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SIMPLE EXPONENTIAL STABILITY CRITERION AND GUARANTEED DOMAIN OF ATTRACTION FOR A CLASS OF UNCERTAIN T-S FUZZY TIME-VARYING SYSTEMS

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Abstract

In this paper, the exponential stability with guaranteed domain of attraction (DA) is studied for a class of uncertain Takagi-Sugeno (T-S) fuzzy time-varying systems. Based on the time-domain approach, a simple criterion is derived to guarantee the exponential stability of such systems. An estimate of the exponential decay rate of such stable systems is also presented. Besides, we will provide a simple way to calculate the guaranteed DA by finding the unique positive root of a polynomial equation. Finally, numerical simulations are provided to illustrate the feasibility and effectiveness of the obtained result.

I. Introduction

In recent years, fuzzy systems have attracted much interest and have been widely studied. This is due to theoretical interests as well as to a

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powerful tool for system analysis and practical control design. In particular, T-S fuzzy system models frequently appear in various engineering systems and these models can be viewed as complex nonlinear systems.

During the past several years, the stability analysis and control design of T-S fuzzy systems have been an active area of research and have been extensively discussed; see, for instance, [1-2, 4-6, 8-22], and the references cited therein. Over the past decades, various techniques have been reported in robust stability analysis and control design of T-S fuzzy system, such as LMI approach, Lyapunov-based methodology, frequency-domain approach, time-domain approach, and others. Many important results in discrete-time T-S systems have been presented. In [19], the problems of stability analysis and stabilization for a class of discrete-time T-S fuzzy systems have been investigated. Based on a novel fuzzy Lyapunov-Krasovskii functional method, a delay partitioning method has been developed for the delay-dependent stability analysis of discrete-time fuzzy time-varying systems. In [4], the stability analysis and stabilization for a class of discrete-time T-S fuzzy systems has been studied. By defining a new fuzzy Lyapunov functions and by making use of novel techniques, a delay-dependent stability condition has been established in terms of LMIs, which is dependent on the lower and upper delay bounds. Besides, a new method has been developed in [22] to test stability of discrete-time fuzzy large-scale systems based on piecewise quadratic Lyapunov functions.

It is well known that both exponential stability criterion and guaranteed DA are of importance in checking the stability of closed-loop control systems. Meanwhile, in practical uncertain discrete-time T-S fuzzy time-varying systems, all of searching the exponential stability criterion, estimation of guaranteed DA, and calculation of the exponential decay rate are not easy tasks. This motivated us to investigate such problems simultaneously for a class of uncertain T-S fuzzy systems. In this paper, the robust stability with guaranteed DA for a class of uncertain discrete-time T-S fuzzy time-varying systems will be investigated. Based on the time-domain approach, a simple criterion will be derived to guarantee the exponential stability of such systems. An estimate of the exponential decay rate of such stable systems is also presented. Furthermore, a simple way will be provided to calculate the guaranteed DA. Finally, a numerical example is provided to illustrate the use of the main result.

This paper is organized as follows. The problem description, necessary notation, and main result are presented in Section II. Numerical simulations are given in Section III to illustrate the main result. Finally, conclusion is made in Section IV.

II. Problem Formulation and Main Result

Nomenclature

Z^+	the set of all non-negative integers
$\Re^{m\times n}$	the set of all real m by n matrices
$\lVert x \rVert$	the Euclidean norm of the vector $x \in \Re^{n\times 1}$
$\lambda_{\max}(A)$	the maximum eigenvalue of the matrix A with real eigenvalues
A^*	the conjugate transpose of the matrix A
$\lVert A \rVert$	the induced Euclidean norm of the matrix A, $\lVert A \rVert = [\lambda_{\max}(A^* A)]^{1/2}$
$\underline{m}$	$\{1, 2, ..., m\}$
$\overline{m}$	$\{0, 1, ..., m\}$

Consider the following uncertain discrete-time nonlinear time-varying systems:

$$x(k+1) = \Delta f(x(k), k), \quad \forall k \in Z^+, \tag{1a}$$

$$x(0) = x_0, \tag{1b}$$

where $x \in \Re^{n\times 1}$, $\Delta f \in \Re^{n\times 1}$ is the uncertain term with $\Delta f(0, \cdot) = 0$, and x_0 represents the initial condition.

The exponential stability, guaranteed DA, and the exponential decay rate of the *n*th order system (1) are defined as follows.

Definition 1. The equilibrium point $x^* = 0$ is said to be *exponentially stable*, if there exist three positive numbers k_1, β and α, with $0 < \alpha < 1$ such that every solution $x(k)$ to system (1) with $\| x(0) \| \le \beta$ satisfies

$$\| x(k) \| \le k_1 \cdot \alpha^k, \quad \forall k \in Z^+.$$

In this case, the positive number α is called the *exponential decay rate* and the set of $\Omega = \{x \in \Re^{n\times 1} \,|\, \| x \| \le \beta\}$ is called the *guaranteed DA*.

Takagi and Sugeno have proposed a fuzzy model to represent complex nonlinear systems. We consider, in this paper, the following uncertain discrete-time fuzzy time-varying systems, which is represented by a T-S fuzzy model and composed by a set of fuzzy implications. Each implication is expressed by the discrete-time uncertain time-varying systems and the *i*th rule of the T-S model is written in the following form:

Rule *j*. If $z_1(t)$ is about $S_{1,j}$, $z_2(t)$ is about $S_{2,j}$, ..., and $z_r(t)$ is about $S_{r,j}$, then

$$x(k+1) = \Delta A_j(k)x(k) + \Delta f_j(x(k), k), \quad \forall k \in Z^+, \tag{2a}$$

$$x(0) = x_0, \tag{2b}$$

where $z_1(t), z_2(t), ..., z_r(t)$ are premise variables, $S_{i,j}$, $\forall i \in \underline{r}$, $j \in \underline{m}$, are fuzzy sets, m is the number of If-then rules, $x \in \Re^{n\times 1}$ is the state, $\Delta A_j \in \Re^{n\times n}$, $\forall j \in \underline{m}$, are uncertain matrices, $\Delta f_j \in \Re^{n\times 1}$, $\forall j \in \underline{m}$, are system uncertainties or parameter mismatchings, and x_0 is the initial value.

The following assumption is made on the nth order system (2) throughout this paper.

(A1) There exist nonnegative constants a_j, $b_{i,j}$ and $p \in N$ such that for every $k \in Z^+$, $j \in \underline{m}$,

$$\| \Delta A_j(k) \| \leq a_j,$$

$$\| \Delta f_j(x(k), k) \| \leq \sum_{i=1}^{p} b_{i,j} \| x(k) \|^i.$$

For brevity, let us define

$$\hat{a} := \max_{1 \leq j \leq m} a_j \quad \text{and} \quad \hat{b}_i := \max_{1 \leq j \leq m} b_{i,j}, \; \forall i \in \underline{p}. \tag{3}$$

If we use the standard fuzzy inference method [11], i.e., minimum fuzzy inference and singleton fuzzifier, system (2) is inferred as follows:

$$x(k+1)$$

$$= \frac{1}{\sum_{i=1}^{m} u_i(z(t))} \cdot \sum_{j=1}^{m} u_j(z(t)) \cdot [\Delta A_j(k)x(k) + \Delta f_j(x(k), k)], \quad \forall k \in Z^+, \tag{4}$$

where $u_i(z(t)) = \prod_{j=1}^{r} \Phi_{ij}(z_j(t))$ and $\Phi_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in fuzzy set S_{ij}. Define $\lambda_j(z(t)) = \frac{u_j(z(t))}{\sum_{i=1}^{m} u_i(z(t))}$, $\forall j \in \underline{m}$ and we assume, in this paper, $u_j(z(t)) \geq 0$ for each $j \in \underline{m}$ and $\sum_{j=1}^{m} u_j(z(t)) > 0$.

Thus, the system (4) can be represented as

$$x(k+1) = \sum_{j=1}^{m} \lambda_j(z(t)) \cdot [\Delta A_j(k) x(k) + \Delta f_j(x(k), k)], \quad \forall k \in Z^+, \tag{5a}$$

$$x(0) = \sum_{j=1}^{m} [\lambda_j(z(t)) \cdot x_0] \tag{5b}$$

with $0 \le \lambda_j(z(t)) \le 1$, $\forall j \in \underline{m}$ and $\sum_{j=1}^{m} \lambda_j(z(t)) = 1$.

Remark 1. Let

$$g(x) := \hat{b}_p x^p + \hat{b}_{p-1} x^{p-1} + \hat{b}_{p-2} x^{p-2}$$

$$+ \cdots + \hat{b}_2 x^2 - \frac{1 - \hat{a} - \hat{b}_1}{2} x, \quad x \ge 0, \tag{6}$$

where $0 \le \hat{a} + \hat{b}_1 < 1$ and p is defined in (A1). In the case of $\sum_{i=2}^{p} \hat{b}_i^2 \ne 0$, by Descartes' rule of signs [7], it is easy to see that the polynomial equation $g(x) = 0$ has a unique positive root, denoted as η.

The objective of this paper is to derive a simple criterion with guaranteed DA such that the exponential stability for a class of uncertain T-S fuzzy time-varying systems can be achieved. Now we are in a position to present the main result.

Theorem 1. *The uncertain T-S fuzzy time-varying system* (2) *with* (A1) *is exponentially stable provided that*

$$\hat{a} + \hat{b}_1 < 1. \tag{7}$$

In this case, the guaranteed exponential decay rate is given by

$$\alpha := \frac{1 + \hat{a} + \hat{b}_1}{2}. \tag{8}$$

Furthermore, the guaranteed DA is given by

$$\Omega := \{x \in \Re^{n\times 1} \,|\, \| x \| \le \beta\}, \tag{9}$$

where

$$\beta := \begin{cases} \eta, & \text{if } \sum_{i=2}^{p} \hat{b}_i^2 \ne 0, \\ \infty, & \text{if } \sum_{i=2}^{p} \hat{b}_i^2 = 0 \end{cases} \tag{10}$$

and η is defined in Remark 1.

Proof. Assume that $x_0 \in \Omega$. Then it can be readily obtained that $x(0) \in \Omega$, in view of (5b). Two cases are separably discussed as follows:

(i) In case of $\sum_{i=2}^{p} \hat{b}_i \ne 0$ and $\hat{a} + \hat{b}_1 < 1$, by Descartes' rule of signs in (6), it is obvious that the polynomial equation $g(x) = 0$ has a unique positive root, denoted as η. Moreover, it is easy to see that

$$g(0) = 0,\ g(x) < 0,\ \forall 0 < x < \beta,\ g(\beta) = 0,\ \text{and}\ g(x) > 0,\ \forall x > \beta. \tag{11}$$

(ii) In case of $\sum_{i=2}^{p} \hat{b}_i = 0$ and $\hat{a} + \hat{b}_1 < 1$, by (6), we have

$$g(0) = 0 \quad \text{and} \quad g(x) < 0, \quad \forall 0 < x \le \beta. \tag{12}$$

Hence, from (11) and (12), one has $g(x) \le 0$, $\forall 0 \le x \le \beta$. It follows that

$$\hat{b}_p \| x \|^p + \hat{b}_{p-1} \| x \|^{p-1} + \hat{b}_{p-2} \| x \|^{p-2}$$

$$+ \cdots + \hat{b}_2 \| x \|^2 - \frac{1 - \hat{a} - \hat{b}_1}{2} \| x \| \le 0, \quad \forall x \in \Omega. \tag{13}$$

From (5), (8), (13) with (A1), it is easy to see that

$$
\begin{aligned}
\| x(k+1) \| &= \left\| \sum_{j=1}^{m} \lambda_j(z(t)) \cdot [\Delta A_j(k) x(k) + \Delta f_i(x(k), k)] \right\| \\
&\le \sum_{j=1}^{m} \lambda_j(z(t)) \cdot [\| \Delta A_j(k) \| \cdot \| x(k) \| + \| \Delta f_j(x(k), k) \|] \\
&\le \sum_{j=1}^{m} \lambda_j(z(t)) \cdot \left[a_j \cdot \| x(k) \| + \sum_{i=1}^{p} b_{i,j} \| x(k) \|^i \right] \\
&\le \left(\sum_{j=1}^{m} \hat{a} \cdot \lambda_j(z(t)) \cdot \| x(k) \| \right) + \sum_{j=1}^{m} \lambda_j(z(t)) \cdot \left[\sum_{i=1}^{p} b_{i,j} \| x(k) \|^i \right] \\
&= \hat{a} \| x(k) \| + \sum_{j=1}^{m} \lambda_j(z(t)) \cdot \left[\sum_{i=1}^{p} b_{i,j} \| x(k) \|^i \right] \\
&\le \hat{a} \| x(k) \| + \sum_{i=1}^{p} \hat{b}_i \| x(k) \|^i \\
&= (\hat{a} + \hat{b}_1) \| x(k) \| + \sum_{i=2}^{p} \hat{b}_i \| x(k) \|^i \\
&\le \frac{1 + \hat{a} + \hat{b}_1}{2} \| x(k) \| \\
&= \alpha \| x(k) \|, \ \forall k \ge 0, \ \| x(k) \| \in \Omega. \qquad (14)
\end{aligned}
$$

Consequently, from (14), we conclude that

$$
\| x(k) \| \le \alpha \| x(k-1) \| \le \alpha^2 \| x(k-2) \| \le \cdots \le \alpha^k \| x(0) \|, \quad \forall k \in Z^+.
$$

This completes the proof. □

Remark 2. In case of $\sum_{i=2}^{p} \hat{b}_i \neq 0$ and $\hat{a} + \hat{b}_1 < 1,$ it is not difficult to see that $g\left(1 + \frac{1 - \hat{a} - \hat{b}_1}{2\sum_{i=2}^{p} \hat{b}_i}\right) > 0.$ It follows that the value of η can be directly evaluated by using the Newton's method in $g(x) = 0$ with the starting value $x_1 = 1 + \frac{1 - \hat{a} - \hat{b}_1}{2\sum_{i=2}^{p} \hat{b}_i}.$

Remark 3. In what follows, we present an algorithm to find the exponential decay rate and the guaranteed DA stated in Theorem 1.

INPUT: a class of uncertain T-S fuzzy time-varying systems (2).

OUPUT #1: the exponential decay rate of (8).

OUPUT #2: the guaranteed DA of (9).

Step 1. Determiner n and m, from (2).

Step 2. Choose a_j, $b_{i,j}$, and $p \in N$ such that (A1) is satisfied.

Step 3. Calculate $\hat{a}$ and $\hat{b}_i$ from (3).

Step 4. If $\hat{a} + \hat{b}_1 \geq 1,$ then OUTPUT= "Search for another stability criteria." and go to Step 7, otherwise, calculate the parameter β from (10) with Remark 1.

Step 5. OUPUT #1: the exponential decay rate is given by $\alpha = \frac{1 + \hat{a} + \hat{b}_1}{2}.$

Step 6. OUPUT #2: the guaranteed DA is given by $\Omega = \{x \in \Re^{n \times 1} \,|\, \| x \| \leq \beta\}.$

Step 7. Stop.

Step 8. Search for another stability criteria.

III. Numerical Simulations

A T-S fuzzy model of predator-prey interactions ([3, 17]) is given by

Rule 1. If $x_2(k)$ is about 0.1, then

$$x(k+1)$$

$$= \begin{bmatrix} \Delta c_1(k)x_1(k) + \Delta c_2(k)x_2(k) + \Delta c_7(k)[\sin(k)]x_2^8(k) \\ \Delta c_3(k)x_1(k) + \Delta c_4(k)x_2(k) + \Delta c_8(k)x_1^{10}(k) \end{bmatrix}, \quad \forall k \in Z^+. \quad (15a)$$

Rule 2. If $x_2(k)$ is about 0.5, then

$$x(k+1)$$

$$= \begin{bmatrix} \Delta c_5(k)x_1(k) + \Delta c_6(k)x_2(k) - \Delta c_9(k)x_2^{10}(k) \\ \Delta c_7(k)[\cos(k)]x_1(k) + \Delta c_8(k)x_2(k) + \Delta c_{10}(k)x_1^8(k) \end{bmatrix}, \quad \forall k \in Z^+, \quad (15b)$$

where $x(k) = [x_1(k) \;\; x_2(k)]^T \in \Re^{2\times 1}$,

$y_1(t) :=$ the population at time t of a species of fish called *prey*,

$y_2(t) :=$ the population at time t of a species of fish called the *predator*,

$y_{1,eq} :=$ the nominal population of the prey,

$y_{2,eq} :=$ the nominal population of the predator,

$$x_1(t) := y_1(t) - y_{1,eq}; \quad x_2(t) := y_2(t) - y_{2,eq}$$

with

$$-0.5 \le \Delta c_i(k) \le 0.5, \quad \forall i \in \{1, 2, 4, 7, 10\},$$

$$-0.4 \le \Delta c_i(k) \le 0.4, \quad \forall i \in \{3, 6, 8, 9\},$$

$$-0.6 \le \Delta c_5(k) \le 0.6.$$

Step 1. Comparison of (15) with (2), it yields

$n = 2, \quad m = 2,$

$$\Delta A_1(k) = \begin{bmatrix} \Delta c_1(k) & \Delta c_2(k) \\ \Delta c_3(k) & \Delta c_4(k) \end{bmatrix}, \quad \Delta A_2(k) = \begin{bmatrix} \Delta c_5(k) & \Delta c_6(k) \\ \Delta c_7(k)\cos(k) & \Delta c_8(k) \end{bmatrix},$$

$$\Delta f_1(x(k), k) = \begin{bmatrix} \Delta c_7(k)[\sin(k)]x_2^8(k) \\ \Delta c_8(k)x_1^{10}(k) \end{bmatrix}, \quad \Delta f_2(x(k), k) = \begin{bmatrix} -\Delta c_9(k)x_2^{10}(k) \\ \Delta c_{10}(k)x_1^8(k) \end{bmatrix}.$$

Step 2. Clearly, we have

$\| \Delta A_1(k) \| \le 0.9525, \quad \| \Delta A_2(k) \| \le 0.9635,$

$a_1 = 0.9525, \quad a_2 = 0.9635,$

$b_{8,1} = 0.5, \quad b_{10,1} = 0.4, \quad b_{i,1} = 0, \quad \forall i \in \{1, 2, 3, 4, 5, 6, 7, 9\},$

$b_{8,2} = 0.5, \quad b_{10,2} = 0.4, \quad b_{i,2} = 0, \quad \forall i \in \{1, 2, 3, 4, 5, 6, 7, 9\},$

$p = 10.$

Step 3. From (3), it can be readily obtained that

$$\hat{a} = \max_{i \in \underline{2}} a_i = 0.9635, \quad \hat{b}_8 = \max_{i \in \underline{2}} b_{8,i} = 0.5, \quad \hat{b}_{10} = \max_{i \in \underline{2}} b_{10,i} = 0.4,$$

$\hat{b}_i = 0, \quad \forall i \in \{1, 2, 3, 4, 5, 6, 7, 9\}.$

Step 4. Clearly, one has $\hat{a} + \hat{b}_1 = 0.9635 < 1$. This shows that (7) is evidently satisfied. By (6), we obtain

$$g(x) = 0.4x^{10} + 0.5x^8 - 0.0183x.$$

The unique positive solution of $g(x) = 0$ is given by $\eta = 0.6012$. From (10) with Remark 1, we deduce $\beta = 0.6012$.

Step 5. From (8), the guaranteed exponential decay rate is given by $\alpha = 0.9818$.

Step 6. From (9), the guaranteed DA is given by $\Omega = \{x \in \Re^{2\times 1} \,|\, \| x \| \leq 0.6012\}$.

Finally, it can be easily deduced that

$$\lim_{k\to\infty}\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \lim_{k\to\infty}\begin{bmatrix} x_1(k) + y_{1,eq} \\ x_2(k) + y_{2,eq} \end{bmatrix} = \begin{bmatrix} y_{1,eq} \\ y_{2,eq} \end{bmatrix}, \quad \forall x_0 \in \Omega,$$

in view of

$$\lim_{k\to\infty}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = 0, \quad \forall x_0 \in \Omega.$$

The typical state trajectories of the system (15) are depicted in Figures 1-2. From the foregoing simulations results, it is seen that the system (15) is exponentially stable. It is noted that the exponential stability of system (15) cannot be guaranteed by the main results of [4], [17], [18], and [21].

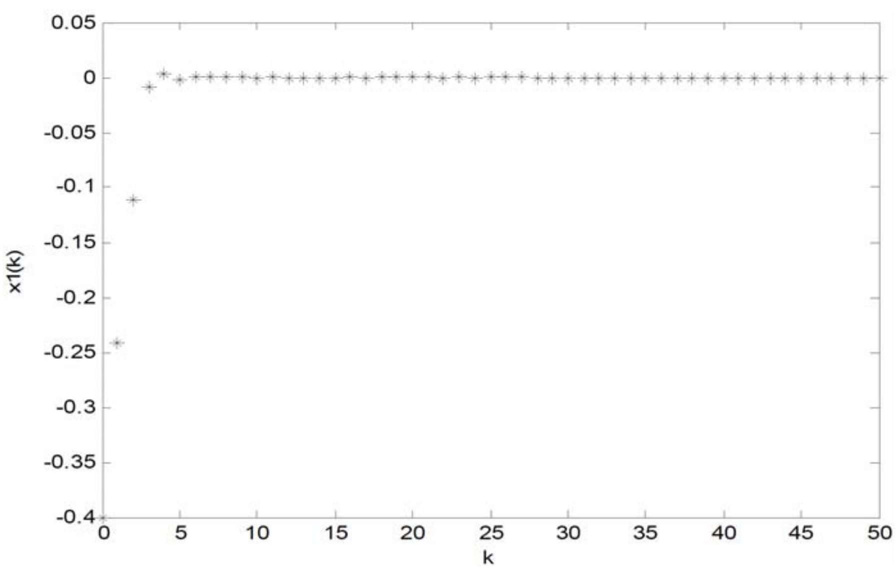

Figure 1. $x_1(k)$ of the system (15).

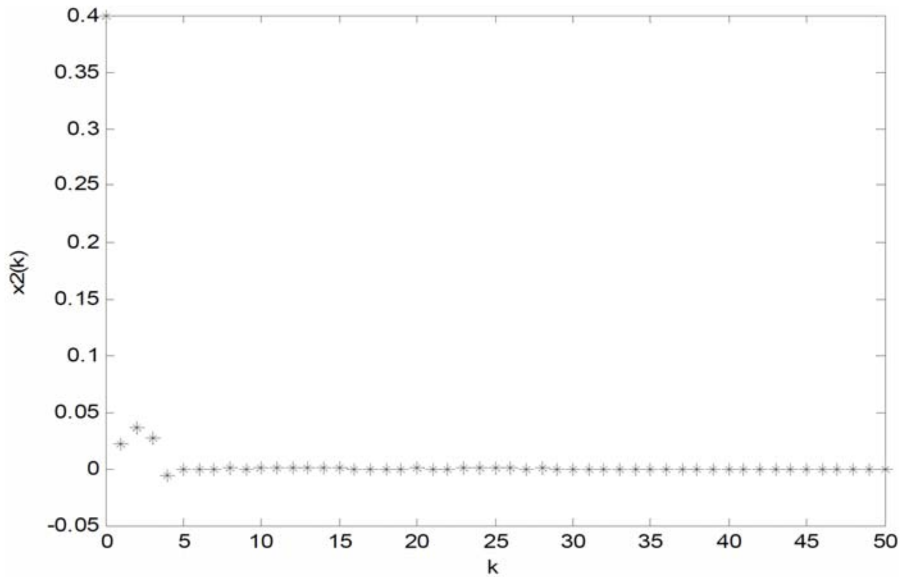

Figure 2. $x_2(k)$ of the system (15).

IV. Conclusion

In this paper, the exponential stability with guaranteed DA for a class of uncertain T-S fuzzy time-varying systems has been investigated. Based on the time-domain approach, a simple criterion has been derived to guarantee the exponential stability of such systems. An estimate of the exponential decay rate of such stable systems has also been presented. Besides, we have provided a simple way to calculate the guaranteed DA by finding the unique positive root of a polynomial equation. Finally, numerical simulations have also been provided to illustrate the feasibility and effectiveness of the obtained result.

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