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A BAYESIAN INFERIORITY INDEX FOR MEAN FROM NORMAL DISTRIBUTIONS

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Abstract

In drug developments, it is common that using evaluation variable to evaluate the effectiveness of new drugs. General evaluation methods are *t*-test or Analysis of Variance. These methods were on the frequency theory framework, and including the problems like multiplicity. Kawasaki and Miyaoka [1] proposed an index that shows superiority of binomial proportions in the Bayesian framework. In this study, we present new index expanded Kawasaki and Miyaoka [1] method to an index comparing mean of the normal distributions, also show usability of new index in the example.

1. Introduction

In drug development, it is common that using evaluation variable to evaluate the effectiveness of new drugs. General evaluation methods are *t*-test (Student [2]) or Analysis of Variance. However, usage of these methods is limited by strong supposition. Many test statistics and confidence intervals for inferiority test are based on the frequentist framework. However, research on inferiority in the Bayesian framework is limited.

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Kawasaki and Miyaoka [1] proposed an index $\theta = \mathrm{P}(\pi_{1,\,\mathrm{post}} > \pi_{2,\,\mathrm{post}} \mid X_1X_2)$, $\pi_{1,\,\mathrm{post}}$ and $\pi_{2,\,\mathrm{post}}$ denote binominal proportions following posterior density. They provided approximate and exact expressions for by using the beta prior. Finally, they suggested that θ can potentially provide useful information in a clinical trial. In this paper, we propose an index $\kappa = \mathrm{P}(\mu_{1,\,\mathrm{post}} > \mu_{2,\,\mathrm{post}} \mid \underline{X}_1\underline{X}_2)$.

In this paper, we applied the concept of the θ to new index, and propose the new index $\kappa = \mathrm{P}(\mu_{1,\,\mathrm{post}} > \mu_{2,\,\mathrm{post}} \mid \underline{X}_1\underline{X}_2)$. This index shows superiority of posterior mean of normal distribution, where $\underline{X}_i = (X_{i1}, X_{i2}, X_{i3}, \ldots, X_{ini})$ denotes the mean μ_i and posterior variance σ_i^2 for normal distribution, respectively, and μ_i denotes mean $\mu_{i,\mathrm{pre}}$, and variance $\sigma_{i,\,\mathrm{pre}}^2$ for normal distribution $(i = 1, 2)$.

The remainder of this article is organized as following: We show Notation and computation expression of κ in Section 2, and provide the example in Section 3. Finally, we conclude this article in Section 4.

2. Method

Let $\underline{X}_1 = (X_{11}, X_{12}, X_{13}, \ldots, X_{1n1})$ and $\underline{X}_2 = (X_{21}, X_{22}, X_{23}, \ldots, X_{2n2})$ denote random variables for normal distribution with trials n_1 and n_2 and parameters (μ_1, σ_1^2) and (μ_2, σ_2^2), respectively. Further, we assume that μ_1 is independent from μ_2. The conjugate prior density for $\mu_i (i = 1, 2)$ is the normal distribution with parameters $\mu_{i,\mathrm{pre}}$ and $\sigma_{i,\,\mathrm{pre}}^2$.

2.1. Expression for κ

The new index κ can be calculated

$$\kappa = P(\mu_{1,\text{post}} > \mu_{2,\text{post}} \mid X_1, X_1)$$

$$= P(\mu_{1,\text{post}} - \mu_{2,\text{post}} > 0 \mid X_1, X_1) = \Phi\left(\frac{\mu_{1,p} - \mu_{2,p}}{\sqrt{\frac{\sigma_{1,p}^2}{n_1} + \frac{\sigma_{2,p}^2}{n_2}}}\right),$$

where $\Phi(\bullet)$ is the cumulative distribution function of the standard normal distribution and

$$\mu_{i,p} = \frac{\frac{n_i \bar{x}_i}{\sigma_i^2} + \frac{\mu_{i,\text{pre}}}{\sigma_{i,\text{pre}}^2}}{\frac{n_i}{\sigma_i^2} + \frac{1}{\sigma_{i,\text{pre}}^2}} \quad \text{and} \quad \sigma_{i,p}^2 = \frac{1}{\frac{1}{\sigma_i^2} + \frac{1}{\sigma_{i,\text{pre}}^2}}$$

denote the posterior mean and variance of μ_i $(i = 1, 2)$.

3. Example

In this section, we show example of clinical trial, purpose of this trial was investigate difference in the means between the active drug group and the placebo group. In Table 1, we showed only summary statistics of the active drug group and the placebo group.

The purpose of this clinical trial is to show the mean of active drug group as to whether or not superior to the mean of placebo group. The main analysis method in this clinical trial was *t*-test, and the results of using *t*-test afford *p*-value = 0.0321. Therefore, it was above one-sided level of significance level 0.025, we obtained the result that null hypothesis cannot reject. On the other hand, we calculated κ with non-informative prior, and the probability of κ was 0.954. Consequently, it suggests clearly that mean of the active drug group is high on a probability of 95.4%.

Table 1. The summery of result of end point in a clinical trial

Drug	Number	Means	S.D.	Min	Max
New	8	76.63	16.78	44	94
Placebo	8	59.13	12.23	35	75

4. Conclusion

We proposed an index κ to compare the mean of normal distribution. This new index is given in framework of Bayesian, and can be easily and intuitively understood. Also, information regarding a previous clinical trials and results can compare the mean by using the empirical Bayes method. We believe that an index κ is utility. In addition, this index is easily to use in comparing multi-groups and comparing means of repeated measurements, without consider the multiplicity.

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References

[1] Y. Kawasaki and E. Miyaoka, A Bayesian inference of $P(\pi_1 > \pi_2)$ for two proportions, J. Biopharm. Statist. 22(3) (2012), 425-437.

[2] Student, The probable error of a mean, Biometrika 6(1) (1908), 1-25.