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THERMAL RADIATION AND RADIATION ABSORPTION EFFECTS ON UNSTEADY MHD DOUBLE DIFFUSIVE FREE CONVECTION FLOW OF KUVSHINSKI FLUID PAST A MOVING POROUS PLATE EMBEDDED IN A POROUS MEDIUM WITH CHEMICAL REACTION AND HEAT GENERATION

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Abstract

The objective of the present work is to analyze the effects of chemical reaction, thermal radiation and radiation absorption on unsteady double diffusive free convection flow of Kuvshinski fluid past a moving porous plate with heat generation under the influence of a uniform transverse magnetic field. The plate moves with constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. The governing equations for this investigation are formulated and solved using perturbation technique. Non-dimensional velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are discussed through graphs for different values of parameters entering into the problem.

Nomenclature

g	Acceleration due to gravity
t^*	Time
C_p	Specific heat at constant pressure
λ_1^*	Coefficient of Kuvshinski fluid
T^*	Temperature
T_ω^*	Temperature of the fluid at the surface
T_∞^*	Temperature of the fluid in the free stream
q_r^*	Radiative heat flux
K^*	Permeability of the porous medium
B_0	Magnetic field strength
D_m	Coefficient of chemical molecular diffusivity
C^*	Species concentration in the fluid

C_{ω}^{*}	Species concentration at the surface
C_{∞}^{*}	Species concentration in the free stream
R^{*}	Reaction rate constant
R_{1}^{*}	Radiation absorption coefficient
Q_{0}	Heat source/sink constant
U_{0} and n^{*}	Constants
Gr	Thermal Grashof number
Gm	Mass Grashof number
M	Magnetic parameter
K	Porosity parameter
S	Heat source/sink parameter
Pr	Prandtl number
Ra	Radiation absorption parameter
R	Thermal radiation parameter
Kr	Chemical reaction parameter
Sc	Schmidt number
λ_{1}	Visco-elastic parameter

Greek symbols

β	Coefficient of volume expansion of heat transfer
β^{*}	Coefficient of volume expansion of mass transfer
σ	Electrical conductivity
μ	Viscosity of fluid
ν	Kinematic viscosity
ρ	Density

κ	Thermal conductivity
θ	Dimensionless fluid temperature
ϕ	Dimensionless species concentration
τ	Shearing stress
ε	Scalar constant ($<<1$)

Introduction

Unsteady free convection flows in a porous medium have received much attention in recent time due to its wide applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. Moreover, considerable interest has been shown in radiation interaction with convection for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly, in free convection problems involving absorbing emitting fluids. The unsteady fluid flow past a moving plate in the presence of free convection and radiation were studied by Mansour [1], Raptis and Perdikis [2], Ganesan and Loganathan [3], Makinde [4] and Abdus-Sattar and Hamid Kalim [5]. All these studies have been confined to unsteady flow in a non-porous medium. From the porous literature survey about unsteady fluid flow, we observe that little papers are done in porous medium. The effect of radiation on MHD flow and heat transfer must be considered when high temperatures are reached. El-Hakiem [6] studied the unsteady MHD oscillatory flow on free convection radiation through a porous medium with vertical infinite surface that absorbs the fluid with a constant velocity. Ghaly [7] employed a symbolic computation software Mathematica to study the effect of radiation on heat and mass transfer over stretching sheet in the presence of magnetic field. Raptis et al. [8] studied the effect of radiation on 2D steady MHD optically thin gray gas flow along an infinite vertical plate taking into account the induced magnetic field. Takhar et al. [9] described the radiation effects on MHD free convection flow past a semi-infinite vertical plate. Kim [10] studied unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Chamkha [11] investigated unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption. Shanker et al. [12] presented a numerical solution for radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption using Galerkin finite element method. Hady et al. [13] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect. Hossain et al. [14] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/ absorption.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Chamkha [15] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Muthucumaraswamy and Ganesan [16] investigated the effects of a chemical reaction on the unsteady flow past an impulsively started semi-infinite vertical plate which is subjected to uniform heat flux. Ibrahim et al. [17] analyzed the effects of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Mohamed [18] has discussed double diffusive convection radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects.

In spite of all these studies, the unsteady MHD double diffusive free convection flow of Kuvshinski fluid with thermal radiation, radiation absorption, chemical reaction and heat generation has received little attention. Hence, the main objective of the present investigation is to study the effect of thermal radiation and radiation absorption on unsteady MHD double diffusive free convection flow of Kuvshinski fluid past a moving porous plate with chemical reaction and heat generation in the presence of mass blowing or suction. It is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. It is also assumed that the free stream to consist of a mean velocity and temperature over which are superimposed an exponentially varying with time.

Mathematical Formulation

In this problem, we consider two-dimensional unsteady double diffusive free convection flow of an incompressible, electrically conducting, visco-elastic fluid (Kuvshinski fluid) past a semi-infinite vertical moving porous plate embedded in a uniform porous medium under the influence of a uniform magnetic field in the presence of chemical reaction, heat generation, thermal radiation and radiation absorption. Let x^*-axis is taken along the porous plate in the upward direction and y^*-axis is normal to it. The fluid is assumed to be gray, absorbing-emitting but non-scattering medium. The radiative heat flux in the x^*-direction is considered negligible in comparison with that in the y^*-direction. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy resistance terms are taken into account in the constant permeability porous medium. The MHD term is derived from an order of magnitude analysis of the full Navier-Stokes equation. It is assumed that the whole size of the porous plate is significantly larger than characteristic microscopic length scale of the porous medium. We regard the porous medium as an assemblage of small identical spherical particles fixed in space. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant of the linear momentum equation which is approximation. The fluid properties are assumed to be constant except that the influence of density variation with temperature and concentration has been considered in the body force term. The magnetic and viscous dissipations are neglected. Due to the assumption that the plate in x^*-direction is of infinite length, all the flow variables except pressure are functions of y^* and t^* only. Under the above assumptions, the governing equations are

$$\frac{\partial V^*}{\partial y^*} = 0, \tag{1}$$

$$\rho\left\{\left(1+\lambda_1^*\frac{\partial}{\partial t^*}\right)\frac{\partial u^*}{\partial t^*} + V^*\frac{\partial u^*}{\partial y^*}\right\} = -\left(1+\lambda_1^*\frac{\partial}{\partial t^*}\right)\frac{\partial p^*}{\partial x^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$- \rho g - \left(\sigma B_0^2 + \frac{\mu}{K^*}\right)\left(1+\lambda_1^*\frac{\partial}{\partial t^*}\right)u^*, \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + V^*\frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Q_0}{\rho C_p}(T^* - T_\infty^*)$$

$$- \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} - \frac{R_1^*}{\rho C_p}(C^* - C_\infty^*), \tag{3}$$

$$\frac{\partial e^*}{\partial t^*} + V^*\frac{\partial c^*}{\partial y^*} = D_m\frac{\partial^2 C^*}{\partial y^{*2}} - R^*(C^* - C_\infty^*). \tag{4}$$

The boundary conditions are

$$u^* = u_p^*,\ T^* = T_\omega^* + \varepsilon(T_\omega^* - T_\infty^*)/e^{n^*t^*},\ C^* = C_\omega^* + \varepsilon(C_\omega^* - C_\infty^*)e^{n^*t^*}$$

$$\text{at } y = 0, \tag{5}$$

$$u^* \to u_\infty^* = U_0(1+\varepsilon e^{n^*t^*}), \quad T^* \to T_{\infty,}^* \quad C^* \to C_\infty^*. \tag{6}$$

It is assumed that the porous plate moves with a constant velocity u_p^* in the direction of the fluid flow and the free stream velocity U_∞^* follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time.

It is clear from equation (1) that the suction velocity at the plate surface is a function of time only. Assuming that it takes the following form:

$$V^* = -V_0(1 + \varepsilon A e^{n^* t^*}), \tag{7}$$

where A is a real positive constant, ε and εA are real numbers less than unity, and V_0 is a scale of suction velocity which has non-zero positive constant.

In the free stream, from equation (2) we get

$$\rho \frac{dU_\infty^*}{dt^*} = -\frac{\partial p^*}{\partial x^*} - \rho_\infty g - \sigma B_0^2 U_\infty^* - \frac{\mu}{K^*} U_\infty^*. \tag{8}$$

Eliminating $\frac{\partial P^*}{\partial x^*}$ from equations (2) and (8), we obtain

$$\rho\left\{\left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right)\frac{\partial u^*}{\partial t^*} + V^* \frac{\partial u^*}{\partial y^*}\right\} = \rho\left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right)\frac{dU_\infty^*}{dt^*}$$

$$+ \mu \frac{\partial^2 U^*}{\partial y^{*2}} + (\rho_\infty - \rho) g + \left(\sigma B_0^2 + \frac{\mu}{K^*}\right)\left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right)(U_\infty^* - u^*). \tag{9}$$

By making use of the equation of state

$$(\rho_\infty - \rho) = \rho\beta(T^* - T_\infty^*) + \rho\beta^*(C^* - C_\infty^*). \tag{10}$$

Substituting equation (10) into equation (9), we obtain

$$\left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right)\frac{\partial u^*}{\partial t^*} + V^* \frac{\partial u^*}{\partial y^*} = \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right)\frac{\partial U_\infty^*}{dt^*}$$

$$+ \nu \frac{\partial^2 U^*}{\partial y^{*2}} + g\beta(T^* - T^*_\infty)$$

$$+ g\beta^*(C^* - C^*_\infty) + \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K^*}\right)\left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right)(U^*_\infty - u^*), \tag{11}$$

where ρ_∞ is the density of the fluid far away the surface.

The radiation flux on the basis of the Rosseland diffusion model for radiation heat transfer is expressed as

$$q_r^* = \frac{-4\sigma^*}{3K_1^*} \frac{\partial T^{*4}}{\partial y^*}, \tag{12}$$

where σ^* and K_1^* are, respectively, the Stefan–Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding in Taylor series about T^*_∞ and neglecting higher order terms, thus

$$T^{*4} \cong 4T^{*3}_\infty T^* - 3T^{*4}_\infty. \tag{13}$$

By using equations (12) and (13), equation (3) is reduced to

$$\frac{\partial T^*}{\partial t^*} + V^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Q_0}{\phi C_p}(T^* - T^*_\infty)$$

$$- \frac{16\sigma^* T^{*3}_\infty}{3\rho C_p K_1^*} \frac{\partial^2 T^*}{\partial {y^*}^2} - \frac{R_1^*}{\rho C_p}(C^* - C^*_\infty). \tag{14}$$

Introducing the dimensionless variables as follows:

$$u = \frac{u^*}{U_0},\ v = \frac{v^*}{V_0},\ y = \frac{y^* V_0}{\nu},\ U^*_\infty = U_\infty U_0,$$

$$u_p^* = U_p U_0,\ t = \frac{t^* V_0^2}{\nu},\ n = \frac{n^* \nu}{V_0^2},$$

$$\theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*},\ \phi = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*},\ Gr = \frac{\nu g \beta (T_w^* - T_\infty^*)}{V_0^2 U_0},$$

$$Gm = \frac{\nu g \beta^* (C_w^* - C_\infty^*)}{V_0^2 U_0},\ M = \frac{\sigma B_0^2 \nu}{\rho V_0^2},\ K = \frac{K^* V_0^2}{\nu^2},\ S = \frac{\phi_0 \nu^2}{k V_0^2},$$

$$Pr = \frac{\mu C_p}{\kappa},\ Ra = R_1^* \left(\frac{C_\omega^* - C_\infty^*}{T_\omega^* - T_\infty^*} \right) \frac{\nu^2}{V_0^2},\ \lambda_1 = \frac{\lambda_1^* V_0^2}{\nu},$$

$$R = \frac{4\sigma^* T_\infty^{*^3} \nu^2}{K_1 \kappa V_0^2},\ Kr = \frac{R^* \nu}{\nu_0^2},\ Sc = \frac{\nu}{D_m}. \tag{15}$$

Definition of Parameters

Prandtl number (Pr)

It is an important dimensionless parameter dealing with the properties of a fluid. It is defined as the ratio of viscous force to thermal force of a fluid.

Hartmann number or magnetic parameter (M)

The dimensionless quantity denoted by M is known as the Hartmann number. It was first introduced by Hartmann in 1930, in the study of plane Poiseuille flow of an electrically conducting fluid in the presence of transverse magnetic field. It is defined as the ratio of magnetic force to viscous force.

Thermal Grashof number (Gr)

The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. It plays a significant role in free convection heat and mass transfer.

Mass Grashof number (Gm)

Another non-dimensional parameter usually occurs in natural convection problem is mass Grashof number which is defined as the ratio of the species buoyancy force to the viscous hydrodynamic force.

Schmidt number (Sc)

The Schmidt number Sc embodies the ratio of the momentum diffusivity to the mass (species) diffusivity.

Substituting equation (15) into equations (11), (14) and (4) and taking into account equation (7), we obtain

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial u}{\partial y}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{dU_\infty}{dt}$$

$$+\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gm\phi+N\left(1+\lambda_1\frac{\partial}{\partial t}\right)(U_\infty-u), \tag{16}$$

$$\frac{\partial\theta}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{S}{Pr}\theta+\frac{4R}{3Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{D_f}{Pr}\frac{\partial^2\phi}{\partial y^2}-\frac{R_q}{Pr}\phi, \tag{17}$$

$$\frac{\partial\theta}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\left(1+\frac{4R}{3}\right)\frac{\partial^2\theta}{\partial y^2}+\frac{S}{Pr}\theta+\frac{D_f}{Pr}\frac{\partial^2\phi}{\partial y^2}-\frac{R_q}{Pr}\phi,$$

$$\frac{\partial\phi}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-Kr\phi. \tag{18}$$

The corresponding boundary conditions

$$u=U_P,\ \theta=1+\varepsilon e^{nt},\ \phi=1+\varepsilon e^{nt}\ \text{ at }\ y=0, \tag{19}$$

$$u\to U_\infty=1+\varepsilon e^{nt},\ \theta\to 0,\ \phi\to 0\ \text{ as }\ y\to\infty, \tag{20}$$

where $N=M+\dfrac{1}{K}$.

Solution of the Problem

To solve equations (16) to (18) subject to the boundary conditions (19) and (20), we apply the perturbation technique. Let the velocity, temperature and concentration fields be as

$$u = u_0(y) + \varepsilon e^{nt} u_1(y) + 0(\varepsilon^2),$$

$$\theta = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + 0(\varepsilon^2),$$

$$\phi = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + 0(\varepsilon^2). \tag{21}$$

Using (21) in equations (16)-(18) and equating the coefficients of the same degree terms and neglecting terms of $0(\varepsilon^2)$, the following ordinary differential equations are obtained:

$$u_0^{11} + u_0^1 - Nu_0 = -Gr\theta_0 - Gm\phi_0 - N, \tag{22}$$

$$u_1^{11} + u_1^1 - Lu_1 = -L - Au_0^1 - Gr\theta_1 - Gm\phi_1, \tag{23}$$

$$N_1\theta_0^{11} + Pr\theta_0^1 + S\theta_0 = Ra\phi_0, \tag{24}$$

$$N_1\theta_1^{11} + Pr\theta_1^1 + (S - nPr)\theta_1 = -PrA\theta_0^1 + Ra\phi_1, \tag{25}$$

$$\phi_0^{11} + Sc\phi_0^1 - KrSc\phi_0 = 0, \tag{26}$$

$$\phi_1^{11} + Sc\phi_1^1 - Sc(Kr + n)\phi_1 = -ScA\phi_0^1, \tag{27}$$

where the primes denote differentiation with respect to *y* only.

The boundary conditions (19) and (20) reduce to

$$u_0 = U_P,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 1,\ \phi_0 = 1,\ \phi_1 = 1 \text{ at } y = 0, \tag{28}$$

$$u_0 = 1,\ u_1 = 1,\ \theta_0 \to 0,\ \theta_1 \to 0,\ \phi_0 \to 0,\ \phi_1 \to 0 \text{ at } y \to \infty. \tag{29}$$

The solutions of (22) to (27) under the transformed boundary conditions (28) and (29) yield

$$u_0(y) = 1 + L_1 e^{-l_1 y} + P_7 e^{-R_1 y} + P_8 e^{-R_3 y}, \tag{30}$$

$$u_1(y) = 1 + L_2 e^{-l_2 y} + P_9 e^{-l_1 y} + P_{10} e^{-R_1 y}$$
$$+ P_{11} e^{-R_2 y} + P_{12} e^{-R_3 y} + P_{13} e^{-R_4 y}, \tag{31}$$

$$\theta_0(y) = (1 - P_2) e^{-R_3 y} + P_2 e^{-R_1 y}, \tag{32}$$

$$\theta_1(y) = P_3 e^{-R_1 y} + P_4 e^{-R_2 y} + P_5 e^{-R_3 y} + P_6 e^{-R_4 y}, \tag{33}$$

$$\phi_0(y) = e^{-R_1 y}, \tag{34}$$

$$\phi_1(y) = (1 - P_1) e^{-R_2 y} + P_1 e^{-R_1 y}. \tag{35}$$

In view of equations (30) to (35) and (21), the expressions for velocity, temperature and concentration field are obtained as

$$u(y, t) = (1 + L_1 e^{-l_1 y} + P_7 e^{-R_1 y} + P_8 e^{-R_3 y}) + \varepsilon e^{nt} (1 + L_2 e^{-l_2 y} + P_9 e^{-l_1 y}$$
$$+ P_{10} e^{-R_1 y} + P_{11} e^{-R_2 y} + P_{12} e^{-R_3 y} + P_{13} e^{-R_4 y}), \tag{36}$$

$$\theta(y, t) = (P_2 e^{-R_1 y} + (1 - P_2) e^{-R_3 y}) + \varepsilon e^{nt} (P_3 e^{-R_1 y}$$
$$+ P_4 e^{-R_2 y} + P_5 e^{-R_3 y} + P_6 e^{-R_4 y}), \tag{37}$$

$$\phi(y, t) = e^{-R_1 y} + \varepsilon e^{nt} ((1 - P_1) e^{-R_2 y} + P_1 e^{-R_1 y}), \tag{38}$$

where

$$R_1 = [Sc + \sqrt{Sc^2 + 4KrSc}]/2,$$

$$R_2 = [Sc + \sqrt{Sc^2 + 4Sc(Kr + n)}]/2, \; N_1 = 1 + \frac{4R}{3},$$

$$P_1 = (AScR_1)/(R_1^2 - ScR_1 - Sc(Kr + n)),$$

$$B_2 = (1 - P_1)Ra, \; P_2 = Ra/(N_1 R_1^2 - PrR_1 + S),$$

$$R_3 = (Pr + \sqrt{Pr^2 - 4SN_1})/2, \; B_1 = P_1 Ra + AR_1 PrR_2,$$

$$R_4 = [Pr + \sqrt{Pr^2 - 4N_1(S - nPr)}]/2,$$

$$B_3 = (1 - P_2)AR_3Pr,\ P_3 = B_1/(N_1R_1^2 - PrR_1 + S - nPr),$$

$$N = M + \frac{1}{K},\ P_4 = B_2/(N_1R_2^2 - PrR_2 + S - nPr),$$

$$P_5 = B_3/(N_1R_3^2 - PrR_3 + S - nPr),\ B_5 = -Gr(1 - P_2),$$

$$P_6 = 1 - (P_3 + P_4 + P_5),\ B_4 = -(GrP_2 + Gm),\ P_7 = B_4/(R_1^2 - R_1 - N),$$

$$P_8 = B_5/(R_3^2 - R_3 - N),\ l_1 = (1 + \sqrt{1 + 4N})/2,\ L_1 = U_p - P_7 - P_8 - 1,$$

$$L = (1 + n\lambda_1)(N + n),\ l_2 = (1 + \sqrt{1 + 4L})/2,$$

$$P_9 = Al_1(U_p - 1 - P_7 - P_8)/(l_1^2 - l_1 - L),$$

$$P_{10} = (AR_1P_7 - GrP_3 - GmP_1)/(R_1^2 - R_1 - L),$$

$$P_{11} = (GmP_1 - Gm - GrP_4)/(R_2^2 - R_2 - L),$$

$$P_{12} = (AR_3P_8 - GrP_5)/(R_3^2 - R_3 - L),\ P_{13} = (-GrP_6)/(R_4^2 - R_4 - L),$$

$$L_2 = -(1 + P_9 + P_{10} + P_{11} + P_{12} + P_{13}).$$

Shear Stress

The skin friction at the plate $y = 0$ in terms of shear stress is given by

$$\tau = \frac{-\partial u}{\partial y}\bigg|_{y=0} = (l_1L_1 + R_1P_7 + R_3P_8)$$

$$+ \varepsilon e^{nt}(l_2L_2 + l_1P_9 + R_1P_{10} + R_2P_{11} + R_3P_{12} + R_4P_{13}). \quad (39)$$

Nusselt Number

The rate of heat transfer coefficient at the plate $y = 0$ in terms of

Nusselt number is given by

$$Nu = \frac{-\partial\theta}{\partial y}\bigg|_{y=0} = R_1P_2 + R_3(1 - P_2) + \varepsilon e^{nt}(R_1P_3 + R_2P_4 + R_3P_5 + R_4P_6). \quad (40)$$

Sherwood Number

The rate of mass transfer coefficient at the plate $y = 0$ in terms of Sherwood number is given by

$$Sh = \frac{-\partial\phi}{\partial y}\bigg|_{y=0} = R_1 + \varepsilon e^{nt}(R_1P_1 + R_2(1 - P_1)). \quad (41)$$

Results and Discussion

To get a physical insight into the problem, the numerical evaluation of the analytical results reported in the previous section was performed and a set of results is reported graphically in Figures 1-6 for the cases cooling $Gr > 0$ of the plate. These results are obtained to illustrate the effects of various physical parameters like magnetic parameter *M*, Prandtl number *Pr*, radiation parameter *R*, chemical reaction parameter *Kr*, heat source parameter *S*, radiation absorption parameter *Ra* and Schmidt number *Sc* on the velocity, temperature and concentration profiles.

Figure 1 describes the velocity profiles due to variation in magnetic field parameter *M* and *Pr* for cooling of the plate $(Gr > 0)$. It is observed that with an increase in *M* the velocity decreases for both water $(Pr = 7)$ and conducting air $(Pr = 0.71)$. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. It is also observed that the maximum velocity occurs for the conducting air while minimum velocity occurs for the water. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly. Figure 2 shows the effects of *R*, *S* and *Ra* on the velocity profile *u*. It is observed that the velocity increases as the radiation parameter R or heat source parameter S increases whereas the velocity decreases with increase of radiation absorption parameter Ra. Thus, thermal radiation enhances convective flow and we noted that the heat is generated the buoyancy force increases which induce the flow rate to the increase in the velocity profiles. The effects of chemical reaction parameter Kr, thermal Grashof number Gr and mass Grashof number Gm on the velocity field are shown in Figure 3. From this figure, we observed that the velocity decreases with the increase of Kr and increases with the increase of Gr or Gm. This is due to the fact that buoyancy force enhances fluid velocity and increases the boundary layer thickness and hydrodynamics boundary layer becomes thin as the chemical reaction parameter increases.

The effects of Prandtl number and Schmidt number on the temperature field are shown in Figure 4. This figure shows that the temperature decreases with the increase of both the Prandtl number and Schmidt number. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore, heat is able to differ away from the heated surface more rapidly than for higher values of Pr. Hence, the boundary layer is thicker and the rate of heat transfer is reduced, for gradient has been reduced. The effects of radiation parameter, heat absorption parameter and absorption radiation parameter on the temperature field are shown in Figure 5. It is noticed that the temperature increases with an increase in radiation parameter and heat source parameter while it decreases with an increase in absorption radiation parameter.

Figure 6 shows the effects of Schmidt number Sc and chemical reaction parameter Kr on concentration profile. It is observed that an increase in Schmidt number Sc or chemical reaction parameter Kr decreases in concentration. Also, it is noticed that the concentration boundary layer becomes thin as the Schmidt number or chemical reaction parameter increases.

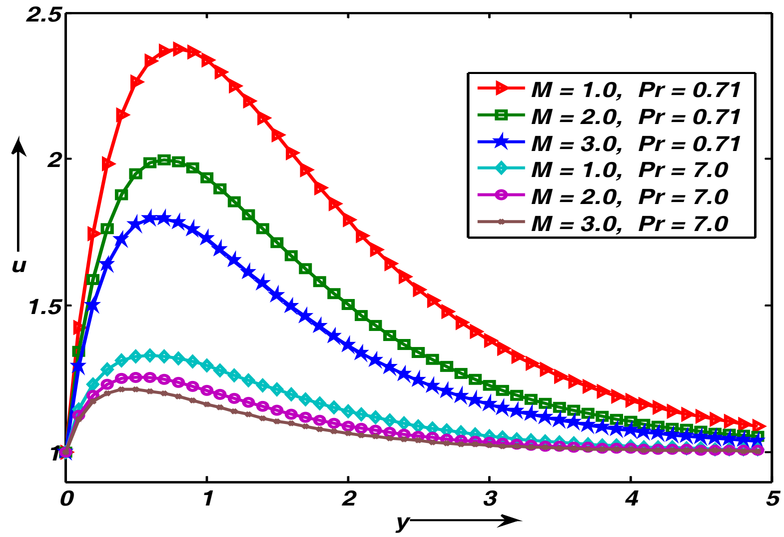

Figure 1. Velocity profiles when $n = 0.2$, $A = 1$, $\varepsilon = 0.002$, $t = 1$, $S = 0.01$, $Ra = 1$, $R = 1$, $Sc = 0.60$, $Gr = 5$, $Gm = 3$, $Kr = 1$, $Up = 1$, $K = 5$, $V = 1$.

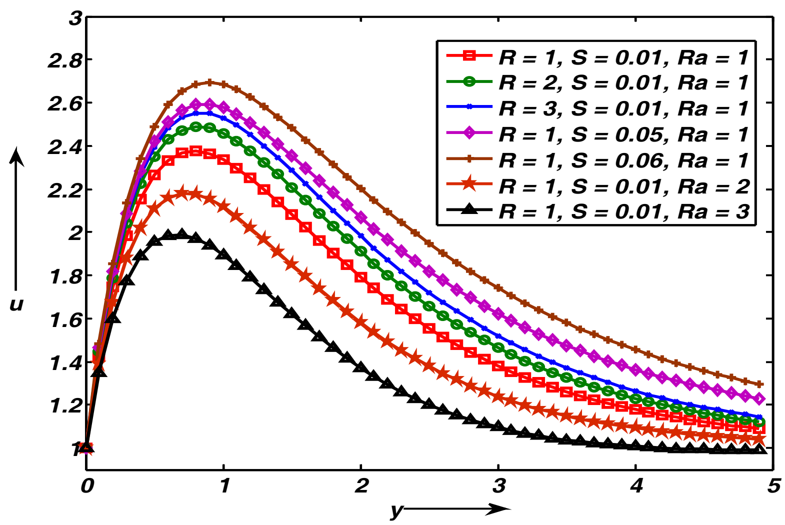

Figure 2. Velocity profiles when $n = 0.2$, $A = 1$, $\varepsilon = 0.002$, $t = 1$, $Pr = 0.71$, $M = 1$, $Sc = 0.60$, $Gr = 5$, $Gm = 3$, $Kr = 1$, $Up = 1$, $K = 5$, $V = 1$.

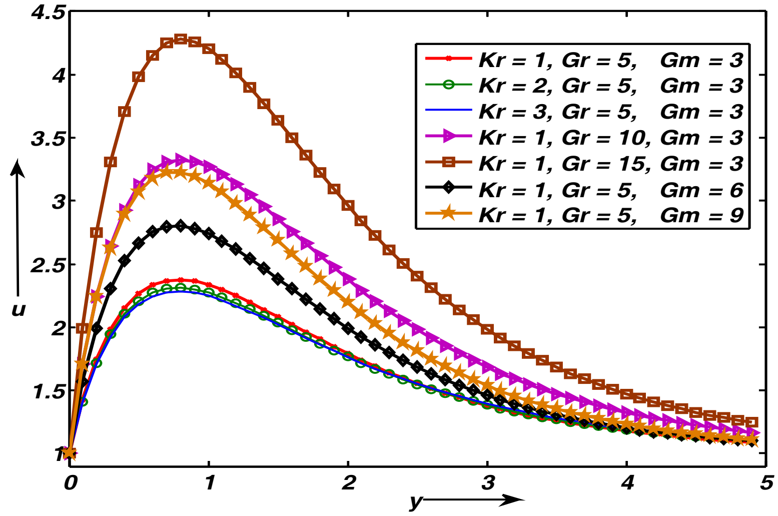

Figure 3. Velocity profiles when $n = 0.2$, $A = 1$, $\varepsilon = 0.002$, $t = 1$, $Pr = 0.71$, $M = 1$, $Sc = 0.60$, $S = 0.01$, $R = 1$, $Ra = 1$, $Up = 1$, $K = 5$, $V = 1$.

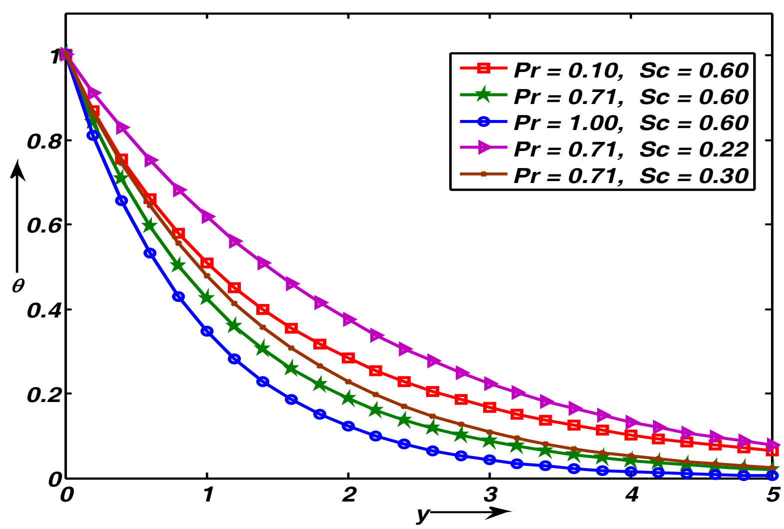

Figure 4. Temperature profiles when $n = 0.2$, $A = 1$, $\varepsilon = 0.002$, $t = 1$, $Kr = 1$, $S = 0.01$, $Ra = 1$, $R = 1$.

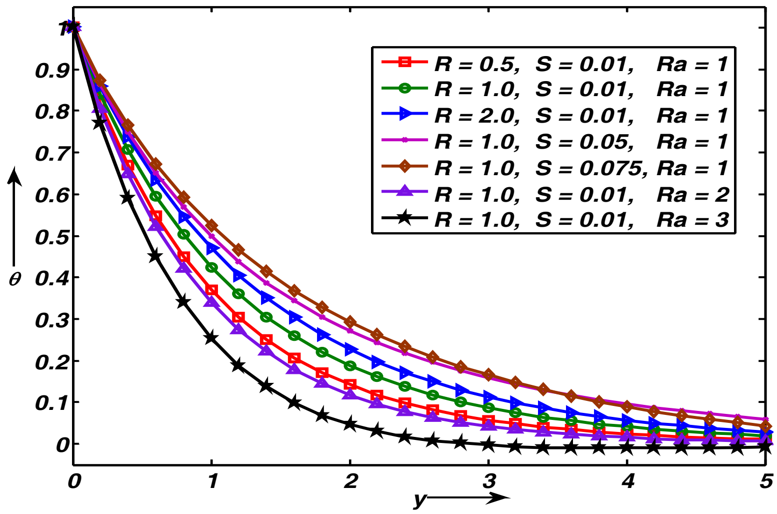

Figure 5. Temperature profiles when $n = 0.2$, $A = 1$, $\varepsilon = 0.002$, $t = 1$, $Kr = 1$, $Sc = 0.60$, $Pr = 0.71$.

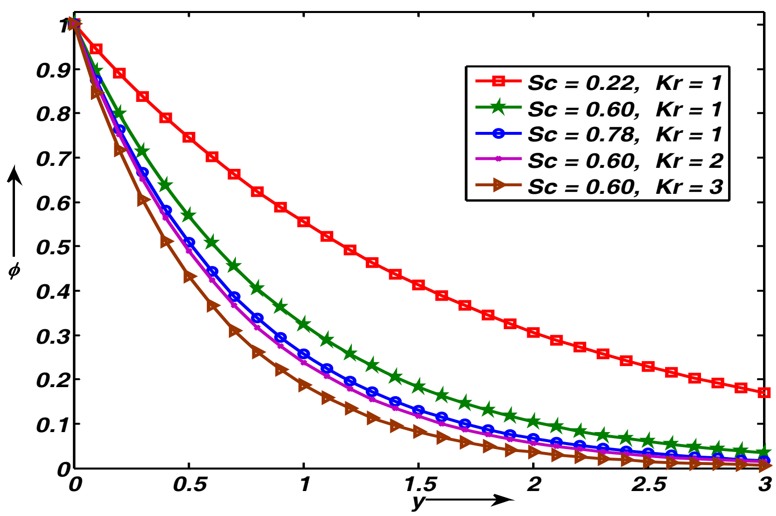

Figure 6. Concentration profiles when $n = 0.2$, $A = 1$, $\varepsilon = 0.002$, $t = 1$.

In the absence of chemical reaction, thermal radiation, heat source, radiation absorption effects and $\lambda_1 = 0$, all flow and heat mass transfer solutions reported above are consistent with those reported earlier by Chamkha [11].

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