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CHAOS SYNCHRONIZATION APPROACH FOR COUPLED OF ARBITRARY 3-D QUADRATIC DYNAMICAL SYSTEMS IN DISCRETE-TIME

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Abstract

In this paper, a new chaos synchronization method is proposed for coupled of arbitrary 3-D quadratic dynamical systems in discrete-time. The synchronization scheme, based on nonlinear controllers and Lyapunov stability theory, is theoretically rigorous. Numerical simulation shows the effectiveness and the feasibility of the new method.

1. Introduction

Over the last decades, synchronization of chaotic systems has attracted more and more attention from many areas of science and technology, due to its potential application in secure communications [15, 16].

Many types of synchronization phenomenon have been presented such as complete synchronization [2], anti-synchronization [14], generalized

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synchronization [13], projective synchronization [10], generalized projective synchronization [12], etc. Various methods and techniques for chaos synchronization have been reported to investigate some types of chaos synchronization in continuous-time systems, such as OGY method [3], PC method [5], feedback approach [7], active and adaptive control [8, 14], sliding mode approach [9], backstepping design technique [11, 19], etc. In fact, many mathematical models of biological processes, physical processes and chemical processes were defined using discrete-time dynamical systems. Recently, more attentions were paid to the chaos synchronization in discrete-time dynamical systems [17].

In this paper, using new controller law and Lyapunov stability theory, a general method in discrete-time is proposed to achieve synchronization between two arbitrary 3-D quadratic chaotic systems. In order to verify the effectiveness of the new approach, the proposed scheme is applied to two discrete-time hyper-chaotic systems: the 3-D generalized Hénon system [17] and the 3-D discrete-time Baier-Klein system [1].

The rest of this paper is organized as follows: In Section 2, the new discrete chaos synchronization approach is introduced. In Section 3, the proposed criterion is applied to achieve synchronization between two discrete-time quadratic chaotic systems in 3-D and numerical simulation is used to verify its effectiveness. In Section 4, conclusion is followed.

2. A New Chaos Synchronization Approach

Consider the following drive and response chaotic systems:

$$x_i(k+1) = \sum_{j=1}^{3} a_{ij} x_j(k) + X^T(k) C^i X(k) + \delta_i, \quad 1 \leq i \leq 3, \tag{1}$$

$$y_i(k+1) = \sum_{j=1}^{3} b_{ij} y_j(k) + Y^T(k) D^i Y(k) + \gamma_i + u_i, \quad 1 \leq i \leq 3, \tag{2}$$

where $X(k) = (x_i(k))_{1 \leq i \leq 3} \in \mathbb{R}^3$, $Y(k) = (y_i(k))_{1 \leq i \leq 3} \in \mathbb{R}^3$ are the state vectors of the drive system and the response system, respectively,

$(a_{ij}) \in \mathbb{R}^{3\times 3}$, $(b_{ij}) \in \mathbb{R}^{3\times 3}$, $C^i = (c^i_{pq}) \in \mathbb{R}^{3\times 3}$ $(i = 1, 2, 3)$, $D^i = (d^i_{pq})$ $\in \mathbb{R}^{3\times 3}$ $(i = 1, 2, 3)$, $(\delta_i, \gamma_i)_{1\le i\le 3}$ are real numbers and

$$U = (u_i(X(k), Y(k)))_{1\le i\le 3} \in \mathbb{R}^3$$

is the vector controller to be determined. Let us define the synchronization errors as

$$e_i(k) = y_i(k) - x_i(k), \quad 1 \le i \le 3. \tag{3}$$

From the definition (3), the synchronization errors between the drive system (1) and the response system (2) can be derived as follows:

$$e_i(k+1) = \sum_{j=1}^{3} b_{ij} e_j(k) + u_i + L_i + N_i, \quad 1 \le i \le 3, \tag{4}$$

where

$$L_i = \sum_{j=1}^{3} (b_{ij} - a_{ij}) x_j(k), \quad 1 \le i \le 3 \tag{5}$$

and

$$N_i = Y^T(k) D^i Y(k) - X^T(k) C^i X(k) + \gamma_i - \delta_i, \quad 1 \le i \le 3. \tag{6}$$

To achieve synchronization between systems (1) and (2), we can choose the vector controller $U = (u_i)_{1\le i\le 3}$ as follows:

$$u_i = \sum_{j=1}^{3} (\lambda_{ij} - b_{ij}) e_j(k) - L_i - N_i, \quad 1 \le i \le 3, \tag{7}$$

where $\lambda_{11} = \frac{1}{b_{11}^2 + l_1}$, $\lambda_{12} = \frac{1}{b_{22}^2 + l_2}$, $\lambda_{13} = -\frac{1}{b_{33}^2 + l_3}$, $\lambda_{21} = \frac{1}{b_{11}^2 + l_1}$, $\lambda_{22} = \frac{1}{b_{22}^2 + l_2}$, $\lambda_{23} = \frac{1}{b_{33}^2 + l_3}$, $\lambda_{31} = \frac{1}{b_{11}^2 + l_1}$, $\lambda_{32} = -\frac{2}{b_{22}^2 + l_2}$, $\lambda_{33} = 0$ and $(l_i)_{1\le i\le 3}$ are unknown constants to be determined later.

By substituting equation (7) into equation (4), the synchronization errors can be written as:

$$\begin{cases} e_1(k+1) = \dfrac{1}{b_{11}^2 + l_1} e_1(k) + \dfrac{1}{b_{22}^2 + l_2} e_2(k) - \dfrac{1}{b_{33}^2 + l_3} e_3(k), \\ e_2(k+1) = \dfrac{1}{b_{11}^2 + l_1} e_1(k) + \dfrac{1}{b_{22}^2 + l_2} e_2(k) + \dfrac{1}{b_{33}^2 + l_3} e_3(k), \\ e_3(k+1) = \dfrac{1}{b_{11}^2 + l_1} e_1(k) - \dfrac{2}{b_{22}^2 + l_2} e_2(k). \end{cases} \tag{8}$$

Now, we have the following result.

Theorem 1. *If* $(l_i)_{1 \leq i \leq 3}$ *are chosen such that*

$$l_1 > \sqrt{3},\ l_2 > \sqrt{6} \ \text{ and } \ l_3 > \sqrt{2}, \tag{9}$$

then the drive system (1) *and the response system* (2) *are globally synchronized under the controller law* (7).

Proof. For stability analysis, let us consider the following quadratic Lyapunov function:

$$V(e(k)) = \sum_{i=1}^{3} e_i^2(k), \tag{10}$$

we obtain:

$$\begin{aligned} \Delta V(e(k)) &= V(e(k+1)) - V(e(k)) \\ &= \sum_{i=1}^{3} e_i^2(k+1) - \sum_{i=1}^{3} e_i^2(k) \\ &= \left(\frac{3}{(b_{11}^2 + l_1)^2} - 1 \right) e_1^2(k) + \left(\frac{6}{(b_{22}^2 + l_2)^2} - 1 \right) e_2^2(k) \\ &\quad + \left(\frac{2}{(b_{33}^2 + l_3)^2} - 1 \right) e_3^2(k). \end{aligned}$$

Using (9), we can prove that

$$\frac{3}{(b_{11}^2 + l_1)^2} < 1, \ \frac{6}{(b_{22}^2 + l_2)^2} < 1 \text{ and } \frac{2}{(b_{33}^2 + l_3)^2} < 1,$$

then we get: $\Delta V(e(k)) < 0.$ Thus, by Lyapunov stability, it is immediate that

$$\lim_{k \to \infty} e_i(k) = 0, \quad (i = 1, 2, 3). \tag{11}$$

We conclude that the drive system (1) and the response system (2) are globally synchronized. □

3. Numerical Simulation

In this section, numerical example is considered to validate the proposed chaos synchronization approach.

The drive system is the 3-D generalized Hénon system [17]:

$$\begin{cases} x_1(k+1) = -bx_2(k), \\ x_2(k+1) = 1 + x_3(k) - ax_2^2(k), \\ x_3(k+1) = bx_2(k) + x_1(k), \end{cases} \tag{12}$$

where $a = 1.07,\ b = 0.3.$

The response system is the 3-D discrete-time Baier-Klein system [1]:

$$\begin{cases} y_1(k+1) = -0.1y_3(k) - y_2^2(k) + 1.76 + u_1, \\ y_2(k+1) = y_1(k) + u_2, \\ y_3(k+1) = y_2(k) + u_3, \end{cases} \tag{13}$$

where $U = (u_1, u_2, u_3)^T$ is the vector controller.

According to equation (7), the synchronization controllers can be written as:

$$\begin{cases} u_1 = \dfrac{1}{l_1} e_1(k) + \dfrac{1}{l_2} e_2(k) - \left(\dfrac{1}{l_3} - 0.1\right) e_3(k) - R_1, \\ u_2 = \left(\dfrac{1}{l_1} - 1\right) e_1(k) + \dfrac{1}{l_2} e_2(k) + \dfrac{1}{l_3} e_3(k) - R_2, \\ u_3 = \dfrac{1}{l_1} e_1(k) - \left(\dfrac{2}{l_2} + 1\right) e_2(k) - R_3, \end{cases} \tag{14}$$

where

$$\begin{cases} -R_1 = -bx_2(k) + 0.1x_3(k) + y_2^2(k) - 1.76, \\ -R_2 = -x_1(k) + x_3(k) + ax_2^2(k) - 1, \\ -R_3 = x_1(k) + (b-1)x_2(k) \end{cases} \tag{15}$$

and $(l_i)_{1 \leq i \leq 3}$ are unknown constants to be determined.

Finally, the synchronization errors between (12) and (13) can be derived as:

$$\begin{cases} e_1(k+1) = \dfrac{1}{l_1} e_1(k) + \dfrac{1}{l_2} e_2(k) - \dfrac{1}{l_3} e_3(k), \\ e_2(k+1) = \dfrac{1}{l_1} e_1(k) + \dfrac{1}{l_2} e_2(k) + \dfrac{1}{l_3} e_3(k), \\ e_3(k+1) = \dfrac{1}{l_1} e_1(k) - \dfrac{2}{l_2} e_2(k). \end{cases} \tag{16}$$

Corollary 2. *For the two coupled 3-D generalized Hénon and 3-D discrete-time Baier-Klein systems, if we choose* $(l_i)_{1 \leq i \leq 3}$ *such that* $l_1 = 5$, $l_2 = 4$ *and* $l_3 = 3$, *then they are globally synchronized.*

By using Matlab, we get the numeric result that is shown in Figure 1.

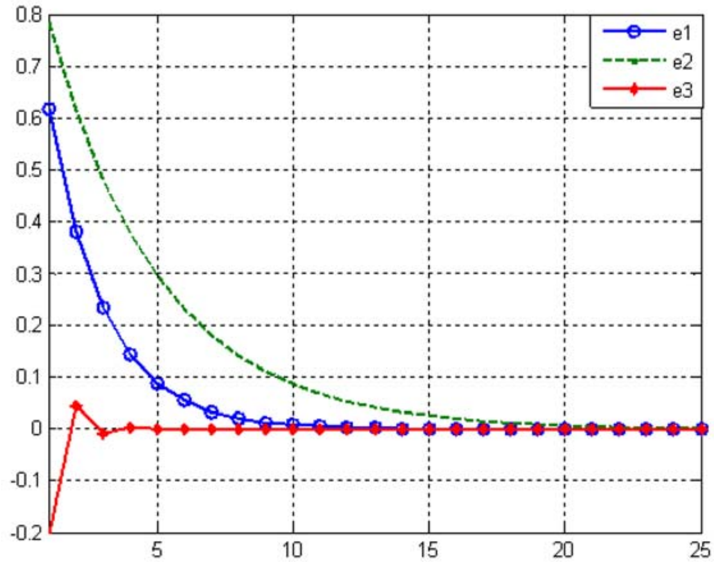

Figure 1. Time evolution of synchronization errors between 3-D generalized Hénon system and Baier-Klein system.

4. Conclusion

In this paper, a new control scheme was designed to achieve synchronization for coupled of 3-D quadratic chaotic systems in discrete-time. It was showed that the proposed controllers guarantee the asymptotic convergence to zero of the errors between the drive and the response systems. Finally, numerical simulation was provided to illustrate the effectiveness of our approach.

References

[1] G. Baier and M. Klein, Maximum hyperchaos in generalized Hénon maps, Phys. Lett. A 51(6-7) (1990), 281-284.

[2] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares and C. S. Zhou, The synchronization of chaotic systems, Phys. Rep. 366 (2002), 1-101.

[3] E. Ott, C. Grebogi and J. A. Yorke, Controlling chaos, Phys. Rev. Lett. 64 (1990), 1196-1199.

[4] K. Pyragas, Continuous control of chaos by self-controlling feedback, Phys. Lett. A 170 (1992), 421-428.

[5] L. M. Pecora and T. L. Carrol, Synchronization in chaotic systems, Phys. Rev. Lett. 64 (1990), 821-824.

[6] M. C. Ho and Y. C. Hung, Synchronization of two different chaotic systems using generalized active control, Phys. Lett. A 301 (2002), 424-428.

[7] X. Wu, X. Han and J. Lu, Adaptive feedback synchronization of unified chaotic systems, Phys. Lett. A 329 (2004), 327-333.

[8] S. H. Chen and J. Lu, Synchronization of an uncertain unified systems via adaptive control, Chaos Soliton Fractals 14 (2002), 643-647.

[9] H. T. Yau, Design of adaptive sliding mode controller for chaos synchronization with uncertainties, Chaos Soliton Fractals 22 (2004), 341-347.

[10] J. Qiang, Projective synchronization of a new hyperchaotic Lorenz system chaotic systems, Phys. Lett. A 370 (2007), 40-45.

[11] X. Tan, J. Zhang and Y. Yang, Synchronizing chaotic systems using backstepping design, Chaos Solitons Fractals 16 (2003), 37-45.

[12] X. Li, Generalized projective synchronization using nonlinear control method, International Journal of Nonlinear Science 8 (2009), 79-85.

[13] Y. W. Wang and Z. H. Guan, Generalized synchronization of continuous chaotic systems, Chaos Solitons Fractals 27 (2006), 97-101.

[14] X. Zhang and H. Zhu, Anti-synchronization of two different hyper-chaotic systems via active and adaptive control, International Journal of Nonlinear Science 6 (2008), 216-223.

[15] L. Kocarev and U. Parlitz, General approach for chaotic synchronization with applications to communication, Phys. Rev. Lett. 74 (1995), 5028-5030.

[16] Y. Tao, Chaotic secure communication systems - history and new results, Telecommun. Review 9 (1999), 597-634.

[17] Zhenya Yan, Q-S synchronization in 3D Hénon-like map and generalized Hénon map via a scalar controller, Phys. Lett. A 342(4) (2005), 309-317.

[18] M. Itoh, T. Yang and L. O. Chua, Internat. J. Bifur. Chaos Appl. Sci. Engrg. 11 (2001), 551.

[19] J. Zhao and J. Lu, Parameter identification and backstepping control of uncertain Lu system, Chaos Soliton Fractals 17 (2003), 721-729.