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SHIP HEADING CONTROL OF CORVETTE-SIGMA WITH DISTURBANCES USING MODEL PREDICTIVE CONTROL

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Abstract

Ship heading control has been a representative control problem for marine application and has attracted considerable attention from the control community. The large yaw rate can produce other motions (such as sway and roll) that can cause seasickness and cargo damage, enforcing yaw rate constraints while maneuvering in seaways becomes an important design consideration in surface vessel. In this paper, the Sigma class corvette ship is adopted as an example. To address the constraint violation for ship heading control in wave fields, the model predictive control (MPC) controller has been proposed to satisfy the state constraints in the presence of environmental disturbances. The simulation results show that the performance of proposed controller in terms of satisfying yaw rate and actuator saturation constraint.

I. Introduction

The problem in ship control system is heading control, or so-called

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course keeping, which is the main system in autopilot control [1]. In a ship heading control, Nomoto model is the most commonly used model. This model considers only one degree of freedom (DOF) dynamic system, namely *r* (yaw rate), and a control input [2]. Some methods have been used in the automatic control system on ship's heading control. A fuzzy logic controller for ship path control in restricted waters is developed by Parson et al. [3]. McGookin et al. developed genetic algorithms to optimize the performance of the complete system under various operating conditions by optimizing the parameters of the sliding mode controller [4]. Proportional-Integral-Derivative controller (PID) algorithms are applied by Jagannathan to control rudder angle [5].

In this paper, model predictive control is used, where by using an optimization approach to deal with systems that have constraints on input and state. This type of control is included in the category of controller-based process model, which explicitly models the process used to design the controller, by minimizing a criterion function. Moreover, MPC can also combine all the objectives into a single objective function [2].

It is essential to take into account the environmental disturbance in the design of ship control, such as: wave and wind. In this paper, the environmental disturbance factor is wave and model predictive control (MPC) will be employed to ship heading control.

II. Model Predictive Control

Prediction horizon in MPC refers to the measures that used to predict the output. In prediction horizon, the previous control input becomes the guideline to determine predictive control input that will be used to predict the next output. In MPC, control signals change at any time, so the response can produce the better value on the system.

The mathematical model of the ship heading control is a linear discrete state and can be described by [6, 7]:

$$\tilde{\mathbf{x}}(k+1|k) = \mathbf{A}\tilde{\mathbf{x}}(k) + \mathbf{B}\tilde{\mathbf{u}}(k)$$

$$\tilde{\mathbf{y}}(k) = \tilde{\mathbf{x}}(k), \tag{1}$$

where

$\tilde{\mathbf{x}}(k)$: state variable vector,

$\tilde{\mathbf{y}}(k)$: output variable vector,

$\tilde{\mathbf{u}}(k)$: input variable vector.

The state equation (1) is an ideal condition, where there is no disturbance. In this paper, there is environmental disturbance that is ocean wave. In the case of linear MPC with disturbance, the form of equation (1) becomes [2]:

$$\tilde{\mathbf{x}}(k+1|k) = \mathbf{A}\tilde{\mathbf{x}}(k) + \mathbf{B}\tilde{\mathbf{u}}(k) + \tilde{\mathbf{w}}(k), \tag{2}$$

where $\tilde{\mathbf{w}}(k)$ is disturbance vector. At each time k we define the stage cost [8]:

$$\mathbf{s}(\tilde{\mathbf{x}}(k), \tilde{\mathbf{u}}(k)) = \begin{bmatrix} \tilde{\mathbf{x}}(k) \\ \tilde{\mathbf{u}}(k) \end{bmatrix}^T \begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}(k) \\ \tilde{\mathbf{u}}(k) \end{bmatrix}, \tag{3}$$

where $\mathbf{Q}$ is error weighting matrix on the state and $\mathbf{R}$ is control weighting matrix. The state and control constraints are defined as:

$$\mathbf{F}_1\tilde{\mathbf{x}}(k) \le \mathbf{f}_1,$$

$$\mathbf{F}_2\tilde{\mathbf{u}}(k) \le \mathbf{f}_2. \tag{4}$$

In MPC, the control $\tilde{\mathbf{u}}(k)$ is found at each step by first solving quadratic programming [8] as follows:

$$J = \sum_{j=k}^{k+N-1} \mathbf{s}(\tilde{\mathbf{x}}(j+1), \tilde{\mathbf{u}}(j)) \tag{5}$$

subject to

$$\mathbf{F}_1\tilde{\mathbf{x}}(j+1) \le \mathbf{f}_1,\ j = k, k+1, \ldots, k+N-1 \tag{6}$$

$$\mathbf{F}_2\tilde{\mathbf{u}}(j) \le \mathbf{f}_2,\ j = k, k+1, \ldots, k+N-1 \tag{7}$$

$$\tilde{\mathbf{x}}(k+1|k) = \mathbf{A}\tilde{\mathbf{x}}(k) + \mathbf{B}\tilde{\mathbf{u}}(k) + \tilde{\mathbf{w}}(k) \tag{8}$$

$$\tilde{\mathbf{x}}(j+2|j) = \mathbf{A}\tilde{\mathbf{x}}(j+1) + \mathbf{B}\tilde{\mathbf{u}}(j+1),\ j = k,\ k+1,\ \ldots,\ k+N-2. \tag{9}$$

The objective function in equation (5) can be written in the form of quadratic programming as:

$$J = \mathbf{z}^{\mathbf{T}}\mathbf{H}\mathbf{z} + \mathbf{g}^{\mathbf{T}}\mathbf{z} \tag{10}$$

subject to

$$\mathbf{P}\mathbf{z} \leq \mathbf{h} \tag{11}$$

$$\mathbf{Y}\mathbf{z} = \mathbf{b}, \tag{12}$$

where $\mathbf{z} = (u(t),\ x(t+1),\ u(t+1),\ \ldots,\ x(t+T-1),\ u(t+T-1))$. At each time k, the MPC policy takes

$$\tilde{\mathbf{u}}(k|k) = \tilde{\mathbf{u}}^{*}(k|k). \tag{13}$$

III. Mathematical Model of Ship Heading Control with Disturbance

For marine vehicles, the 6 degree of freedom (DOF) is defined as: surge, sway, heave, roll, pitch, and yaw. In general, there are two kinds of ship's movement in the ocean, namely translation and rotation motion [1]. Translation motion of the ship is divided into three types: surge (motion in the *x*-direction), sway (motion in the *y*-direction) and heave (motion in the *z*-direction). While rotation motion of the ship is divided into three types: roll (rotation on the *x*-axis), pitch (rotation on the *y*-axis), and yaw (rotation on the *z*-axis).

Ship heading control, or the so-called course keeping, is the primary task of autopilots. The control objective of ship heading control is to obtain [2]:

$$\psi \to \psi_d, \tag{14}$$

where ψ is ship's actual heading angle, and ψ_d is the desired ship heading angle, which is normally assumed to be constant. Notice that $\dot{\psi} = r$, where r is the yaw rate. For the ship heading control design, the Nomoto model is by far the most commonly employed one in the literature [1]. The Nomoto model considers 1 DOF ship dynamics, namely, the yaw rate r, and one control input, namely, the rudder angle δ. The illustration of ship heading and rudder angle is shown in Figure 1. In this study, the other degrees of freedom of the ship have been neglected.

These disturbances form a sinusoidal wave in yaw motion of the vessel that satisfies the equations:

$$\psi = \psi_a \sin(\varpi t), \tag{15}$$

$$\psi = \varpi \psi_a \sin(\varpi t), \tag{16}$$

where ϖ is the frequency of the waves against the ship dynamic system, and ψ_a is the amplitude of the waves after the value is multiplied by a RAO (Response Amplitude Operator) factor to the ship's dynamic system. From equations (15)-(16), we obtain the disturbance vector of ship's dynamical system as:

$$\widetilde{\mathbf{w}}(k) = [\dot{\psi};\ \psi] = [\varpi \psi_a \sin(\varpi t);\ \psi_a \sin(\varpi t)].$$

The value of the disturbance depends on the sea state where the ship is operated at that time. In marine field, sea state is a condition of the sea surface which is closely related to the waves, and specific time and location. Sea state can be seen from the statistical data which consists of wave height, wave period, and their characteristics.

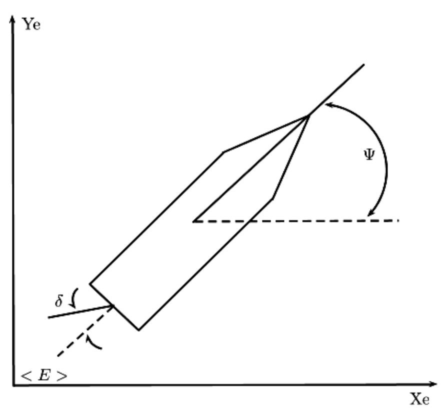

Figure 1. Ship heading control and rudder angle.

IV. Computational Result

In this paper, the Sigma class corvette ship is used as study case. This ship can work up to sea state 5. Table 1 shows the parameter data of the ship.

Table 1. Parameter data of Sigma class corvette ship

Parameter	**Value**	**Parameter**	**Value**
ρ (kg/m^3)	1024	T (m)	3.7
L (m)	101.07	C_B	0.65
U (m/s)	15.4	X_G (m)	5.25
B (m)	14	m (ton)	2423

Based on the above parameters, the second-order transfer function of Nomoto model can be described as:

$$\frac{r(s)}{\delta_R(s)} = \frac{1715.3329 \cdot s + 502.3033}{9.5323 s^2 + 4.7013 s + 1}.$$

The next step is to change the transfer function into the state and discretize the state matrix using a finite difference scheme with sampling time $\Delta t = 0.1$, so we get the equation:

$$\begin{bmatrix} \tilde{\mathbf{x}}_1(k+1) \\ \tilde{\mathbf{x}}_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.9629 & -0.00503 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_1(k) \\ \tilde{\mathbf{x}}_2(k) \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \tilde{\mathbf{u}}(k). \tag{17}$$

A system is said to be *controllable* if for any state $\mathbf{x}_0$, there is input $\mathbf{u}(k)$ which is not limited to any transfer state $\mathbf{x}_0$ to the last state $\mathbf{x}_k$ with finite last time k. Based on equation (17) with a value of $\mathbf{A}$ and $\mathbf{B}$ is known, it can be shown if rank $[\mathbf{B} \mid \mathbf{AB}] = 2$, which means that the system is controllable. This allows the application of the linear MPC in the ship heading control.

The constraints of ship heading control depend on the maximum value of the rudder angle (δ) and the maximum value of the yaw rate. The maximum limit for the rudder angle (δ) is equal to $35° = 35\pi/180$ rad, while the maximum limit of the yaw rate is 0.0932 rad/sec. Therefore, by using (4) we obtain:

$$\mathbf{F}_1 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \quad \mathbf{f}_1 = \begin{bmatrix} 0.0932 \\ 0.0932 \end{bmatrix}, \quad \mathbf{F}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{f}_2 = \begin{bmatrix} 35\pi/180 \\ 35\pi/180 \end{bmatrix},$$

where $\mathbf{Q} = \begin{bmatrix} 300 & 0 \\ 0 & 300 \end{bmatrix}$ and $\mathbf{R} = 1$.

From the analysis of the MPC controller that has been described previously, the simulation will be shown. We give an initialization $\tilde{\mathbf{x}}(0) = [0, 30°]$, and $\tilde{\mathbf{u}}(0) = 0$. The purpose of ship heading control with the MPC method is to keep the yaw rate and the rudder angle of the ship are within the constraints, and the ship's heading angle towards 0°. The ship is assumed to move at a constant speed of 30 knots at surge. When the heading angle was at 0°, the ship just did a straightforward motion with a constant surge speed of 30 knots.

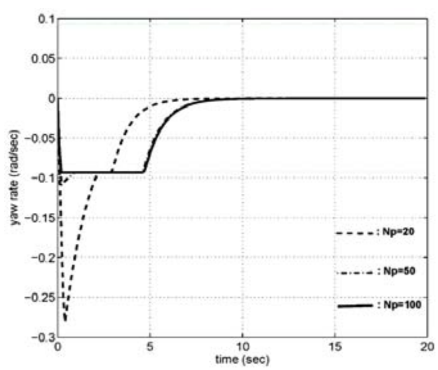

Figure 2. Yaw rate without disturbance for varying predictive horizon.

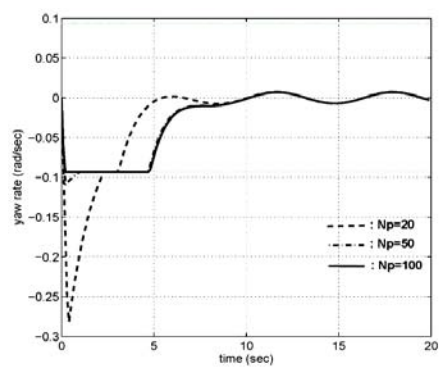

Figure 3. Yaw rate with disturbance for varying predictive horizon.

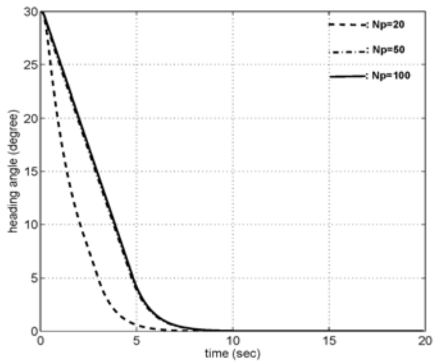

Figure 4. Heading angle without disturbance for varying predictive horizon.

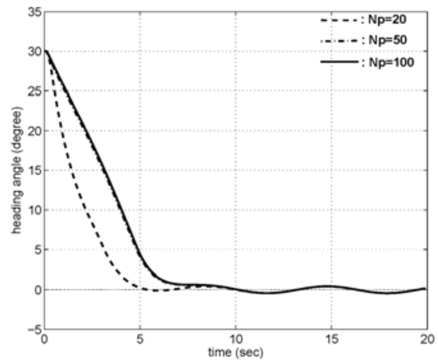

Figure 5. Heading angle with disturbance for varying predictive horizon.

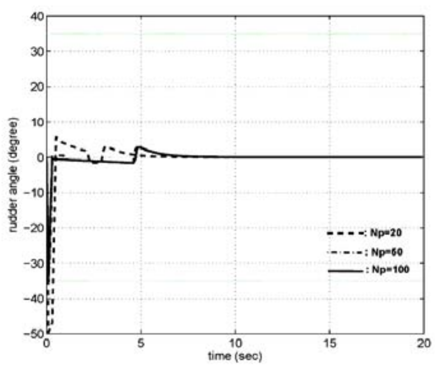

Figure 6. Rudder angle without disturbance for varying predictive horizon.

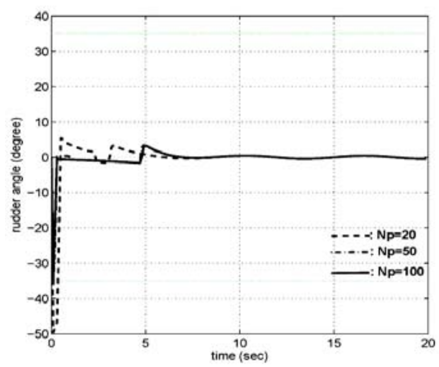

Figure 7. Rudder angle with disturbance for varying predictive horizon.

A. The simulation without disturbance

The simulation without disturbance begins with varying prediction horizon values N on MPC controller. Other parameters are made fixed. The simulation time is 20 seconds, with a sampling time $\Delta t = 0.1$ seconds. The simulation results can be seen on Figures 2, 4 and 6.

It can be seen that the controller will get better results if the value of the prediction horizon (N) increases. On Figure 2, by giving the value of $N = 20,$ it can be seen that the value of the yaw rate violates the constraint. Then by increasing the value of N to be 50, the value of yaw rate constraint is still outside but it gets better. When the value of N is enlarged again to 100, the value of the yaw rate satisfies the specified constraints.

The control of system is also becoming increasingly well with the increasing value of N. On Figure 6 with a value of $N = 20,$ it can be seen that the value of the ship's rudder angle violates the constraint. Then by increasing the value of N to be 50, the value of the ship rudder angle just slightly outside constraint. When the value of N is enlarged again to 100, the value of the ship rudder angle is within the constraints. It can be concluded that the greater value of N, the control system gets better and therefore the system becomes more stable.

B. The simulation with disturbance

In this simulation, the disturbance is considered in the model. The simulation time is 20 seconds, with a sampling time $\Delta t = 0.1$ with value $\varpi = 0.1,$ and $\psi_a = 0.001.$ The simulation results can be seen on Figures 3, 5 and 7. It can be seen that the controller will get better results if the value of the prediction horizon (N) increases. On Figure 3, by giving value $N = 20,$ it is seen that the value of the yaw rate is far from the limit specified constraint. Then by increasing the value of N to 50, the value of yaw rate is still outside the constraint but better than before. When the value of N is increasing to 100, the value of the yaw rate is within the constraint. Overall, the value of the yaw rate cannot be really towards 0 by using the MPC method, but this controller can keep the system within the constraint boundaries for large value of N.

On the other hand, the control system is also becoming increasingly well by increasing the value of N. On Figure 7 with a value of $N = 20,$ it can be seen that the value of the ship's rudder angle is still outside the specified constraint. Then by increasing the value of N into 50, the rudder angle is just slightly outside constraint. When the value of N is increasing into 100, the rudder angle is within the constraint.

C. The simulation with varying disturbances

There are two types of disturbances which are:

Disturbance type 1: $\tilde{\mathbf{w}}(k) = [0.00008\cos(0.08t);\ 0.001\sin(0.08t)]$.

Disturbance type 2: $\tilde{\mathbf{w}}(k) = [0.0001\cos(0.1t);\ 0.001\sin(0.1t)]$.

Other parameters are fixed as described in the previous section. The value of N is 100. Figures 8-10 show that the MPC controller works well with some type disturbances. Figure 8 shows that the yaw rate is under the constraint of 0.0932 rad/sec. The control values as shown in Figure 10 suggest that the rudder angle is within the constraint of 35°. MPC controller is good enough to keep the system is in the constraint.

V. Conclusion

From the analysis of the MPC controller system and its simulation, it is concluded that the model predictive control (MPC) works well for the ship heading control with disturbances. It can be seen from the simulation results that the system is within the constraint limits with the large predictive horizon, either on the system without disturbance, or the system with the disturbance.

The simulation results of MPC without disturbance show that the yaw rate and rudder angle of the ship are within the constraint limits and converge to 0 and also the heading angle converges to 0°. For the MPC with disturbance, the simulation results show that the yaw rate and rudder angle are within the constraint limits, but its value cannot converge to 0. It is also seen for the heading angle. The heading angle oscillates following the shape of the given disturbance equation.

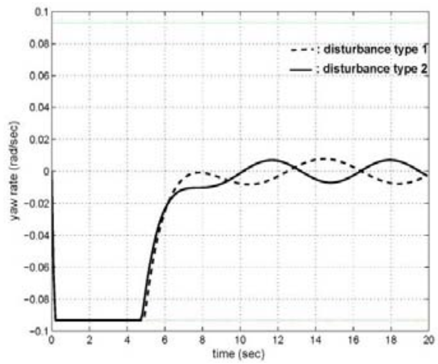

Figure 8. Yaw rate with disturbance types 1 and 2.

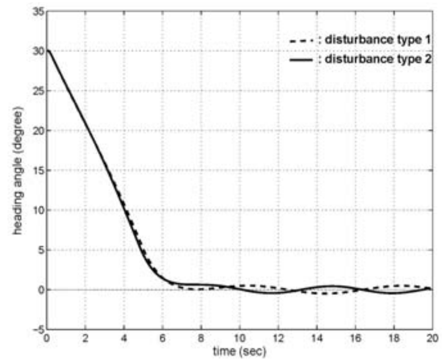

Figure 9. Heading angle with disturbance types 1 and 2.

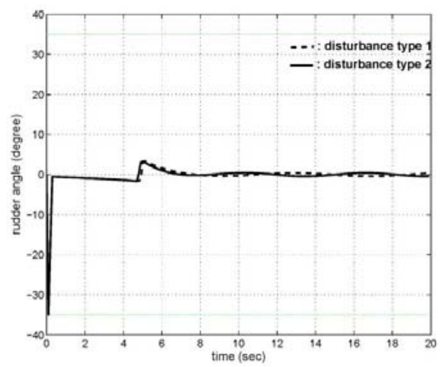

Figure 10. Rudder angle with disturbance types 1 and 2.

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