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ANALYTICAL SOLUTIONS FOR A DUSTY FLUID FLOW THROUGH A NARROWING CHANNEL IN A POROUS MEDIUM

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Abstract

The flow of an unsteady incompressible fluid with uniform distribution of dust particles through a narrowing system has been investigated analytically. In the proposed problem, Laplace transform techniques reduce the governing equations to zeroth order Bessel differential equation. The solutions thus obtained are in the form of Bessel functions and yield the velocity distributions of the fluid and dust particles. Expressions for shear stress are obtained for various cases and the solutions are plotted graphically to appreciate the effect of different parameters like Reynolds number, number density and the permeability of the porous medium on the velocity of dust and fluid phase.

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1. Introduction

In the recent years, considerable research efforts have been expended to enhance the scientific understanding of the physical process governing the mechanics of fluids and developing a mathematical model describing the fluid flow. In particular, fluids embedded with particles are encountered frequently in nature demanding a devoted study towards the understanding of flow mechanics of dusty fluids. Moreover, the analysis of fluid flow through a porous medium has drawn the attention of geophysical scientists specially in the context of the recovery of crude oil from the pores of reservoir rocks [1].

Prasad and Ramacharyulu [2], Mitra and Bhattacharyya [3], Michael and Miller [4] and Debnath and Ghosh [5] have investigated the flow of dusty fluids due to its growing applications in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, polymer technology and electrostatic precipitation. Saffman [6] has discussed the stability of laminar flow of dusty fluid and has formulated the governing equations for the flow of dusty fluid. Michael and Miller [4] have investigated the motion of dusty gas with uniform distribution of the dust particles which occupied the semi-infinite space above a rigid plane boundary. Samba Siva Rao [7] has obtained the solution for unsteady flow of a dusty viscous liquid through circular cylinder. Amos [8] has studied the magnetic effect on pulsatile flow in a constricted axis-symmetric tube. Rukmangadachari and Arunachalam [9] have obtained the results on dusty viscous flow through a cylinder of triangular cross-section. Further, the kinematical properties of fluid flows were studied by Kanwal [10], Truesdell [11], Indrasena [12], Purushotham and Indrasena [13], Bagewadi et al. [14-16] by applying differential geometry techniques and recently they have studied two-dimensional dusty fluid flow in Frenet frame field system.

Parallel to the study of dusty fluid flow in diverse situations, the analysis of fluid flow in narrowing systems has gained significant momentum with regard to its applications in many engineering problems like narrowing of pipeline network in drinking water distribution systems and sewage systems. Moreover, analysis of fluid flow in narrowing systems helps one to understand the mechanism of migration of suspended heavy organic particles towards the walls in oil-producing wells and pipelines. The fluid flow dynamics through narrowing channels are specially being explored because of its relationship with stenosis in blood flow and the optimal design of artificial organs. In this context, several bio-mathematicians like Verma et al. [17, 18], Ponalagusamy [19], Chaturani et al. [20-22] have integrated the concept of the narrowing system in the study of blood flow through a stenosised artery by using different mathematical techniques. More recently, an analytical solution of fluid flow through narrowing system was derived by Patel et al. [23].

This paper addresses the problem of fluid flow suspended with dust particles through a narrowing channel in a porous medium. Analytical solutions are obtained for both fluid and dust velocities by assuming the number density of the dust particles to be a constant throughout the flow. The equations of motion are solved by considering the fluid and dust particles to be at rest initially. A few particular cases, i.e., flow under an impulsive pressure gradient, flow under transition motion and flow in finite time, are considered.

2. Equations of Motion

The Navier-Stokes equations for an incompressible dusty fluid flow through a porous medium are [6]:

For *fluid phase*

$$\nabla \cdot \vec{u} = 0, \tag{2.1}$$

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{u} + \frac{KN}{\rho}(\vec{v} - \vec{u}) - \frac{\mu \vec{u}}{\eta}. \tag{2.2}$$

For *dust phase*

$$\nabla \cdot \vec{v} = 0, \tag{2.3}$$

$$\frac{\partial \vec{v}}{\partial t} = \frac{K}{m}(\vec{u} - \vec{v}). \tag{2.4}$$

We have following nomenclature:

u - velocity of the fluid phase, *v* - velocity of dust phase, *p* - the fluid pressure, *m* - mass of the particle, $K = 6\pi\mu r_1$ - Stokes resistance coefficient with μ being the viscosity of the fluid and r_1 is the radius of the spherical particle, *N* - number density of the particle, *t* - the time, ρ - mass density of the particle, $\nu = \frac{\mu}{\rho}$ - the kinematic viscosity of the fluid, η - permeability of the porous medium.

3. Formulation and Solution of the Problem

Consider an unsteady laminar flow of an incompressible viscous fluid with uniform distribution of dust particles through porous medium in a long circular cylinder in which the fluid is at rest initially. The flow is due to the influence of time dependent pressure gradiance imposed along the axis of the cylinder. It is assumed that the dust particles are spherical in shape and uniform in size and number density of the dust particles is taken to be constant throughout the flow. Let *z* be the direction of the axis of cylinder along which the flow takes place and let *r* be the radial direction outward from the *z* axis. Assumption is made that the channel is narrow due to the depositions of thickness δ on the wall of the cylinder. The elevation of thickness due to deposition is given by [23]:

$$R = R_0 - \frac{\delta}{2}\left(1 + \cos\frac{\pi z}{z_0}\right). \tag{3.1}$$

Here, R_0 is the distance from the axis of the cylindrical boundary and *z* is the distance from $z = 0$ to the point of calculation *P*.

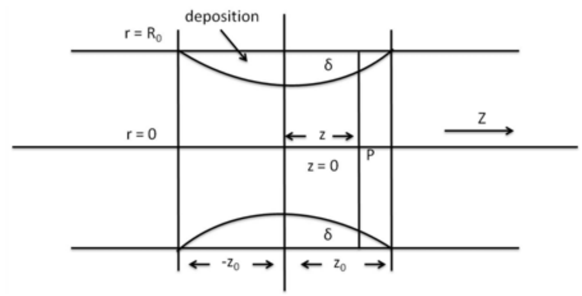

Figure a. Geometry of the flow.

The axis of the channel is along z axis and the velocity components of both fluid and dust particles are, respectively, given by:

$$u_r = 0; \quad u_\theta = 0; \quad u_z = (r,\, t),$$

$$v_r = 0; \quad v_\theta = 0; \quad v_z = (r,\, t). \tag{3.2}$$

By virtue of the above equations, we can rewrite (2.2) and (2.4) as

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right) + \frac{KN}{\rho}(v_z - u_z) - \frac{\mu u_z}{\eta}, \tag{3.3}$$

$$\frac{\partial v_z}{\partial t} = \frac{K}{m}(u_z - v_z). \tag{3.4}$$

The initial and the boundary conditions imposed on the system are

$$\text{for } t < 0; \quad u = 0, \quad v = 0,$$

$$\text{for } t > 0; \quad u = 0, \quad v = 0 \text{ at } r = R,$$

$$u = \mathit{finite} \quad v = \mathit{finite} \text{ at } r = 0. \tag{3.5}$$

By introducing the following nondimensional quantities:

$$r^* = \frac{r}{R_0}, \quad R^* = \frac{R}{R_0}, \quad z^* = \frac{z}{R_0}, \quad p^* = \frac{p}{\rho U^2}, \quad t^* = \frac{tU}{R_0}, \quad u = \frac{u_z}{U},$$

$$v = \frac{v_z}{U}, \quad \delta^* = \frac{\delta}{R_0}, \quad z_0^* = \frac{z_0}{R_0},$$

equations (3.3) and (3.4) can be expressed as

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \beta(v - u) - \lambda_1 u, \tag{3.6}$$

$$\frac{\partial v}{\partial t} = \lambda(u - v), \tag{3.7}$$

where

$$Re = \frac{UR_0}{\nu}, \quad \beta = \frac{KNR_0}{\rho U}, \quad \lambda_1 = \frac{\mu R_0}{\eta U}, \quad \lambda = \frac{KR_0}{mU}.$$

Accordingly, equations (3.1) and (3.5) assume a nondimensional form

$$R = 1 - \frac{\delta}{2}\left(1 + \cos\frac{\pi z}{z_0}\right),$$

$$\text{for } t > 0; \quad u = 0, \quad v = 0 \text{ at } r = R,$$

$$u = \textit{finite} \quad v = \textit{finite} \text{ at } r = 0.$$

Let $\wp(t)$ be the time dependent pressure gradient to be imposed on the system. So we can write

$$-\frac{\partial p}{\partial z} = \wp(t).$$

Applying Laplace transform to equations (3.6) and (3.7), one arrives at

$$s\bar{u} = P(s) + \frac{1}{Re}\left[\frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial r}\right] + \beta(\bar{v} - \bar{u}) - \lambda_1 \bar{u}, \tag{3.8}$$

$$s\bar{v} = \lambda(\bar{u} - \bar{v}), \tag{3.9}$$

where $\bar{u}$ and $\bar{v}$ are the Laplace transforms defined by

$$\bar{u} = \int_0^\infty e^{-st} u\,dt \quad \text{and} \quad \bar{v} = \int_0^\infty e^{-st} v\,dt,$$

and $P(s)$ is the Laplace transform of $\wp(t)$.

After Laplace transform, the boundary conditions become

$$\bar{u} = 0, \quad \bar{v} = 0 \text{ at } r = R,$$

$$\bar{u} = Finite, \bar{v} = Finite, \text{ at } r = 0. \tag{3.10}$$

Equation (3.8) can be rearranged as

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - Q^2\bar{u} = -ReP(s), \tag{3.11}$$

where $Q^2 = Re\left(s + \beta + \lambda_1 - \frac{\beta\lambda}{s + \lambda}\right)$.

The above equation resembles a modified second order Bessel equation of zeroth order whose solutions are

$$\bar{u} = AI_0(Qr) + BK_0(Qr) + \frac{ReP(s)}{Q^2}. \tag{3.12}$$

$I_0(Qr)$ and $K_0(Qr)$ are the Bessel functions of the first kind and second kind, respectively. The property $K_0(Qr) \to \infty$ as $r \to 0$ demands that $B = 0$ thereby reducing equation (3.12) to

$$\bar{u} = AI_0(Qr) + \frac{ReP(s)}{Q^2}. \tag{3.13}$$

Subject to the conditions (3.10), one obtains

$$\bar{u} = \frac{ReP(s)}{Q^2}\left[1 - \frac{I_0(Qr)}{I_0(QR)}\right]. \tag{3.14}$$

Further, it is evident that from (3.9) that

$$\bar{v} = \frac{ReP(s)\lambda}{Q^2(s + \lambda)}\left[1 - \frac{I_0(Qr)}{I_0(QR)}\right]. \tag{3.15}$$

Let us now consider the following cases.

4. Particular Cases

Case 1. Impulsive motion. In the case of impulsive motion, the pressure gradient is given by

$$\wp(t) = p_0\delta(t),$$

where $\delta(t)$ is Dirac delta function and p_0 is a constant.

Equations (3.14) and (3.15) become

$$\bar{u} = \frac{p_0 Re}{Q^2}\left(\frac{I_0(QR) - I_0(Qr)}{I_0(QR)}\right),$$

$$\bar{v} = \frac{p_0 Re\lambda}{(s+\lambda)Q^2}\left(\frac{I_0(QR) - I_0(Qr)}{I_0(QR)}\right).$$

Laplace inverse of $\bar{u}$ and $\bar{v}$ gives

$$u = \sum_{k_m=1}^{\infty}\frac{2Rp_0 Re}{k_m}\left(\frac{I_0(k_m) - I_0\left(\frac{k_m r}{R}\right)}{I_1(k_m)}\right) \times\left\{\frac{e^{y_1 t}(y_1+\lambda)^2}{[(y_1+\lambda)^2+\lambda\beta]Re} + \frac{e^{y_2 t}(y_2+\lambda)^2}{[(y_2+\lambda)^2+\lambda\beta]Re}\right\}, \tag{4.1}$$

$$v = \sum_{k_m=1}^{\infty}\frac{2Rp_0 Re\lambda}{k_m}\left(\frac{I_0(k_m) - I_0\left(\frac{k_m r}{R}\right)}{I_1(k_m)}\right) \times\left\{\frac{e^{y_1 t}(y_1+\lambda)}{[(y_1+\lambda)^2+\lambda\beta]Re} + \frac{e^{y_2 t}(y_2+\lambda)}{[(y_2+\lambda)^2+\lambda\beta]Re}\right\}, \tag{4.2}$$

where $(m = 1, 2, 3, \ldots)$ are the positive roots of $I_0(k) = 0$.

Shear stress (Skin friction)

The shear stress at the boundary at $r = R$ for impulsive motion is given by

$$D_{r,R} = -2p_0 Re\left\{\frac{e^{y_1 t}(y_1+\lambda)^2}{[(y_1+\lambda)^2+\lambda\beta]Re} + \frac{e^{y_2 t}(y_2+\lambda)^2}{[(y_2+\lambda)^2+\lambda\beta]Re}\right\}.$$

Case 2. Transition motion. For transition motion, we have

$$\wp(t) = p_0 H(t) e^{-wt},$$

where $H(t)$ is Heaviside step function. The solutions in this case are

$$u = \sum_{k_m=1}^{\infty} \frac{2Rp_0 Re}{k_m}\left(\frac{I_0(k_m) - I_0\left(\frac{k_m r}{R}\right)}{I_1(k_m)}\right)$$

$$\times\left\{\frac{e^{y_1 t}(y_1+\lambda)^2}{[(y_1+\lambda)^2+\lambda\beta](y_1+w)Re} + \frac{e^{y_2 t}(y_2+\lambda)^2}{[(y_2+\lambda)^2+\lambda\beta](y_2+w)Re}\right\}$$

$$+ \frac{Rep_0 e^{-wt}}{Q_1^2}\left[\frac{I_0(Q_1 R) - I_0(Q_1 r)}{I_0(Q_1 R)}\right],$$

$$v = \sum_{k_m=1}^{\infty} \frac{2Rp_0 Re\lambda}{k_m}\left(\frac{I_0(k_m) - I_0\left(\frac{k_m r}{R}\right)}{I_1(k_m)}\right)$$

$$\times\left\{\frac{e^{y_1 t}(y_1+\lambda)}{[(y_1+\lambda)^2+\lambda\beta](y_1+w)Re} + \frac{e^{y_2 t}(y_2+\lambda)}{[(y_2+\lambda)^2+\lambda\beta](y_2+w)Re}\right\}$$

$$+ \frac{\lambda Rep_0 e^{-wt}}{Q_1^2(\lambda - w)}\left[\frac{I_0(Q_1 R) - I_0(Q_1 r)}{I_0(Q_1 R)}\right].$$

Shear stress (Skin friction)

The shear stress at $r = R$,

$$D_{r,R} = -2p_0 Re\left\{\frac{e^{y_1 t}(y_1+\lambda)^2}{[(y_1+\lambda)^2+\lambda\beta](y_1+w)Re} + \frac{e^{y_2 t}(y_2+\lambda)^2}{[(y_2+\lambda)^2+\lambda\beta](y_2+w)Re}\right\}$$

$$- \frac{Rep_0 e^{-wt}}{Q_1}\frac{I_1(Q_1 R)}{I_0(Q_1 R)}.$$

Case 3. Motion for a finite time. In this case, we consider

$$\wp(t) = p_0[H(t) - H(t - T)].$$

The solutions are given by

$$u = \sum_{k_m=1}^{\infty} \frac{2Rp_0Re}{k_m}\left(\frac{I_0(k_m) - I_0\left(\frac{k_m r}{R}\right)}{I_1(k_m)}\right)$$

$$\times \left\{\frac{e^{y_1 t}(y_1 + \lambda)^2(1 - e^{-y_1 T})}{y_1[(y_1 + \lambda)^2 + \lambda\beta]Re} + \frac{e^{y_2 t}(y_2 + \lambda)^2(1 - e^{-y_2 T})}{y_2[(y_2 + \lambda)^2 + \lambda\beta]Re}\right\},$$

$$v = \sum_{k_m=1}^{\infty} \frac{2Rp_0Re\lambda}{k_m}\left(\frac{I_0(k_m) - I_0\left(\frac{k_m r}{R}\right)}{I_1(k_m)}\right)$$

$$\times \left\{\frac{e^{y_1 t}(y_1 + \lambda)(1 - e^{-y_1 T})}{y_1[(y_1 + \lambda)^2 + \lambda\beta]Re} + \frac{e^{y_2 t}(y_2 + \lambda)(1 - e^{-y_2 T})}{y_2[(y_2 + \lambda)^2 + \lambda\beta]Re}\right\}.$$

Shear stress (Skin friction)

At $r = R,$ the shear stress becomes

$$D_{r,R} = -2p_0Re\left\{\frac{e^{y_1 t}(y_1 + \lambda)^2(1 - e^{-y_1 T})}{Re[(y_1 + \lambda)^2 + \lambda\beta]y_1} + \frac{e^{y_2 t}(y_2 + \lambda)^2(1 - e^{-y_2 T})}{Re[(y_2 + \lambda)^2 + \lambda\beta]y_2}\right\}.$$

5. Conclusion

Considering that the flow takes place in a porous medium, an analytical solution for the velocity distributions for both fluid and dust in a narrowing channel has been derived. Based on the solutions obtained in the form of modified Bessel functions, various plots are depicted below for different values of Reynolds number Re and number density N and the permeability η of the porous medium. It is evident from the graphs that the velocity distributions are paraboloid in nature and the flow of fluid is parallel to that of dust. Observations from the graph reveal that the velocity profiles decrease with the increase in Reynolds number and number density of the dust particles. However, with the increase in the permeability of the porous medium, the velocity tends to increase. Further, it is observed that if the dust is very fine, i.e., mass of the dust particles is negligibly small, then the relaxation time $\tau = \frac{m}{K}$ of the dust particles decreases and as $\tau \to 0$, fluid and dust velocities will be same. Also, the fluid particles reach the steady state earlier than the dust particles. This difference is due to the fact that the time dependent pressure gradient is directly exerted on the fluid.

The notations used during the above discussion are given by the following expressions:

$$y_1 = \frac{-x_1 + x_2}{2}, \quad y_2 = \frac{-x_1 - x_2}{2}, \quad x_1 = \lambda R^2 + \beta R^2 + \lambda_1 R^2 - k_m^2,$$

$$x_2 = \sqrt{x_1^2 - 4R^2(\lambda_1 \lambda R^2 + \lambda k_m^2)}, \quad Q_1^2 = Re\left(-w - \frac{\beta s}{\lambda - w} + \lambda_1\right).$$

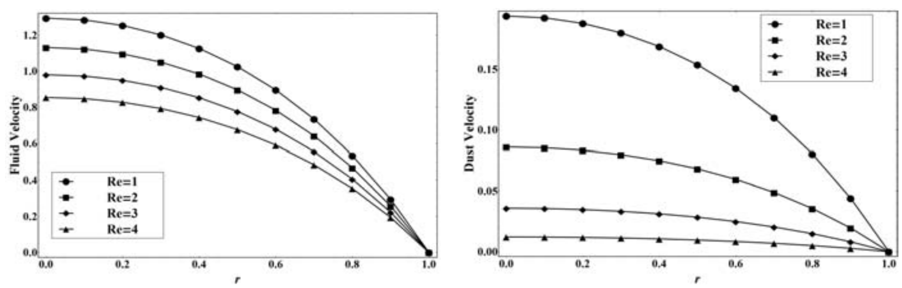

Figure 1. Variation of fluid and dust phase velocity with *r* (for different *Re*, Case 1).

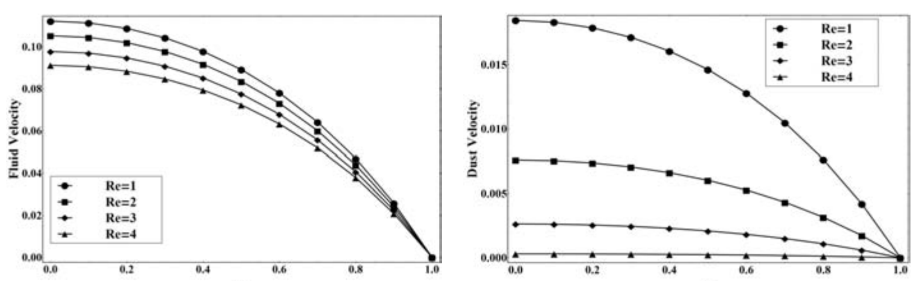

Figure 2. Variation of fluid and dust phase velocity with *r* (for different *Re*, Case 2).

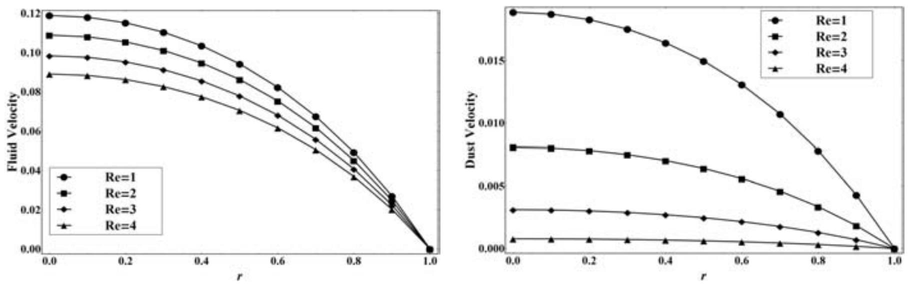

Figure 3. Variation of fluid and dust phase velocity with r (for different Re, Case 3).

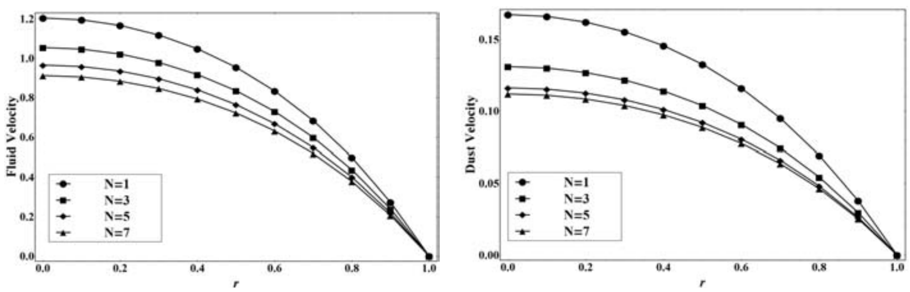

Figure 4. Variation of fluid and dust phase velocity with r (for different N, Case 1).

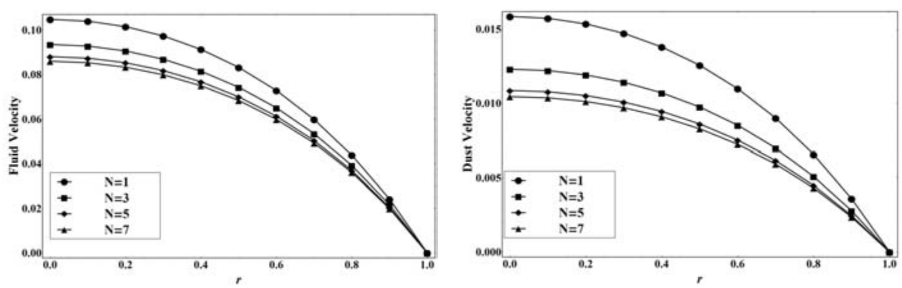

Figure 5. Variation of fluid and dust phase velocity with r (for different N, Case 2).

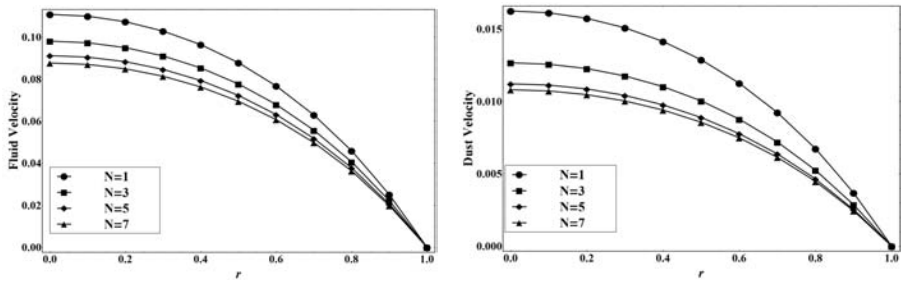

Figure 6. Variation of fluid and dust phase velocity with *r* (for different *N*, Case 3).

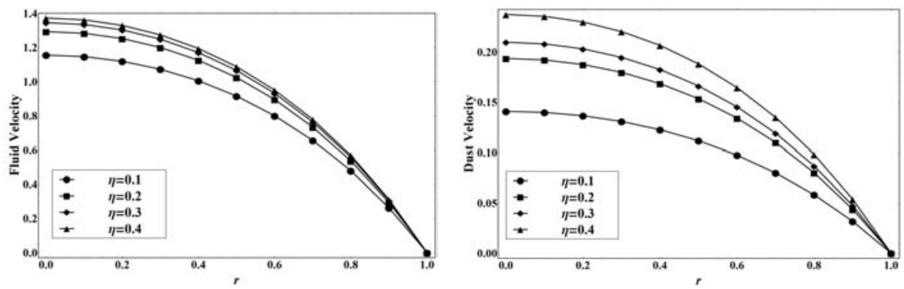

Figure 7. Variation of fluid and dust phase velocity with *r* (for different η, Case 1).

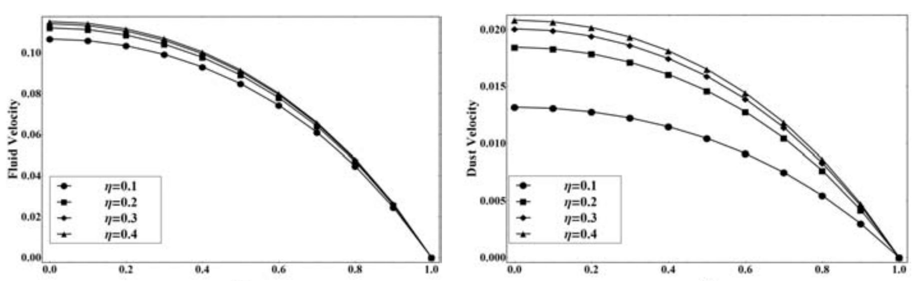

Figure 8. Variation of fluid and dust phase velocity with *r* (for different η, Case 2).

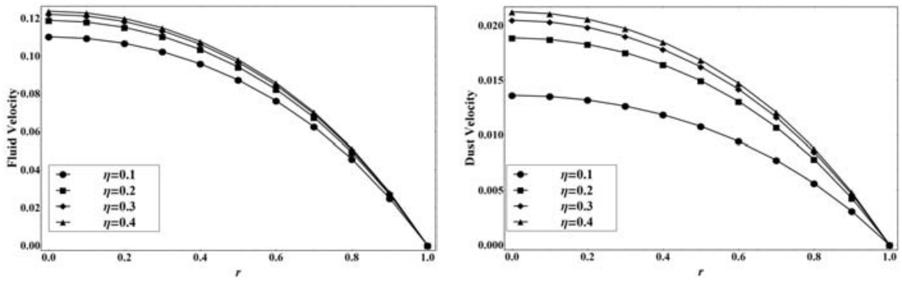

Figure 9. Variation of fluid and dust phase velocity with r (for different η, Case 3).

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