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HEAT TRANSFER CHARACTERISTICS OF AN UNSTEADY SHRINKING SHEET WITH MASS TRANSFER IN A ROTATING FLUID

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Abstract

The problem of unsteady laminar boundary layer flow and heat transfer over a permeable shrinking sheet in a rotating fluid is considered. The transformed boundary layer equations are solved numerically using an implicit finite-difference scheme, namely the Keller-box method. Numerical results for different values of the Prandtl number, suction, unsteadiness and rotation parameters on the heat transfer characteristics are obtained and discussed.

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Introduction

Very recently, the problem of boundary layer flow induced by a shrinking sheet has become popular and attracted many researchers to investigate further. Shrinking film is one of the examples of shrinking problem in industries whereby it is very useful in packaging of bulk products. Shrinking sheet problem has been initiated by Wang [1] by taking the case of stretching decelerating surface and discovered a flow reversal. Later, Wang [2] studied the stagnation flow towards a shrinking sheet and reported that non-alignment destroys the symmetric stagnation flow for two-dimensional case. The viscous flow induced by a shrinking sheet with suction was considered by Miklavcic and Wang [3]. Sajid and Hayat [4] and Fang and Zhang [5] studied MHD flow of a viscous and electrically conducting fluid due to a shrinking sheet and obtained the exact series solution using homotopy analysis method and showed a closed-form exact solution of the full Navier-Stokes equations, respectively.

Unsteady boundary layer flow towards a stretching sheet has been studied by many researchers, see [6-14]. On the other hand, not many works have been done for the problem of an unsteady boundary layer flow induced by a shrinking sheet. Namely, Fang et al. [15] studied the boundary layer flow over a continuously shrinking sheet with time dependent deceleration taken into account, while Ali et al. [16] extended this idea to the case of a rotating fluid. Ali et al. [17] also considered the problem of unsteady axisymmetric boundary layer flow and heat transfer induced by a permeable shrinking sheet with radiation effect. The unsteady MHD boundary layer flow on a shrinking sheet dependent on a dimensionless magnetic parameter reported by Merkin and Kumaran [18]. Therefore, the present paper aims to extend the problem considered in Ali et al. [16] by adding the heat transfer characteristic to the problem. The ordinary differential equations are solved numerically for some values of the governing parameters using a finite-difference scheme.

Basic Equations

Consider the unsteady laminar flow and heat transfer over a permeable shrinking sheet in a rotating fluid. The fluid motion becomes three-dimensional due to the Coriolis force. The Cartesian coordinates are (x, y, z) with the axes rotating at an angular velocity $\Omega(t)$ in the z-direction, as shown in Ali et al. [16], where t represents the time. It is assumed that the sheet shrinking velocities in the (x, y) directions are $u_w(x, t)$ and $v_w(x, t)$, and the wall mass flux velocity in the z-direction is $w_w(x, t)$, which will be detailed later. Under these conditions, the Navier-Stokes and energy equations in this Cartesian coordinate system are (see Wang [19])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\Omega v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} - 2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v, \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w, \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\nabla^2 T \tag{5}$$

subject to the boundary conditions

$$t < 0 : u = v = w = 0,\ T = T_\infty \ \text{for any } x, y, z,$$

$$t \geq 0 : u = u_w(x, t),\ v = v_w(x, t),\ w = w_w(x, t),\ T = T_w \ \text{at}\ z = 0,$$

$$u \to 0,\ v \to 0,\ T \to T_\infty \ \text{as}\ z \to \infty, \tag{6}$$

where u, v, w are the velocity components in the x-, y- and z-directions, respectively, ρ is the fluid density, ν is the kinematic viscosity, T is the fluid temperature, T_w is the surface temperature, T_∞ is the ambient temperature, α is the thermal diffusivity, ∇^2 is the Laplacian operator in the three-dimensional coordinates (x, y, z) and $w_w(x, t) < 0$ for suction and $w_w(x, t) > 0$ for injection.

We assume that $u_w(x, t)$, $v_w(x, t)$ and $\Omega(t)$ have the following form:

$$u_w(x, t) = -\frac{ax}{1-\alpha_1 t}, \quad v_w(x, t) = -\frac{ax}{1-\alpha_1 t}, \quad \Omega(t) = \frac{\omega}{1-\alpha_1 t}, \tag{7}$$

where $a(> 0)$ represents the stretching rate, Ω is the constant angular velocity of the shrinking sheet and α_1 is a parameter showing the unsteadiness of the problem. Therefore, we introduce the following similarity transformation:

$$u = \frac{ax}{1-\alpha_1 t} f'(\eta), \quad v = \frac{ax}{1-\alpha_1 t} g(\eta), \quad \sqrt{\frac{a\nu}{1-\alpha_1 t}} f(\eta),$$

$$\eta = \sqrt{\frac{a}{\nu(1-\alpha_1 t)}} z, \quad \theta(\eta) = (T - T_\infty)/(T_w - T_\infty). \tag{8}$$

Thus, $w_w(x, t)$ should have the form of

$$w_w(x, t) = -\sqrt{\frac{a\nu}{1-\alpha_1 t}} f(0) = s, \tag{9}$$

where s is the constant wall mass transfer parameter with $s > 0$ for suction and $s < 0$ for injection, respectively.

By substituting equations (8) and (9) into equations (2), (3) and (5), we obtain the following ordinary differential equations:

$$f''' + ff'' - f'^2 + 2\lambda g - A\left(f' + \frac{\eta}{2} f'' \right) = 0, \tag{10}$$

$$g'' + fg' - f'g - 2\lambda f' - A\left(g + \frac{\eta}{2} g' \right) = 0, \tag{11}$$

$$\frac{1}{\Pr}\theta'' + f\theta' - A\frac{\eta}{2}\theta' = 0 \tag{12}$$

subject to the boundary conditions (6) which become

$$f(0) = s, \quad f'(0) = -1, \quad g(0) = -1, \quad \theta(0) = 1,$$
$$f'(\eta) \to 0, \quad g(\eta) \to 0, \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty, \qquad (13)$$

where $\lambda = \omega/a$ and $A = \alpha_1/a$ are non-dimensional parameters signifying the relative importance of rotation rate to stretching rate and the unsteadiness parameter, respectively. Primes denote the differentiation with respect to η. The pressure term can be determined from equation (4). For the present solution, we assume a decelerating shrinking sheet with $A \le 0$.

In this study, the physical quantities of interest are the skin friction in x and y directions, (C_{fx}, C_{fy}) and the local Nusselt number Nu_x which are defined as

$$C_{fx} = \frac{2\tau_{wx}}{\rho u_w^2}, \quad C_{fy} = \frac{2\tau_{wy}}{\rho u_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \qquad (14)$$

where τ_{wx} and τ_{wy} are the surface shear stress in the directions of x and y, respectively, and q_w represents the surface heat flux, which are given by

$$\tau_{wx} = -\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}, \quad \tau_{wy} = -\mu\left(\frac{\partial v}{\partial z}\right)_{z=0}, \quad q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} \qquad (15)$$

with μ and k are the dynamic viscosity and thermal conductivity of the fluid, respectively. Using equation (8), we obtain

$$-2\,\mathrm{Re}_x^{1/2}\, C_{fx} = f''(0), \quad -2\,\mathrm{Re}_y^{1/2}\, C_{fy} = g'(0), \quad \mathrm{Re}_x^{-1/2}\, Nu_x = -\theta'(0), \quad (16)$$

with $\mathrm{Re}_x = u_w\, x/\nu$ and $\mathrm{Re}_y = u_w\, y/\nu$ are the local Reynolds number.

Results and Discussion

Equations (10)-(12) subject to the boundary conditions (13) have been solved numerically via the Keller-box method, as described in the book by Cebeci and Bradshaw [20]. Table 1 and Figure 1 show the heat transfer rate at the surface and the temperature profiles for various Prandtl number when other parameters are fixed. The heat transfer rate at the surface increases, while the temperature profiles and the thermal boundary layer thickness reduce with Prandtl number. This is because higher Prandtl number reduces the thermal conductivity and the conduction is also reduced, which in turn thinning the thermal boundary layer thickness. Therefore, this implies an increase in the heat transfer rate at the surface.

Table 1. Variation of $-\theta'(0)$ with Pr when $\lambda = 2$, $s = 2.5$ and $A = -2$

Pr	0.7	1	3	6.8	10
$-\theta'(0)$	1.8967	2.6267	7.5567	17.0243	25.0159

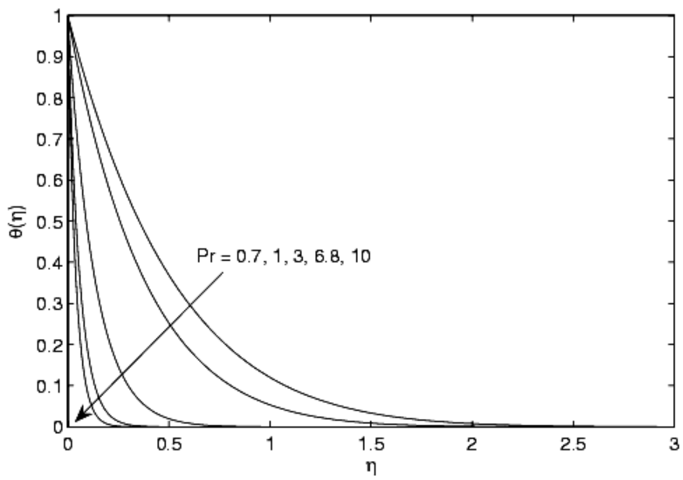

Figure 1. Temperature profiles for various Pr when $\lambda = 2$, $A = -2$ and $s = 2.5$.

The effect of the constant wall mass transfer parameter s on the temperature profile is illustrated in Figure 2. It can be seen clearly that the temperature profiles reduce as s increases. The constant wall mass transfer parameter s has also increased the wall temperature gradient. Hence s has increased the wall shear stress, therefore increases the local Nusselt number. Figures 3 and 4 show the effect of the rotation parameter λ and unsteadiness parameter A on the temperature profiles, respectively. Both λ and A decrease the temperature profiles, where the effect of λ is not very significant. It is also found that the heat transfer coefficient is also increased with λ and A. From Figures 1-4, it is observed that the temperature profiles are affected the most by the Prandtl number and the boundary conditions (13) for temperature are also satisfied.

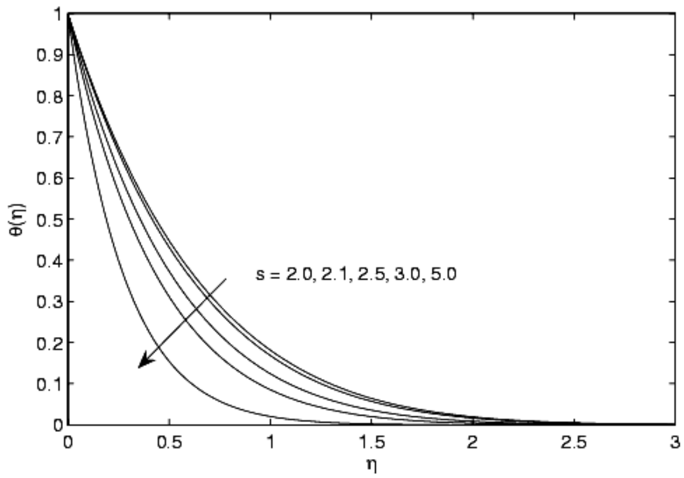

Figure 2. Temperature profiles for various s when $\lambda = 1$, $A = -2$ and $\mathrm{Pr} = 0.7$.

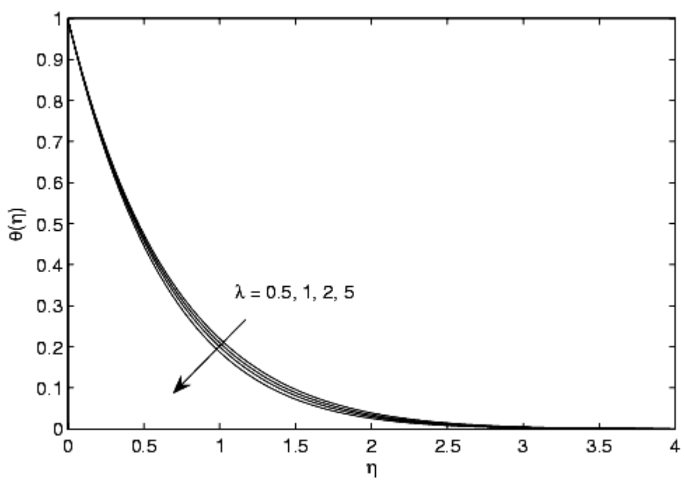

Figure 3. Temperature profiles for various λ when $A = -1$, $s = 2.2$ and $\mathrm{Pr} = 0.7$.

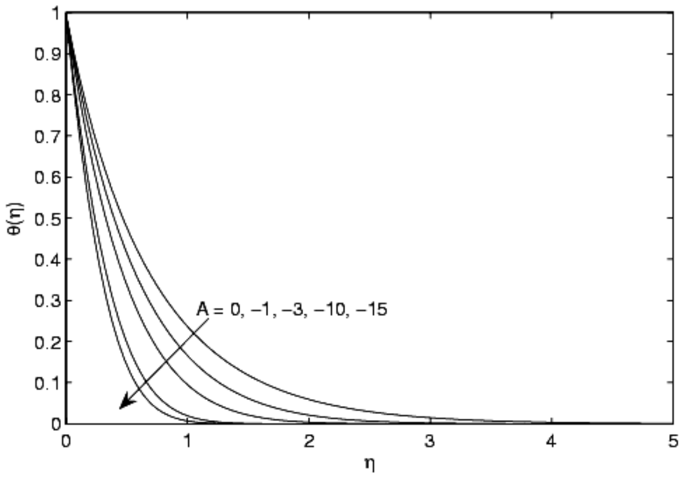

Figure 4. Temperature profiles for various A when $\lambda = 1$, $s = 2.5$ and $\Pr = 0.7$.

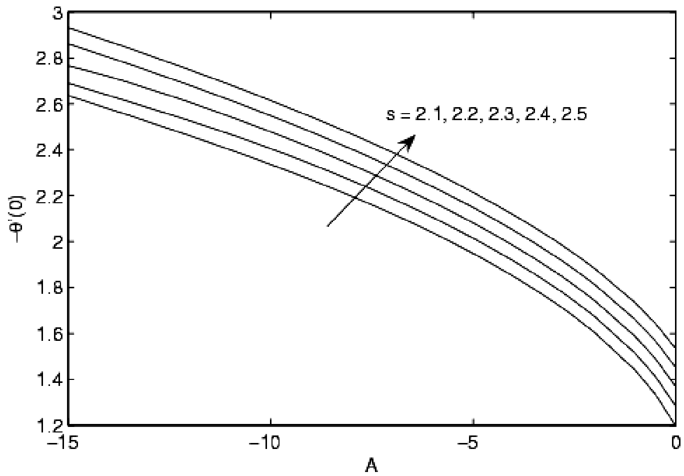

Figure 5. Variation of the heat transfer coefficient $-\theta'(0)$ with A for various s when $\lambda = 1$ and $\Pr = 0.7$.

Figure 5 illustrates the local Nusselt number for various suction parameter, s and unsteadiness parameter, A. It can be seen that the heat transfer coefficients increase with both s and A. Physically, suction increases the surface shear stress and slows down the fluid motion which consequently thinning the thermal boundary layer thickness and as a result, the local Nusselt number increases.

Conclusion

A numerical study is performed for the problem of unsteady laminar boundary layer flow and heat transfer over a permeable shrinking sheet in a rotating fluid. It is found that the temperature profiles decrease with Prandtl number, as well as the thermal boundary layer thickness. The same effect can be observed when the constant wall mass transfer, the rotation and the unsteadiness parameters are applied. It is also observed that heat transfer rate at the surface increases with Prandtl number, suction and unsteadiness parameters.

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