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A ϕ-CONTRACTION PRINCIPLE IN PARTIAL METRIC SPACES WITH SELF-DISTANCE TERMS

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Abstract

We prove a generalized contraction principle with control function in complete partial metric spaces. The contractive type condition used allows the appearance of self-distance terms. The obtained result generalizes some previously obtained results such as the very recent Ilić et al. [15]. An example is given to illustrate the generalization and its properness. Our presented example does not verify the contractive

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type conditions of the main results proved recently by Romaguera in [17] and by Altun et al. in [8]. Therefore, our results have an advantage over the previously obtained.

1. Introduction and Preliminaries

Banach contraction mapping principle is considered to be the key of many extended fixed point theorems. It has widespread applications in many branches of mathematics, engineering and computer. Previously, many authors were able to generalize this principle [11-14, 20-23]. After the appearance of partial metric spaces as a place for distinct research work into flow analysis, non-symmetric topology and domain theory [5, 1], many authors started to generalize this principle to these spaces (see [2-4, 6-10, 17-19, 24-26]). However, the contraction type conditions used in those generalizations do not reflect the structure of partial metric space apparently. Later, the authors in [15] proved a more reasonable contraction principle in partial metric space in which they used self-distance terms. In this article, we present a ϕ-contraction principle in partial metric spaces. The presented contractive condition allows the self-distance to appear so that completeness, rather than the 0-completeness, of the partial metric space is needed.

We recall some definitions of partial metric spaces and state some of their properties. A partial metric space (PMS) is a pair $(X, p : X \times X \to \mathbb{R}^+)$ (where $\mathbb{R}^+$ denotes the set of all nonnegative real numbers) such that

(P1) $p(x, y) = p(y, x)$ (symmetry);

(P2) if $0 \leq p(x, x) = p(x, y) = p(y, y)$, then $x = y$ (equality);

(P3) $p(x, x) \leq p(x, y)$ (small self-distances);

(P4) $p(x, z) + p(y, y) \leq p(x, y) + p(y, z)$ (triangularity)

for all $x, y, z \in X$.

For a partial metric p on X, the function $p^s : X \times X \to \mathbb{R}^+$ given by

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y) \tag{1}$$

is a (usual) metric on X. Each partial metric p on X generates a T_0 topology τ_p on X with a base of the family of open p-balls $\{B_p(x, \varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

Definition 1 [1]. (i) A sequence $\{x_n\}$ in a PMS (X, p) converges to $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

(ii) A sequence $\{x_n\}$ in a PMS (X, p) is called *Cauchy* if and only if $\lim_{n, m\to\infty} p(x_n, x_m)$ exists (and finite).

(iii) A PMS (X, p) is said to be *complete* if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x, x) = \lim_{n, m\to\infty} p(x_n, x_m)$.

(iv) A mapping $T : X \to X$ is said to be *continuous* at $x_0 \in X$, if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $T(B_p(x_0, \delta)) \subset B_p(T(x_0), \varepsilon)$.

Lemma 1 [1]. (a1) *A sequence $\{x_n\}$ is Cauchy in a PMS (X, p) if and only if $\{x_n\}$ is Cauchy in a metric space (X, p^s).*

(a2) *A PMS (X, p) is complete if and only if the metric space (X, p^s) is complete. Moreover,*

$$\lim_{n\to\infty} p^s(x, x_n) = 0 \Leftrightarrow p(x, x) = \lim_{n\to\infty} p(x, x_n) = \lim_{n, m\to\infty} p(x_n, x_m). \tag{2}$$

A sequence $\{x_n\}$ is called 0-*Cauchy* [15] if $\lim_{m,n} p(x_n, x_m) = 0$. The partial metric space (X, p) is called 0-*complete* if every 0-Cauchy sequence in x converges to a point $x \in X$ with respect to p and $p(x, x) = 0$. Clearly, every complete partial metric space is complete. The converse need not be true.

Example 1 (See [15]). Let $X = \mathbb{Q} \cap [0, \infty)$ with the partial metric $p(x, y) = \max\{x, y\}$. Then (X, p) is a 0-complete partial metric space which is not complete.

Definition 2. Let (X, p) be a complete metric space. Set $\rho_p = \inf\{p(x, y) : x, y \in X\}$ and define $X_p = \{x \in X : p(x, x) = \rho_p\}$.

Theorem 1 [15]. *Let (X, p) be a complete metric space, $\alpha \in [0, 1)$ and $T : X \to X$ be a given mapping. Suppose that for each $x, y \in X$ the following condition holds*:

$$p(x, y) \leq \max\{\alpha p(x, y), p(x, x), p(y, y)\}.$$

Then

(1) *the set X_p is nonempty*;

(2) *there is a unique $u \in X_p$ such that $Tu = u$*;

(3) *for each $x \in X_p$ the sequence $\{T^n x\}_{n \geq 1}$ converges with respect to the metric p^s to u.*

The proof of the following lemma can be easily achieved by using the partial metric topology.

Lemma 2 [2, 4]. *Assume $x_n \to z$ as $n \to \infty$ in a PMS (X, p) such that $p(z, z) = 0$. Then $\lim_{n \to \infty} p(x_n, y) = p(z, y)$ for every $y \in X$.*

The following lemma summarizes the relation between certain comparison functions that usually act as control functions in the studied contractive typed mappings in fixed point theory. For such a summary and fixed point theory for ϕ-contractive mappings, we advice for [16].

Lemma 3. *Let* $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ *be a function and relative to the function* ϕ *consider the following conditions*:

- (i) ϕ *is monotone increasing*.
- (ii) $\phi(t) < t$ *for all* $t > 0$.
- (iii) $\phi(0) = 0$.
- (iv) ϕ *is right upper semicontinuous*.
- (v) ϕ *is right continuous*.
- (vi) $\lim_{n\to\infty} \phi^n(t) = 0$ *for all* $t \geq 0$.

Then the following are valid:

- (1) *The conditions* (i) *and* (ii) *imply* (iii).
- (2) *The conditions* (ii) *and* (v) *imply* (iii).
- (3) *The conditions* (i) *and* (vi) *imply* (ii).
- (4) *The conditions* (i) *and* (iv) *imply* (vi).
- (5) *If* ϕ *satisfies* (i), *then* (iv) $\Leftrightarrow$ (v).

2. Main Results

Theorem 2. *Let* (X, p) *be a complete partial metric space. Suppose* $T : X \to X$ *is a given mapping satisfying*:

$$p(Tx, Ty) \leq \max\{\phi(p(x, y)), p(x, x), p(y, y)\}, \tag{3}$$

where $\phi : [0, \infty) \to [0, \infty)$ *is an increasing function such that* $f(t) = t - \phi(t)$ *is increasing with* f^{-1} *is right continuous at* 0. *Also, assume* $\lim_{n\to\infty} \phi^n(t) = 0$ *for all* $t \geq 0$ (*and hence* $\phi(0) = 0$, $\phi(t) < t$ *for* $t > 0$). *Then*:

(1) *the set* X_p *is nonempty*;

(2) *there is a unique* $u \in X_p$ *such that* $Tu = u$;

(3) *for each* $x \in X_p$ *the sequence* $\{T^n x\}_{n \geq 1}$ *converges with respect to the metric* p^s *to* u.

Proof. Let $x \in X$. Then $p(Tx, Tx) \leq p(x, x)$ and therefore $\{p(T^n x, T^n x)\}_{n \geq 0}$ is a nonincreasing sequence. Now define

$$M_x := f^{-1}(p(x, Tx)) + p(x, x),$$

where $f(t) = t - \phi(t)$. Notice that $f(0) = 0$ (and hence $f^{-1}(0) = 0$), $f(t) < t$ for $t > 0$ and hence $f^{-1}(t) > t$ for $t > 0$. Now we prove that

$$p(T^n x, x) \leq M_x, \quad \forall n \geq 0. \tag{4}$$

Inequality (4) is true for $n = 0, 1,$ since: $p(x, x) \leq M_x$ and $p(Tx, x) \leq f^{-1}(p(Tx, x)) \leq M_x$. Now we proceed by induction. Suppose that (4) is true for each $n \leq n_0 - 1$ for some positive integer $n_0 \geq 2$. Then

$$\begin{aligned} p(T^{n_0} x, x) \leq\ & p(T^{n_0} x, Tx) + p(Tx, x) \\ & \leq \max\{\phi(p(T^{n_0 - 1} x, x)),\ p(T^{n_0 - 1} x, T^{n_0 - 1} x),\ p(x, x)\} \\ & \quad + p(Tx, x) \\ & \leq \max\{\phi(p(T^{n_0 - 1} x, x)),\ p(x, x)\} + p(Tx, x). \end{aligned}$$

Case 1.

$$\begin{aligned} p(T^{n_0} x, x) & \leq \phi(p(T^{n_0 - 1} x, Tx)) + p(Tx, x) \\ & \leq \phi(f^{-1}(p(Tx, x)) + p(x, x)) + p(Tx, x) \\ & = f^{-1}(p(Tx, x)) + p(x, x) - f(f^{-1}(p(Tx, x)) \\ & \quad + p(x, x)) + p(Tx, x) \\ & \leq M_x - f(f^{-1}(p(Tx, x))) + p(Tx, x) = M_x. \end{aligned}$$

Case 2.

$$p(T^{n_0}x, x) \leq p(x, x) + p(Tx, x)$$
$$\leq p(x, x) + f^{-1}(p(Tx, x)) = M_x.$$

Thus, we obtain (4). Next, we prove that the sequence $\{p(T^n x, T^n x)\}_{n \geq 0}$ is Cauchy. Equivalently, we show that

$$\lim_{n, m \to \infty} p(T^n x, T^m x) = r_x, \tag{5}$$

where $r_x := \inf_n p(T^n x, T^n x)$. Now clearly $r_x \leq p(T^n x, T^n x) \leq p(T^n x, T^m x)$ for all n, m. Also, given any $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $p(T^{n_0}x, T^{n_0}x) < r_x + \varepsilon$ and $\phi^{n_0}(2M_x) < r_x + \varepsilon$. Therefore, for any $m, n > 2n_0$, we have

$$\begin{aligned}
r_x &\leq p(T^n x, T^m x) \\
&\leq \max\{\phi(p(T^{n-1}x, T^{m-1}x)), p(T^{n-1}x, T^{n-1}x), p(T^{m-1}x, T^{m-1}x)\} \\
&\leq \max\{\phi^2(p(T^{n-2}x, T^{m-2}x)), p(T^{n-2}x, T^{n-2}x), p(T^{m-2}x, T^{m-2}x)\} \\
&\leq \max\{\phi^{n_0}(p(T^{n-n_0}x, T^{m-n_0}x)), \\
&\quad p(T^{n-n_0}x, T^{n-n_0}x), p(T^{m-n_0}x, T^{m-n_0}x)\} \\
&\leq \max\{\phi^{n_0}(p(T^{n-n_0}x, x) + p(T^{m-n_0}x, x)), \\
&\quad p(T^{n-n_0}x, T^{n-n_0}x), p(T^{m-n_0}x, T^{m-n_0}x)\} \\
&< \max\{\phi^{n_0}(2M_x), r_x + \varepsilon, r_x + \varepsilon\} \\
&< r_x + \varepsilon.
\end{aligned}$$

Hence, we obtain (5). Since (X, p) is a complete partial metric space, there exists $z \in X$ such that $p(z, z) = r_x$. Next, we show that $p(z, z) = p(Tz, z)$.

For every $n \in \mathbb{N}$, we have

$$\begin{aligned} p(z, z) &\leq p(Tz, z) \\ &\leq p(Tz, T^n x) + p(T^n x, z) - p(T^n x, T^n x) \\ &\leq \max\{\phi(p(z, T^{n-1}x)), p(T^{n-1}x, T^{n-1}x), p(z, z)\} \\ &\quad + p(T^n x, z) - p(T^n x, T^n x). \end{aligned}$$

Case 1.

$$\begin{aligned} p(Tz, z) &\leq \phi(p(z, T^{n-1}x)) + p(T^n x, z) - p(T^n x, T^n x) \\ &\leq p(z, T^{n-1}x) + p(T^n x, z) - p(T^n x, T^n x) \to p(z, z) \text{ as } n \to \infty. \end{aligned}$$

Case 2.

$$p(Tz, z) \leq p(T^{n-1}x, T^{n-1}x) + p(T^n x, z) - p(T^n x, T^n x) \to p(z, z) \quad \text{as } n \to \infty.$$

Case 3.

$$p(Tz, z) \leq p(z, z) + p(T^n x, z) - p(T^n x, T^n x) \to p(z, z) \text{ as } n \to \infty.$$

Therefore,

$$p(z, z) = p(Tz, z). \tag{6}$$

Now we show that X_p (see Definition 2) is nonempty. For each $k \in \mathbb{N}$, choose $x_k \in X$ with $p(x_k, x_k) < \rho_p + 1/k$, where $x_k = T^k x$. First, we prove that

$$\lim_{m, n \to \infty} p(z_n, z_m) = \rho_p. \tag{7}$$

Given $\varepsilon > 0$, take $n_0 := [f^{-1}(3/\varepsilon)] + 1$. If $k > n_0$, then

$$\begin{aligned} \rho_p &\leq p(Tz_k, Tz_k) \leq p(z_k, z_k) = r_{x_k} \leq p(x_k, x_k) < \rho_p + 1/k \\ &< \rho_p + 1/n_0 < \rho_p + 1/f^{-1}(3/\varepsilon). \end{aligned}$$

Set $U_k := p(z_k, z_k) - p(Tz_k, Tz_k)$. Then $U_k < 1/f^{-1}(3/\varepsilon)$ for $k > n_0$. Thus, if $m, n > n_0$, then by (6) and the fact that f (and hence f^{-1}) is increasing, we have

$$\begin{aligned} p(z_n, z_m) &\leq p(z_n, Tz_n) + p(Tz_n, Tz_m) + p(Tz_m, z_m) \\ &\quad - p(Tz_n, Tz_n) - p(Tz_m, Tz_m) \\ &= U_n + U_m + p(Tz_n, Tz_m) \\ &< 2/f^{-1}(3/\varepsilon) + \max\{\phi(p(z_n, z_m)), p(z_n, z_n), p(z_m, z_m)\} \\ &\leq \max\{f^{-1}(2/f^{-1}(3/\varepsilon)), 3/f^{-1}(3/\varepsilon) + \rho_p\} \\ &\leq \max\{f^{-1}(2\varepsilon/3), \rho_p + \varepsilon\} \\ &\leq \rho_p + \varepsilon + f^{-1}(2\varepsilon/3). \end{aligned}$$

Therefore, if we let $\varepsilon \to 0$, then we get (7). Since (X, p) is a complete partial metric space, there exists $u \in X$ such that

$$p(u, u) = \lim_{m, n \to \infty} p(z_n, z_m) = \rho_p.$$

Consequently, $u \in X_p$ and hence X_p is nonempty.

Now choose an arbitrary $x \in X_p$. Then

$$\rho_p \leq p(Tz, Tz) \leq p(Tz, z) = p(z, z) = r_x = \rho_p,$$

which, using (P2), implies that $Tz = z$. To prove uniqueness of the fixed point, we suppose that $u, v \in X_p$ are both fixed points of T. Then

$$\begin{aligned} \rho_p \leq p(u, v) = p(Tu, Tv) &\leq \max\{\phi(p(u, v)), p(u, u), p(v, v)\} \\ &\leq \max\{\phi(p(u, v)), \rho_p\}. \end{aligned}$$

Case 1.

$$\rho_p \leq p(u, v) \leq \rho_p \Rightarrow p(u, v) = \rho_p = p(u, u) = p(v, v) \Rightarrow u = v.$$

Case 2.

$$\begin{aligned} & p(u, v) \leq \phi(p(u, v)) \\ \Rightarrow\ & p(u, v) - \phi(p(u, v)) \leq 0 \\ \Rightarrow\ & f(p(u, v)) \leq 0 \\ \Rightarrow\ & f(p(u, v)) = 0 \\ \Rightarrow\ & p(u, v) = 0 \\ \Rightarrow\ & u = v. \end{aligned}$$

Thus, the fixed point is unique. □

Clearly, the above theorem does not guarantee uniqueness of the fixed point in X. However, if (3) is replaced by the condition below, then we can show uniqueness.

Theorem 3. *Let (X, p) be a complete partial metric space. Suppose $T : X \to X$ is a given mapping satisfying*:

$$p(Tx, Ty) \leq \max\left\{\phi(p(x, y)), \frac{p(x, x) + p(y, y)}{2}\right\}, \tag{8}$$

where $\phi : [0, \infty) \to [0, \infty)$ is as in Theorem 2. Then there is a unique point $z \in X$ such that $Tz = z$. Furthermore, $z \in X_p$ and for each $x \in X_p$ the sequence $\{T^n x\}_{n \geq 1}$ converges with respect to the metric p^s to z.

Proof. Using Theorem 2, we only need to prove uniqueness. Suppose there exist $u, v \in X$ such that $Tu = u$ and $Tv = v$. Now

$$p(u, v) = p(Tu, Tv) \leq \max\left\{\phi(p(u, v)), \frac{p(u, u) + p(v, v)}{2}\right\}.$$

Case 1.

$$\begin{aligned} & p(u, v) \leq \phi(p(u, v)) \\ \Rightarrow\ & p(u, v) - \phi(p(u, v)) \leq 0 \end{aligned}$$

$$\Rightarrow f(p(u, v)) \leq 0$$

$$\Rightarrow f(p(u, v)) = 0$$

$$\Rightarrow p(u, v) = 0$$

$$\Rightarrow u = v.$$

Case 2.

$$p(u, v) \leq \frac{p(u, u) + p(v, v)}{2}$$

$$\Rightarrow 2p(u, v) - p(u, u) - p(v, v) \leq 0$$

$$\Rightarrow p^s(u, v) = 0$$

$$\Rightarrow u = v. \qquad \square$$

Corollary 1. *Let* (X, p) *be a* 0*-complete partial metric space. Suppose* $T : X \to X$ *is a given mapping satisfying*:

$$p(Tx, Ty) \leq \phi(p(x, y)), \tag{9}$$

where $\phi : [0, \infty) \to [0, \infty)$ *is an increasing function such that* $f(t) = t - \phi(t)$ *is increasing with* f^{-1} *is right continuous at* 0. *Also, assume* $\lim_{n\to\infty} \phi^n(t) = 0$ *for all* $t \geq 0$ (*and hence* $\phi(0) = 0$, $\phi(t) < t$ *for* $t > 0$). *Then there is a unique* $z \in X$ *such that* $Tz = z$. *Also*, $p(z, z) = 0$ *and for each* $x \in X$ *the sequence* $\{T^n x\}$ *converges with respect to the metric* p^s *to* z.

Corollary 2. *If in Theorem* 2 *and Theorem* 3 *the function* $\phi(t) = \alpha t$, $\alpha \in (0, 1]$, *then Theorem* 1 *and Theorem* 3.2 *in* [15] *will follow*.

Example 2. Let $X = [0, 1] \cup [3, 4]$. Define $p : X \times X \to \mathbb{R}$, $T : X \to X$ and $\phi : [0, \infty) \to [0, \infty)$ as follows:

$$p(x, y) = \max\{x, y\},$$

$$T(x) = \begin{cases} \frac{x}{2}, & x \in [0, 1], \\ \frac{7}{5}, & x \in [3, 4], \end{cases}$$

$$\phi(t) = \frac{t}{1+t}.$$

The above definitions satisfy the hypothesis of Theorem 3. In particular, we make the following observations:

• (X, p) is a complete partial metric space.

• We can easily prove by induction that $\phi^n(t) = \frac{t}{1+nt}$ which implies that $\lim_{n\to\infty} \phi^n(t) = 0$.

• T satisfies condition (8):

(1) If $\{x, y\} \cap [3, 4] \neq \varnothing$, then

$$p(Tx, Ty) = \max\{Tx, Ty\} = \frac{7}{5}$$

$$\leq \max\left\{\phi(p(x, y)), \frac{p(x, x) + p(y, y)}{2}\right\}.$$

(2) If $\{x, y\} \subset [0, 1]$, then

$$p(Tx, Ty) = \max\{Tx, Ty\} = \max\left\{\frac{x}{2}, \frac{y}{2}\right\}$$

$$\leq \max\left\{\phi(p(x, y)), \frac{p(x, x) + p(y, y)}{2}\right\}.$$

• By Theorem 3, there is a unique fixed point which is $z = 0$.

• On the other hand, if the partial metric p is replaced by the usual absolute value metric, then it can be easily checked that condition (8) is not satisfied with, for example, $x = 1$ and $y = 3$.

• We remark that this our example does not verify the conditions of the main theorem in [8]. Therefore, our result has a benefit over [8].

• Our example does not verify the conditions of Theorem 4 in [17]. For example, the ϕ-contractive condition appeared there is not satisfied for $x = 3,\ y = 4.$ Thus, it has an advantage over [17].

• Our example does not verify the conditions of Theorem 3 in [17]. Check for $x = 3,\ y = 4.$

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