-1}(\nu)) \le f^{-1}((\tau_i^*, \tau_j^*) - \alpha sCl\,\nu)$ *for each fuzzy set* ν *on Y*.

(5) $f^{-1}((\tau_i^*, \tau_j^*) - \alpha sInt\,\nu) \le (\tau_i, \tau_j) - \alpha sInt(f^{-1}(\nu))$ *for each fuzzy set* ν *on Y*.

Proof. (1) implies (2) Let ν be a $(\tau_i^*, \tau_j^*) - f\alpha sc$ set on Y. Then ν^c is a $(\tau_i^*, \tau_j^*) - f\alpha so$ set on Y. Since f is *fpα-semi-irresolute*, $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$ is a $(\tau_i, \tau_j) - f\alpha so$ set on X. Hence $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\alpha sc$ set on X.

(2) implies (3) Let μ be a fuzzy set on X. Then $f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu)))$ is a $(\tau_i, \tau_j) -$ $f\alpha sc$ set on X. Thus,

$$\begin{aligned}(\tau_i, \tau_j) - \alpha\mathrm{sCl}\,\mu &\leq (\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(f(\mu))) \\ &\leq (\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu)))) \\ &= f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu))).\end{aligned}$$

Hence

$$\begin{aligned}f((\tau_i, \tau_j) - \alpha\mathrm{sCl}\,\mu) &\leq f(f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu)))) \\ &\leq (\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu)).\end{aligned}$$

(3) implies (4) Let ν be a fuzzy set on Y. Then

$$\begin{aligned}f((\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu))) &\leq (\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(f^{-1}(\nu))) \\ &\leq (\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}\,\nu.\end{aligned}$$

Hence

$$\begin{aligned}(\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu)) &\leq f^{-1}(f((\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu)))) \\ &\leq f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}\,\nu).\end{aligned}$$

(4) implies (5) Let ν be a fuzzy set on Y. Then

$$(\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu^c)) \leq f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(\nu^c)).$$

Hence

$$\begin{aligned}f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sInt}\,\nu) &= f^{-1}(((\tau_i^* \tau_j^*) - \alpha\mathrm{sCl}(\nu^c))^c) \\ &\leq ((\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu^c)))^c \\ &= (\tau_i, \tau_j) - \alpha\mathrm{sInt}(f^{-1}(\nu)).\end{aligned}$$

(5) implies (1) Let ν be a $(\tau_i^*, \tau_j^*) -$ *fαso* set on *Y*. Then

$$f^{-1}(\nu) = f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu) \leq (\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)).$$

Hence $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) -$ *fαso* set on *X* and therefore, *f* is *fpα-semi-irresolute*. □

Theorem 3.2. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ *be a bijection. Then, f is fpα-semi-irresolute if and only if* $(\tau_i^*, \tau_j^*) - \alpha sInt(f(\mu)) \leq f((\tau_i, \tau_j) - \alpha sInt\,\mu)$ *for each fuzzy set* μ *on X*.

Proof. Let μ be a fuzzy set on *X*. Then, by Theorem 3.1,

$$f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(\mu))) \leq (\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(f(\mu))).$$

Since *f* is a bijection,

$$\begin{aligned}(\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(\mu)) &= f(f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(\mu)))) \\ &\leq f((\tau_i, \tau_j) - \alpha\text{sInt}\,\mu).\end{aligned}$$

Conversely, let ν be a fuzzy set on *Y*. Then

$$(\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(f^{-1}(\nu))) \leq f((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu))).$$

Recall that *f* is a bijection. Hence

$$\begin{aligned}(\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu &= (\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(f^{-1}(\nu))) \\ &\leq f((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)))\end{aligned}$$

and

$$\begin{aligned}f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu) &\leq f^{-1}(f((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)))) \\ &= (\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)).\end{aligned}$$

Therefore, by Theorem 3.1, *f* is *fpα-semi-irresolute*. □

Definition 3.3. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called:

(1) a *fuzzy pairwise* α-*semiopen irresolute* [*fp*α-*semiopen irresolute*] *mapping* if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - f\alpha so$ set on Y for each $(\tau_i, \tau_j) - f\alpha so$ set μ on X, and

(2) a *fuzzy pairwise* α-*semiclosed irresolute* [*fp*α-*semiclosed irresolute*] *mapping* if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - f\alpha sc$ set on Y for each $(\tau_i, \tau_j) - f\alpha sc$ set μ on X.

It is clear that every *fp*α-*semiopen irresolute* (*fp*α-*semiclosed irresolute*) is *fp*α-*semiopen* (*fp*α-*semiclosed*). But the converses are not true in general.

In fact, in Example 3.2, the identity mapping $i_X : (X, \tau_1^*, \tau_2^*) \to (X, \tau_1, \tau_2)$ is *fp*α-*semiopen* (*fp*α-*semiclosed*) but not *fp*α-*semiopen irresolute* (*fp*α-*semiclosed irresolute*).

Theorem 3.3. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ *be a mapping. Then the following are equivalent*:

(1) *f is fpα-semiopen irresolute*.

(2) $f((\tau_i, \tau_j) - \alpha sInt\,\mu) \leq (\tau_i^*, \tau_j^*) - \alpha sInt(f(\mu))$ *for each fuzzy set* μ *on* X.

(3) $(\tau_i, \tau_j) - \alpha sInt(f^{-1}(\nu)) \leq f^{-1}((\tau_i^*, \tau_j^*) - \alpha sInt\,\nu)$ *for each fuzzy set* ν *on* Y.

Proof. (1) implies (2) Let μ be a fuzzy set on X. Then $f((\tau_i, \tau_j) - \alpha \mathrm{sInt}\,\mu)$ is a $(\tau_i^*, \tau_j^*) - f\alpha so$ set on Y and $f((\tau_i, \tau_j) - \alpha \mathrm{sInt}\,\mu) \leq f(\mu)$. Hence

$$\begin{aligned} f((\tau_i, \tau_j) - \alpha \mathrm{sInt}\,\mu) &= (\tau_i^*, \tau_j^*) - \alpha \mathrm{sInt}(f((\tau_i, \tau_j) - \alpha \mathrm{sInt}\,\mu)) \\ &\leq (\tau_i^*, \tau_j^*) - \alpha \mathrm{sInt}(f(\mu)). \end{aligned}$$

(2) implies (3) Let ν be a fuzzy set on Y. Then

$$f((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu))) \leq (\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(f^{-1}(\nu)))$$

$$\leq (\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu.$$

Hence

$$(\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)) \leq f^{-1}(f((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu))))$$

$$\leq f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu).$$

(3) implies (1) Let μ be a $(\tau_i, \tau_j) -$ $f\alpha so$ set on X. Then

$$\mu = (\tau_i, \tau_j) - \alpha\text{sInt}\,\mu \leq (\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(f(\mu)))$$

$$\leq f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(\mu))).$$

We have

$$f(\mu) \leq f(f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(\mu)))) \leq (\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(\mu)).$$

Hence $f(\mu) = (\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(\mu))$. Consequently, $f(\mu)$ is a $(\tau_i^*, \tau_j^*) -$ $f\alpha so$ set on Y and therefore, f is *fpα-semiopen irresolute*. □

Theorem 3.4. *A mapping* $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ *is fpα-semiclosed irresolute if and only if* $(\tau_i^*, \tau_j^*) - \alpha sCl(f(\mu)) \leq f((\tau_i, \tau_j) - \alpha sCl\,\mu)$ *for each fuzzy set* μ *on X.*

Proof. Let μ be a fuzzy set on X. Then $f((\tau_i, \tau_j) - \alpha\text{sCl}\,\mu)$ is a $(\tau_i^*, \tau_j^*) -$ $f\alpha sc$ set on Y and $f(\mu) \leq f((\tau_i, \tau_j) - \alpha\text{sCl}\,\mu)$. Hence

$$(\tau_i^*, \tau_j^*) - \alpha\text{sCl}(f(\mu)) \leq (\tau_i^*, \tau_j^*) - \alpha\text{sCl}(f((\tau_i, \tau_j) - \alpha\text{sCl}\,\mu))$$

$$= f((\tau_i, \tau_j) - \alpha\text{sCl}\,\mu).$$

Conversely, let μ be a $(\tau_i, \tau_j) -$ *fαsc* set on *X*. Then

$$(\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu)) \leq f((\tau_i, \tau_j) - \alpha\mathrm{sCl}\,\mu)$$

$$= f(\mu).$$

Consequently, $f(\mu)$ is a $(\tau_i^*, \tau_j^*) -$ *fαsc* set on *Y* and therefore, *f* is *fpα-semiclosed irresolute*. □

Theorem 3.5. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ *be a bijection. Then the following are equivalent*:

(1) *f is fpα-semiclosed irresolute*.

(2) $f^{-1}((\tau_i^*, \tau_j^*) - \alpha sCl\nu) \leq (\tau_i, \tau_j) - \alpha sCl(f^{-1}(\nu))$ *for each fuzzy set* ν *on Y*.

(3) *f is fpα-semiopen irresolute*.

(4) f^{-1} *is fpα-semi-irresolute*.

Proof. (1) implies (2) Let ν be a fuzzy set on *Y*. Then, by Theorem 3.4,

$$(\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(f^{-1}(\nu))) \leq f((\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu))).$$

Hence

$$f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(f^{-1}(\nu)))) \leq f^{-1}(f((\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu)))).$$

Since *f* is a bijection,

$$f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}\,\nu) \leq (\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(\nu)).$$

(2) implies (1) Let μ be a fuzzy set on *X*. Then

$$f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu))) \leq (\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(f(\mu))).$$

Hence

$$f(f^{-1}((\tau_i^*, \tau_j^*) - \alpha\mathrm{sCl}(f(\mu)))) \leq f((\tau_i, \tau_j) - \alpha\mathrm{sCl}(f^{-1}(f(\mu)))).$$

Since f is a bijection,

$$(\tau_i^*, \tau_j^*) - \alpha\text{sCl}(f(\mu)) \leq f((\tau_i, \tau_j) - \alpha\text{sCl}\,\mu).$$

Therefore, by Theorem 3.4, f is *fpα-semiclosed irresolute*.

(2) implies (3) Let ν be a fuzzy set on Y. Then

$$f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sCl}(\nu^c)) \leq (\tau_i, \tau_j) - \alpha\text{sCl}(f^{-1}(\nu^c)).$$

Thus,

$$\begin{aligned}(\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)) &= ((\tau_i, \tau_j) - \alpha\text{sCl}(f^{-1}(\nu^c)))^c \\ &\leq f^{-1}(((\tau_i^*, \tau_j^*) - \alpha\text{sCl}(\nu^c))^c) \\ &= f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu).\end{aligned}$$

Hence f is *fpα-semiopen irresolute* from Theorem 3.3.

(3) implies (4) Let μ be a $(\tau_i, \tau_j) - f\alpha so$ set on X. Then $(f^{-1})^{-1}(\mu) = f(\mu)$. Since f is *fpα-semiopen irresolute*, $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - f\alpha so$ set on Y. Hence f^{-1} is *fpα-semi-irresolute*.

(4) implies (2) It is clear from Theorem 3.1. □

We have the following corollaries from Theorem 3.1, Theorem 3.4 and Theorem 3.3.

Corollary 3.6. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then, f is an fpα-semiclosed irresolute and fpα-semi-irresolute if and only if $f((\tau_i, \tau_j) - \alpha sCl\mu) = (\tau_i^*, \tau_j^*) - \alpha sCl(f(\mu))$ for each fuzzy set μ on X.* □

Corollary 3.7. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then, f is fpα-semiopen irresolute and fpα-semi-irresolute if and only if $f^{-1}((\tau_i^*, \tau_j^*) - \alpha sCl\nu) = (\tau_i, \tau_j) - \alpha sCl(f^{-1}(\nu))$ for each fuzzy set ν on Y.* □

A bijection $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is called a *fuzzy pairwise* α*-semihomeomorphism* if f and f^{-1} are *fp*α*-semi-irresolute* continuous mappings.

Theorem 3.8. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ *be a bijection. Then the following are equivalent*:

(1) *f is a fuzzy pairwise* α*-semihomeomorphism*.

(2) f^{-1} *is a fuzzy pairwise* α*-semihomeomorphism*.

(3) *f and* f^{-1} *are fp*α*-semiopen irresolute* (*fp*α*-semiclosed irresolute*).

(4) *f is fp*α*-semi-irresolute and fp*α*-semiopen irresolute* (*fp*α*-semiclosed irresolute*).

(5) $f((\tau_i, \tau_j) - \alpha sCl\, \mu) = (\tau_i^*, \tau_j^*) - \alpha sCl(f(\mu))$ *for each fuzzy set* μ *on X*.

(6) $f((\tau_i, \tau_j) - \alpha sInt\, \mu) = (\tau_i^*, \tau_j^*) - \alpha sInt(f(\mu))$ *for each fuzzy set* μ *on X*.

(7) $f^{-1}((\tau_i^*, \tau_j^*) - \alpha sInt\, \nu) = (\tau_i, \tau_j) - \alpha sInt(f^{-1}(\nu))$ *for each fuzzy set* ν *on Y*.

(8) $(\tau_i, \tau_j) - \alpha sCl(f^{-1}(\nu)) = f^{-1}((\tau_i^*, \tau_j^*) - \alpha sCl\, \nu)$ *for each fuzzy set* ν *on Y*.

Proof. (1) implies (2) It follows immediately from the definition of a fuzzy pairwise α-semihomeomorphism.

(2) implies (3) and (3) implies (4) They follow from Theorem 3.5.

(4) implies (5) It follows from Theorem 3.5 and Corollary 3.6.

(5) implies (6) Let μ be a fuzzy set on X. Then

$$f((\tau_i, \tau_j) - \alpha\text{sInt}\,\mu) = (f((\tau_i, \tau_j) - \alpha\text{sCl}(\mu^c)))^c$$

$$= ((\tau_i^*, \tau_j^*) - \alpha\text{sCl}(f(\mu^c)))^c$$

$$= (\tau_i^*, \tau_j^*) - \alpha\text{sInt } f(\mu).$$

(6) implies (7) Let μ be a fuzzy set on Y. Then

$$f((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)) = (\tau_i^*, \tau_j^*) - \alpha\text{sInt}(f(f^{-1}(\nu)))$$

$$= (\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu.$$

Hence

$$f^{-1}(f((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)))) = f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu).$$

Therefore,

$$(\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu)) = f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}\,\nu).$$

(7) implies (8) Let ν be a fuzzy set on Y. Then

$$(\tau_i, \tau_j) - \alpha\text{sCl}(f^{-1}(\nu)) = (f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sInt}(\nu^c)))^c$$

$$= ((\tau_i, \tau_j) - \alpha\text{sInt}(f^{-1}(\nu^c)))^c$$

$$= f^{-1}((\tau_i^*, \tau_j^*) - \alpha\text{sCl}\,\nu).$$

(8) implies (1) It follows from Theorem 3.5 and Corollary 3.6. □

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