Advances and Applications in Statistics
Published Online: March 2014
Available online at http://pphmj.com/journals/adas.htm
Volume 39, Number 1, 2014, Pages 37-59

ESTIMATION OF POPULATION TOTAL USING NONPARAMETRIC REGRESSION MODELS

El-Housseiny A. Rady and Dalia Ziedan

Institute of Statistical Studies and Research
Cairo University
Egypt

Abstract

In this paper, the estimation for finite population total of a study variable will be considered, and the local linear regression will be used. The study variable is available for the sample and is supplemented by multiple auxiliary variables, which are available for every element in the finite population. Also, the resampling methods will be combined with the local linear regression method to estimate the total. The comparisons between different methods will be performed. These comparisons between the methods are based on the mean squared error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). A simulation study is carried out to assess the effects.

1. Introduction

Survey sampling often supplies information about a study variable only for sampled elements. However, auxiliary information is often available for the entire population. The relationship of the auxiliary information with the study variable across the sample allows inferences about the nonsampled portion of the population. Thus, the use of auxiliary information at the

Received: September 28, 2013; Revised: December 27, 2013; Accepted: January 2, 2014

2010 Mathematics Subject Classification: 62G08, 62D05.

Keywords and phrases: survey sampling, auxiliary variables, multiple local linear regression, bootstrap, jackknife.

estimation stage of a survey improves the precision of the estimation of parameters studied. One approach has used this auxiliary information in the estimation by assuming a working model. This working model describes the relationship between the study variable of interest and the auxiliary variables. Estimators are then derived on the basis of this model.

Usually, a parametric approach is used to represent the relationship between the auxiliary variables and the study variable. But in some situations, the parametric model is not appropriate, and the resulting estimators do not achieve any efficiency gain over purely estimators. A natural alternative first suggested by Kuo [11] for the distribution function, is to adopt a nonparametric approach, which does not place any restrictions on the relationship between the auxiliary data and the study variable. Other important works in this topic are Chambers et al. [4], Dorfman [7], Dorfman and Hall [6] and Rueda and Arcos [12].

Breidt and Opsomer [2] used the traditional local polynomial regression estimator for the unknown regression function $m(x)$. They assumed that $m(x)$ is a smooth function of x and obtained asymptotically design-unbiased and consistent estimators of the finite population total. The local polynomial regression estimator has the form of the generalized regression estimator, but is based on a nonparametric superpopulation model applicable to a much larger class of functions. Breidt et al. [1] considered a related nonparametric model-assisted regression estimator, replacing local polynomial smoothing with penalized splines. Kim et al. [10] extended local polynomial nonparametric regression estimation to two-stage sampling, in which a probability sample of clusters is selected, and then subsamples of elements within each selected cluster are obtained.

In practice, the approach of nonparametric regression in the case of multiple predictor variables is very important. Ruppert and Wand [13] studied the asymptotic bias and variance of multivariate local regression estimator. Ye et al. [18] presented a local linear estimator with variable bandwidth for nonparametric multiple regression models. In this paper, we will concern with the estimation of the finite population total in the presence of multiple auxiliary variables using the local linear regression.

2. Multiple Local Linear Regression

Suppose now that the covariate is d-dimensional, where

$$X_i = (x_{i1}, x_{i2}, \ldots, x_{id})'.$$

In this case,

$$Y = m(x_1, x_2, \ldots, x_d) + \varepsilon.$$

For local linear regression, the kernel function K is now a function of d variables. Given a nonsingular positive definite $d \times d$ bandwidth matrix H, we define

$$K_H(x) = \frac{1}{|H|^{1/2}} K(H^{-1/2}x). \tag{1}$$

Often, one scales each covariate to have the same mean and variance and then we use the kernel

$$h^{-d} K(\|x\|/h), \tag{2}$$

where K is any one-dimensional kernel. Then there is a single bandwidth parameter h.

At a target value $x = (x_1, x_2, \ldots, x_d)'$, the local sum of squares is given by

$$\sum_{i=1}^{n} w_i(x)\left(Y_i - a_0 - \sum_{j=1}^{d} a_j(x_{ij} - x_j)\right)^2, \tag{3}$$

where

$$w_i(x) = K(\|x_i - x\|/h).$$

In this case, the estimator is

$$\hat{m}(x) = \hat{a}_0, \tag{4}$$

where $\hat{a} = (\hat{a}_0, \hat{a}_1, \ldots, \hat{a}_d)'$ is the value of $a = (a_0, a_1, \ldots, a_d)'$ that minimizes the weighted sums of squares. The solution $\hat{a}$ is

$$\hat{a} = (X'WX)^{-1}X'WY, \tag{5}$$

where X in this case is

$$X = \begin{pmatrix} 1 & x_{11} - x_1 & \cdots & x_{1d} - x_d \\ 1 & x_{21} - x_1 & \cdots & x_{2d} - x_d \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} - x_1 & \cdots & x_{nd} - x_d \end{pmatrix}.$$

And W is the diagonal matrix. For more details (see Casella et al. [3]).

3. Estimation of Total Using Multiple Local Linear Regression

Suppose that there is a finite population $U = \{1, ..., j, ..., N\}$ for each $j \in U$. Also, suppose that there are d auxiliary variables known for the entire population, and s is a sample of size n from U, for which Y values are known.

Here, we want to estimate the total $T = \sum_{j=1}^{N} Y_j = \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} Y_j$. Since y values are available to us on s, the problem is essentially to get a reasonable estimate of the remainder of the population, outside s. That is, we want the second sum in T above. Thus, a natural idea is estimating the second term and add these to the first term in T. According to Dorfman [7], the estimate of total is

$$\hat{T} = \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} \hat{Y}_j = \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} \hat{m}(x_j),$$

where $\hat{m}(x_j)$ is local linear regression of the unobserved population. To estimate $m(x_j)$, define the $n \times (d + 1)$ matrix

$$X = \begin{pmatrix} 1 & x_{11} - x_{1j} & \cdots & x_{1d} - x_{dj} \\ 1 & x_{21} - x_{1j} & \cdots & x_{2d} - x_{dj} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} - x_{1j} & \cdots & x_{nd} - x_{dj} \end{pmatrix}.$$

Let $\Delta_{bij} = x_{ib} - x_{bj}$ and $\Delta_{0ij} = 1$, where $b = 0, 1, ..., d$ and $i = 1, 2, ..., n$. Then the transpose matrix of X is

$$X' = \begin{pmatrix} \Delta_{o1j} & \Delta_{o2j} & \cdots & \Delta_{onj} \\ \Delta_{11j} & \Delta_{12j} & \cdots & \Delta_{1nj} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{d1j} & \Delta_{d2j} & \cdots & \Delta_{dnj} \end{pmatrix}.$$

Also, define the $n \times n$ matrix

$$W = \begin{pmatrix} w_{11j} & 0 & \cdots & 0 \\ 0 & w_{22j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{nnj} \end{pmatrix},$$

where $w_{iij} = K\left(\frac{1}{h}\left(\sum_{p=1}^{d}\Delta_{pij}^{2}\right)^{1/2}\right).$

Let e_r represent the rth column of the identity matrix. The local linear regression estimator of $m(x_j)$, based on the entire population, is given by

$$\hat{m}(x) = e_1'(X'WX)^{-1}X'WY,$$

where $(X'WX)$ is well-defined and it is invertible.

Now, we will substitute in the previous equation by X, W and Y to get the estimation of the total. Hence

$$X'W = \begin{pmatrix} \Delta_{o1j}w_{11j} & \Delta_{o2j}w_{22j} & \cdots & \Delta_{onj}w_{nnj} \\ \Delta_{11j}w_{11j} & \Delta_{12j}w_{22j} & \cdots & \Delta_{1nj}w_{nnj} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{d1j}w_{11j} & \Delta_{d2j}w_{22j} & \cdots & \Delta_{dnj}w_{nnj} \end{pmatrix}$$

and

$$X'WX = \begin{pmatrix} \ell_{11j} & \ell_{12j} & \cdots & \ell_{1,d+1,j} \\ \ell_{21j} & \ell_{22j} & \cdots & \ell_{2,d+1,j} \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{d+1,1,j} & \ell_{d+1,2,j} & \cdots & \ell_{d+1,d+1,j} \end{pmatrix},$$

where $\ell_{grj} = \sum_{i=1}^{n} \Delta_{g-1,i,j} \Delta_{r-1,i,j} w_{iij}$, $g = 1, 2, ..., d+1$, $r = 1, 2, ..., d+1$.

Note that $(X'WX)$ is a symmetric matrix.

Let $D_j = |X'WX|$ and let S be the adjoint matrix of $(X'WX)$ as the form

$$S = \begin{pmatrix} s_{11j} & s_{12j} & \cdots & s_{1,d+1,j} \\ s_{21j} & s_{22j} & \cdots & s_{2,d+1,j} \\ \vdots & \vdots & \ddots & \vdots \\ s_{d+1,1,j} & s_{d+1,2,j} & \cdots & s_{d+1,d+1,j} \end{pmatrix}, \quad j = 1, 2, ..., N.$$

Hence, the inverse of the matrix $(X'WX)$ is

$$(X'WX)^{-1} = \frac{1}{D_j}(S).$$

Now, we need to get on $X'WY$

$$X'WY = \begin{bmatrix} \sum_{i=1}^{n} \Delta_{0ij} w_{iij} y_i \\ \sum_{i=1}^{n} \Delta_{1ij} w_{iij} y_i \\ \vdots \\ \sum_{i=1}^{n} \Delta_{dij} w_{iij} y_i \end{bmatrix}.$$

Since our primary interest is to compute an estimate of y_j, by substituting in the equation $\hat{y}_j = e_1'(X'WX)^{-1}X'WY$ we get on

$$\hat{y}_j = \hat{m}(x_j) = \frac{1}{D_j}\sum_{c=0}^{d} s_{1,c+1,j}\sum_{i=1}^{n}\Delta_{c,i,j}w_{ij j}y_i. \tag{6}$$

Now, our main purpose is to estimate the total (T). Therefore, according to Dorfman [7], the estimate of the total is

$$\hat{T} = \sum_{i=1}^{n} y_i + \sum_{j=n+1}^{N}\hat{y}_j. \tag{7}$$

Substituting from equation (6) in (7), the estimated total is

$$\begin{aligned}\hat{T} &= \sum_{i=1}^{n} y_i + \sum_{j\neq i}^{N}\frac{1}{D_j}\sum_{c=0}^{d} s_{1,c+1,j}\sum_{i=1}^{n}\Delta_{c,i,j}w_{iij}y_i \\ &= \sum_{i=1}^{n}\left(1 + \sum_{j\neq i}^{N}\frac{1}{D_j}\sum_{c=0}^{d} s_{1,c+1,j}\Delta_{c,i,j}w_{iij}\right)y_i. \end{aligned} \tag{8}$$

According to the value of d, we can obtain the special cases. These special cases will be studied below when $d = 2$ and $d = 3$, where the special case $d = 1$ was studied by Ziedan et al. [19].

(i) Estimation of total in the case of $d = 2$

In this case, we have two auxiliary variables and the estimation of the total has the following form:

$$\begin{aligned}\hat{T} &= \sum_{i=1}^{n} y_i + \sum_{j\neq i}^{N}\left[\frac{1}{D_j}\sum_{c=0}^{2} s_{1,c+1,j}\sum_{i=1}^{n}\Delta_{c,i,j}w_{iij}y_i\right] \\ &= \sum_{i=1}^{n}\left(1 + \sum_{j\neq i}^{N}\frac{1}{D_j}\sum_{c=0}^{2} s_{1,c+1,j}\Delta_{c,i,j}w_{iij}\right)y_i, \end{aligned} \tag{9}$$

where

$$s_{11j} = \left(\sum_i \Delta_{1ij}^2 w_{iij}\right)\left(\sum_i \Delta_{2ij}^2 w_{iij}\right) - \left(\sum_i \Delta_{1ij}\Delta_{2ij} w_{iij}\right)^2,$$

$$s_{12j} = \left(\sum_i \Delta_{1ij}\Delta_{2ij} w_{iij}\right)\left(\sum_i \Delta_{2ij} w_{iij}\right) - \left(\sum_i \Delta_{1ij} w_{iij}\right)\left(\sum_i \Delta_{1ij}\Delta_{2ij} w_{iij}\right),$$

$$s_{13j} = \left(\sum_i \Delta_{1ij} w_{iij}\right)\left(\sum_i \Delta_{1ij}\Delta_{2ij} w_{iij}\right) - \left(\sum_i \Delta_{1ij}^2 w_{iij}\right)\left(\sum_i \Delta_{2ij} w_{iij}\right).$$

(ii) Estimation of total in the case of $d = 3$

When $d = 3$, the estimation of the total will be more complicated, but we can calculate it as the following:

$$\hat{T} = \sum_{i=1}^{n} y_i + \sum_{j=n+1}^{N} \hat{y}_j,$$

$$\hat{T} = \sum_{i=1}^{n} y_i + \sum_{j \neq i}^{N}\left[\frac{1}{D_j}\sum_{c=0}^{3} s_{1,\,c+1,\,j}\sum_{i=1}^{n}\Delta_{c,\,i,\,j} w_{iij} y_i\right]$$

$$= \sum_{i=1}^{n}\left(1 + \sum_{j \neq i}^{N}\frac{1}{D_j}\sum_{c=0}^{3} s_{1,\,c+1,\,j}\Delta_{c,\,i,\,j} w_{iij}\right) y_i, \qquad (10)$$

where

$$s_{11j} = \left(\sum_{i=1}^{n} \Delta_{1ij}^2 w_{iij}\right)^2\left[\left(\sum_{i=1}^{n} \Delta_{2ij}^2 w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij}^2 w_{iij}\right) - \left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)^2\right]$$

$$+ 2\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)$$

$$- \left(\sum_{i=1}^{n} \Delta_{3ij}^2 w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{2ij} w_{iij}\right)^2 - \left(\sum_{i=1}^{n} \Delta_{2ij}^2 w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)^2,$$

$$s_{12j} = \left(\sum_{i=1}^{n} \Delta_{1ij} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)^2 - \left(\sum_{i=1}^{n} \Delta_{2ij}^2 w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij}^2 w_{iij}\right)\right]$$

$$- \left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{2ij} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij} w_{iij}\right)\right.$$

$$\left.- \left(\sum_{i=1}^{n} \Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij}^2 w_{iij}\right)\right]$$

$$+ \left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{2ij}^2 w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij} w_{iij}\right)\right.$$

$$\left.- \left(\sum_{i=1}^{n} \Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)\right],$$

$$s_{13j} = \left(\sum_{i=1}^{n} \Delta_{1ij} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij}^2 w_{iij}\right)\right.$$

$$\left.- \left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)\right]$$

$$- \left(\sum_{i=1}^{n} \Delta_{1ij}^2 w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij}^2 w_{iij}\right)\right.$$

$$\left.- \left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij} w_{iij}\right)\right]$$

$$+ \left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)\right.$$

$$\left.- \left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{3ij} w_{iij}\right)\right],$$

$$
\begin{aligned}
s_{14j} = {} & \left(\sum_{i=1}^{n} \Delta_{1ij} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{2ij}^{2} w_{iij}\right)\right. \\
& \left. - \left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{2ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)\right] \\
& - \left(\sum_{i=1}^{n} \Delta_{1ij}^{2} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{2ij}^{2} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{2ij} w_{iij}\right)\right. \\
& \left. - \left(\sum_{i=1}^{n} \Delta_{2ij}\Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{2ij} w_{iij}\right)\right] \\
& + \left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{2ij} w_{iij}\right)\left[\left(\sum_{i=1}^{n} \Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)\right. \\
& \left. - \left(\sum_{i=1}^{n} \Delta_{1ij}\Delta_{3ij} w_{iij}\right)\left(\sum_{i=1}^{n} \Delta_{2ij} w_{iij}\right)\right].
\end{aligned}
$$

4. Bootstrapping Multiple Local Linear Regression for Estimating the Total

Efron [8] has developed a new resampling procedure named as "Bootstrap". Bootstrap is a resample which consists of n elements that are drawn randomly from the n original data observations with replacement (Friedl and Stampfer [9]). All the bootstrap samples are n^n, but we choose B bootstrap samples. Bootstrapping can be done by either resampling the residuals, in which the regressors $(x_1, x_2, ..., x_d)$ are assumed to be fixed, or resampling the y_i values and their associated x_i values, in which the regressors are assumed to be random. In our study, we deal with the residuals resampling, where the bootstrap technique with nonparametric regression to estimate the total of the population will be used, the local linear regression will be considered. Suppose we have a univariate response variable Y and d auxiliary variables $(x_1, x_2, ..., x_d)$, then the nonparametric regression model is

$$Y_i = m(x_{1i}, x_{2i}, ..., x_{di}) + \varepsilon_i, \quad i = 1, ..., n$$

and the bootstrap procedure based on the resampling errors can be summarized as following:

(1) Let $Y = (Y_1, Y_2, ..., Y_n)$ denote the sample of observations selected from the generated population. Then based on the sample Y the local linear regression estimator $\hat{m}(x)$ is given by

$$\hat{y}_i = \hat{m}(x_{1i}, x_{2i}, ..., x_{di}) = e_1'(X'WX)^{-1} X'WY.$$

(2) Calculate the residuals as following:

$$\hat{\varepsilon}_i = Y_i - \hat{m}(x_{1i}, x_{2i}, ..., x_{di}), \quad i = 1, 2, ..., n.$$

(3) Define the centered residuals by $\tilde{\varepsilon}_i = \hat{\varepsilon}_i - \frac{1}{n}\sum_{i=1}^{n} \hat{\varepsilon}_i$.

(4) Generate the ε_i^* by sampling with replacement from $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, ..., \tilde{\varepsilon}_n$ calculated in step (3) giving $1/n$ probability for each $\tilde{\varepsilon}_i$ values (see Stine [15, 16] and Wu [17]).

(5) The bootstrap sample of observations is constructed by adding a randomly sampled residual to the original predicted value for each observation. After resampling, new observation is given by

$$Y_i^* = \hat{m}(x_{1i}, x_{2i}, ..., x_{di}) + \varepsilon_i^*.$$

(6) Obtain the local linear estimate from the first bootstrap sample:

$$\hat{Y}_i^{*(1)} = e_1'(X'WX)^{-1} X'WY^*.$$

(7) Repeat steps (4), (5) and (6) for B times.

Then the bootstrap estimate is

$$\hat{Y}_i^* = \frac{1}{B}\sum_{t=1}^{B} \hat{Y}_i^{*(t)}. \tag{11}$$

Now, we will estimate the total using local linear regression estimation with bootstrap method, since we have

$$T = \sum_{j=1}^{N} Y_j = \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} Y_j$$

but $\sum_{j \neq i}^{N} Y_j$ is unknown , so we will estimate it, hence

$$\begin{aligned} \hat{T}^* &= \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} \hat{Y}_j^* \\ &= \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} \frac{1}{B} \sum_{t=1}^{B} \frac{1}{D_j} \sum_{c=0}^{d} s_{1,\, c+1,\, j} \sum_{i=1}^{n} \Delta_{c,\, i,\, j} w_{iij} \hat{Y}_i^{*(t)}. \end{aligned} \tag{12}$$

5. Jackknifing Multiple Local Linear Regression for Estimating the Total

In this section, the algorithm of estimating the total using local linear regression method with jackknife technique will be given. The technique of deleting single case from the original sample (delete-one jackknife) sequentially will be used. Suppose the dataset consists of n vectors $(Y_i, X_{1i}, ..., X_{di})$, where Y_i is the study variable and $(X_{1i}, ..., X_{di})$ are considered auxiliary variables. For simplicity, we will use $x_i = (x_{1i}, ..., x_{di})$ and $z_k = (y_k, x_k)$, $k = 1, 2, ..., n$ denote the values associated with ith observation. In this case, the set of observations is the vector $(z_1, z_2, ..., z_n)$. Then the jackknife procedure based on delete-one is as follows:

(1) Draw n sized sample from population randomly and label the elements of the vector $z_k = (y_k, x_k)$, $k = 1, 2, ..., n.$

(2) Omit first observation of the vector $z_k = (y_k, x_k)$ and label remaining $n - 1$ sized observation set $Y_{(1)}^{(J)} = (y_2, ..., y_n)$ and $X_{(1)}^{(J)} = (x_2, ..., x_n)$ as delete-one jackknife sample $z_1^{(J)}$.

(3) Obtain the local linear regression estimate $\hat{Y}_i^{(J1)}$ from $z_{(1)}^{(J)}$.

(4) Omit the second element of the vector $z_i = (y_i, x_i)$ and label remaining $n-1$ sized observation set $Y_{(2)}^{(J)} = (y_1, y_3, ..., y_n)$ and $X_{(2)}^{(J)} = (x_1, x_3, ..., x_n)$ as $z_2^{(J)}$.

(5) Obtain the local linear regression estimate $\hat{Y}_i^{(J2)}$ from $z_{(2)}^{(J)}$.

(6) Similarly, omit each one of the n observations (there is n samples jackknife each of them has $n-1$ observations) and estimate the local linear regression $\hat{Y}_i^{(Jk)}$, where $\hat{Y}_i^{(Jk)}$ is the jackknife local linear regression estimate after deleting of kth observation from $z_k = (y_k, x_k)$.

(7) Then the jackknife estimate of $\hat{m}(x_j)$ is

$$\hat{Y}_i^{(J)} = \frac{1}{n}\sum_{k=1}^{n} \hat{Y}_i^{(Jk)}. \tag{13}$$

Now, we will estimate the total using local linear regression estimation with jackknife estimate, since we have

$$T = \sum_{j=1}^{N} Y_j = \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} Y_j,$$

$$\hat{T}^{(J)} = \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} \hat{Y}_j^{(J)}$$

$$= \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{D_j} \sum_{c=0}^{d} s_{1, c+1, j} \sum_{i=1}^{n} \Delta_{c, i, j} w_{ij} \hat{Y}_i^{(Jk)}. \tag{14}$$

6. Performance Criteria of the Models

The performance of the model is related with how close are the prediction values to the observed values. Three different consistency criteria are used in order to compare between different methods. These are mean square error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE), respectively. These criteria are defined as follows:

- $MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2.$
- $MAE = \frac{1}{n}\sum_{i=1}^{n}| y_i - \hat{y}_i |.$
- $MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{| y_i - \hat{y}_i |}{| y_i |}(100\%).$

7. Simulation Studies with Two Auxiliary Variables

Sometimes in sampling, we do not usually observe all the survey information. That is, the survey variable Y is not observable for all the population units. Auxiliary variable X is often used to estimate the unobserved survey variables. One way of overcoming the above problem is the super population approach, in which a working model relating the two auxiliary variables is assumed. In this study, we simulate data from four models, which are introduced by Ye et al. [18], each with $Y = m(X_1, X_2) + \delta(X_1)\varepsilon,$ where $\varepsilon \sim N(0, 1).$

Model (1): $m_1(x_1, x_2) = x_1x_2$

$$\delta_1^2(x_1, x_2) = (x_1^2 - 0.04)I_{(x_1^2 > 0.04)} + 0.01.$$

Model (2): $m_2(x_1, x_2) = x_1 \exp(-2x_2^2)$

$$\delta_2^2(x_1, x_2) = 2.5(x_1^2 - 0.04)I_{(x_1^2 > 0.04)} + 0.025.$$

Model (3): $m_3(x_1, x_2) = x_1 + 2\sin(1.5x_2)$

$$\delta_3^2(x_1, x_2) = (x_1^2 - 0.04)I_{(x_1^2 > 0.04)} + 0.01.$$

Model (4): $m_4(x_1, x_2) = \sin(x_1 + x_2) + 2\exp(-2x_2^2)$

$$\delta_4^2(x_1, x_2) = 3(x_1^2 - 0.04) I_{(x_1^2 > 0.04)} + 0.03.$$

The populations of X_1 and X_2 are generated as independent and identically distributed (iid) uniform $(-2, 2)$ random variables.

The simulation experiments will be performed to compare the performance of the local linear regression estimator with the classic linear regression estimator. Also, the effects of the bootstrap and the jackknife techniques on those estimators will be studied. The simulation will be carried out as following: in the first, we generate population of size $N = 1000$ as above. The simple random samples will be chosen from the population, different sizes will be considered $n = 25, 50$ and 100.

Secondly, for each sample, we estimate the total $T = \sum_{i=1}^{n} Y_i + \sum_{j \neq i}^{N} m(x_j)$. The linear regression and the local linear regression will be used to estimate $m(x)$. Also, the bootstrap and the jackknife techniques will be combined with those regression methods to estimate $m(x)$. We consider the normal kernel function with different bandwidth values $h = n^{-1/3}$, $n^{-1/5}$ and $n^{-1/7}$ for the local linear regression, each simulation setting is applied to all four models and repeated $M = 1000$ times.

Thirdly, the mean square error (MSE) of the total (T) under the two types of the regression methods will be calculated. Also, the mean absolute error (MAE) and the mean absolute percentage error (MAPE) will be calculated.

Finally, the effects of the bootstrap and the jackknife techniques on the estimation of total (T) will be studied, these effects are based on the bias, MSE, MAE and MAPE.

Tables from 1 to 4 show the values of the mean squared error (MSE), mean absolute error (MAE) and the mean absolute percentage error (MAPE) of the estimators for the four models, when the sample size (n) has different values $n = 25,$ 50 and 100 and the bandwidth has values $h = n^{-1/3}$, $n^{-1/5}$ and $n^{-1/7}$ in the case of two auxiliary variables.

8. Simulation Studies with Three Auxiliary Variables

Here also, we simulate data from four models, each with $Y = m(X_1, X_2, X_3) + [\delta(X_1) + \delta(X_2)]^{1/2}\varepsilon,$ where $\varepsilon \sim N(0, 1)$.

Model (1): $m_1(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_1x_3$

$$\delta_1^2(x_1) = (x_1^2 - 0.04)I_{(x_1^2 > 0.04)} + 0.01,$$

$$\delta_1^2(x_2) = (x_2^2 - 0.04)I_{(x_2^2 > 0.04)} + 0.01.$$

Model (2): $m_2(x_1, x_2, x_3) = x_1 \exp(-2x_2^2) + x_2 \exp(-2x_3^2) + x_1 \exp(-2x_3^2)$

$$\delta_2^2(x_1) = 2.5(x_1^2 - 0.04)I_{(x_1^2 > 0.04)} + 0.025,$$

$$\delta_2^2(x_2) = 2.5(x_2^2 - 0.04)I_{(x_2^2 > 0.04)} + 0.025.$$

Model (3): $m_3(x_1, x_2, x_3) = x_1 + x_3 + 2\sin(1.5x_3)$

$$\delta_3^2(x_1) = (x_1^2 - 0.04)I_{(x_1^2 > 0.04)} + 0.01,$$

$$\delta_3^2(x_2) = (x_2^2 - 0.04)I_{(x_2 > 0.04)} + 0.01.$$

Model (4): $m_4(x_1, x_2, x_3) = \sin(x_1 + x_2 + x_3) + 2\exp(-2x_3^2)$

$$\delta_4^2(x_1) = 3(x_1^2 - 0.04)I_{(x_1^2 > 0.04)} + 0.03,$$

$$\delta_4^2(x_2) = 3(x_2^2 - 0.04)I_{(x_2^2 > 0.04)} + 0.03.$$

Also, the populations of X_1, X_2 and X_3 are generated as independent and identically distributed (iid) uniform $(-2, 2)$ random variables, and the steps of the simulation as the case of the two auxiliary variables.

Tables from 5 to 8 show the values of the mean squared error (MSE), mean absolute error (MAE) and the mean absolute percentage error (MAPE) of the estimators for the four models, when the sample size (n) has different values $n = 25,$ 50 and 100 and the bandwidth has values $h = n^{-1/3}$, $n^{-1/5}$ and $n^{-1/7}$ in the case of three auxiliary variables.

9. Results of the Simulation Study

Tables from 1-8 summarize the following conclusions about our simulation study:

i. For all the models, the local linear regression estimator dominates the classical linear regression estimator when the regression model is incorrectly specified.

ii. The local linear regression estimator with bootstrap is overall the best choice for all the models and bandwidths under study.

iii. The effect of the bootstrap on the estimator is better than the jackknife at the most.

iv. The bandwidth $h = n^{-1/5}$ is the best choice at the most for all the models.

v. For all estimators, as the sample size increases, the mean squared error (MSE), the mean absolute error (MAE) and the mean absolute percentage error (MAPE) decrease, for the three bandwidths (h) considered and for all the models.

Table 1. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of two auxiliary variables for model 1

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	404.25	29.63	85.5%	397.85	27.24	74.2%	396.56	25.99	27.0%
LLR	332.25	25.85	68.8%	325.87	23.49	57.9%	324.58	22.26	17.8%
LLB	329.22	21.55	42.1%	322.84	19.23	32.2%	321.55	18.01	6.4%
LLJ	336.14	28.14	72.3%	329.76	25.76	61.2%	328.47	24.52	19.6%
$h = n^{-1/5}$									
CLR	373.16	27.35	79.0%	367.25	25.14	68.5%	366.05	24.00	25.0%
LLR	306.69	23.86	63.5%	300.81	21.68	53.4%	299.61	20.55	16.4%
LLB	303.89	19.89	38.9%	298.01	17.75	29.8%	296.81	16.62	5.9%
LLJ	310.28	25.97	66.7%	304.40	23.78	56.5%	303.20	22.63	18.1%
$h = n^{-1/7}$									
CLR	435.35	31.90	92.1%	428.46	29.33	79.9%	427.06	27.99	29.1%
LLR	357.81	27.84	74.1%	350.94	25.30	62.3%	349.55	23.97	19.2%
LLB	354.54	23.21	45.4%	347.68	20.71	34.7%	346.28	19.40	6.9%
LLJ	362.00	30.30	77.8%	355.13	27.74	65.9%	353.74	26.41	21.1%

Table 2. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of two auxiliary variables for model 2

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	513.09	37.60	108.6%	504.97	34.57	94.2%	503.32	32.99	34.3%
LLR	421.70	32.81	87.3%	413.61	29.81	73.4%	411.97	28.25	22.6%
LLB	426.64	35.71	91.7%	418.55	32.69	77.7%	416.90	31.12	24.9%
LLJ	417.85	27.35	53.5%	409.76	24.40	40.9%	408.12	22.86	8.1%
$h = n^{-1/5}$									
CLR	502.83	36.85	106.4%	494.87	33.88	92.3%	493.26	32.33	33.6%
LLR	413.27	32.16	85.5%	405.34	29.22	72.0%	403.73	27.68	22.2%
LLB	409.49	26.80	52.4%	401.57	23.92	40.1%	399.96	22.40	7.9%
LLJ	418.11	35.00	89.9%	410.17	32.04	76.2%	408.56	30.50	24.4%
$h = n^{-1/7}$									
CLR	519.31	38.06	109.9%	511.09	34.99	95.4%	509.43	33.39	34.7%
LLR	426.81	33.21	88.3%	418.62	30.18	74.3%	416.96	28.59	22.9%
LLB	422.92	27.68	54.1%	414.73	24.70	41.4%	413.06	23.14	8.2%
LLJ	431.81	36.15	92.9%	423.62	33.09	78.7%	421.96	31.50	25.2%

Table 3. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of two auxiliary variables for model 3

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	466.44	34.18	98.7%	459.06	31.43	85.7%	457.57	29.99	31.2%
LLR	383.36	29.83	79.4%	376.01	27.10	66.8%	374.51	25.68	20.6%
LLB	379.86	24.86	48.6%	372.51	22.19	37.2%	371.02	20.78	7.4%
LLJ	387.85	32.47	83.4%	380.50	29.72	70.7%	379.00	28.29	22.7%
$h = n^{-1/5}$									
CLR	444.68	32.59	94.1%	437.64	29.96	81.7%	436.21	28.59	29.7%
LLR	365.47	28.44	75.6%	358.46	25.84	63.6%	357.04	24.48	19.6%
LLB	362.14	23.70	46.3%	355.13	21.15	35.5%	353.70	19.81	7.0%
LLJ	369.75	30.95	79.5%	362.74	28.33	67.4%	361.32	26.97	21.6%
$h = n^{-1/7}$									
CLR	478.88	35.10	101.3%	471.30	32.26	87.9%	469.77	30.79	32.0%
LLR	393.59	30.62	81.5%	386.03	27.83	68.5%	384.50	26.37	21.1%
LLB	389.99	25.53	49.9%	382.44	22.78	38.2%	380.91	21.34	7.5%
LLJ	398.20	33.33	85.6%	390.64	30.51	72.5%	389.11	29.05	23.3%

Table 4. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of two auxiliary variables for model 4

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	539.73	41.02	118.4%	550.88	37.71	102.8%	549.08	35.99	37.4%
LLR	440.04	35.79	95.2%	451.21	32.52	80.1%	449.42	30.82	24.7%
LLB	435.84	29.84	58.3%	447.01	26.62	44.6%	445.22	24.94	8.8%
LLJ	445.42	38.96	100.1%	456.60	35.67	84.8%	454.80	33.95	27.2%
$h = n^{-1/5}$									
CLR	522.42	38.29	110.5%	514.15	35.20	95.9%	512.48	33.59	34.9%
LLR	429.37	33.41	88.9%	421.13	30.36	74.8%	419.46	28.76	23.0%
LLB	425.45	27.85	54.4%	417.21	24.85	41.7%	415.54	23.27	8.2%
LLJ	434.40	36.36	93.4%	426.16	33.29	79.1%	424.48	31.69	25.4%
$h = n^{-1/7}$									
CLR	565.95	41.48	119.8%	557.00	38.13	103.9%	555.18	36.39	37.9%
LLR	465.15	36.19	96.3%	456.22	32.89	81.0%	454.41	31.16	24.9%
LLB	470.60	39.39	101.2%	461.67	36.06	85.7%	459.86	34.33	27.5%
LLJ	460.90	30.17	59.0%	451.98	26.92	45.1%	450.17	25.21	8.9%

Table 5. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of three auxiliary variables for model 1

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	376.92	27.63	79.72%	370.96	25.40	69.18%	369.75	24.23	25.17%
LLR	306.10	23.82	63.39%	300.22	21.64	53.34%	299.04	20.51	16.40%
LLB	299.95	19.63	38.36%	294.14	17.52	29.34%	292.96	16.41	12.35%
LLJ	309.25	25.89	66.52%	303.38	23.70	56.30%	302.19	22.56	18.03%
$h = n^{-1/5}$									
CLR	347.93	25.50	73.66%	342.42	23.44	63.87%	341.31	22.38	23.31%
LLR	282.55	21.98	58.50%	277.14	19.97	49.20%	276.03	18.93	15.11%
LLB	276.87	18.12	35.44%	271.52	16.17	27.15%	270.42	15.14	13.87%
LLJ	285.46	23.89	61.36%	280.05	21.88	51.98%	278.94	20.82	16.65%
$h = n^{-1/7}$									
CLR	405.92	29.74	85.87%	399.50	27.35	74.50%	398.19	26.10	27.13%
LLR	329.65	25.65	68.27%	323.32	23.31	57.40%	322.04	22.08	17.69%
LLB	323.02	21.15	41.36%	316.77	18.87	31.62%	315.50	17.68	16.29%
LLJ	333.04	27.88	71.58%	326.72	25.52	60.63%	325.44	24.30	19.41%

Table 6. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of three auxiliary variables for model 2

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	677.79	49.67	143.46%	667.07	45.67	124.44%	664.89	43.58	45.31%
LLR	510.68	39.73	105.72%	500.88	36.10	88.89%	498.90	34.21	27.37%
LLB	474.00	39.67	101.88%	465.01	36.32	86.32%	463.18	34.57	27.66%
LLJ	505.60	33.09	64.74%	495.81	29.52	49.49%	493.83	27.66	25.21%
$h = n^{-1/5}$									
CLR	664.24	48.68	140.55%	653.72	44.76	121.93%	651.60	42.71	44.39%
LLR	500.47	38.95	103.54%	490.87	35.39	87.19%	488.92	33.52	26.88%
LLB	454.94	29.77	58.22%	446.14	26.58	44.55%	444.36	24.89	21.32%
LLJ	505.91	42.35	108.78%	496.31	38.77	92.20%	494.36	36.91	29.52%
$h = n^{-1/7}$									
CLR	686.01	50.28	145.18%	675.15	46.22	126.02%	672.96	44.11	45.84%
LLR	516.87	40.22	106.93%	506.95	36.55	89.98%	504.94	34.62	27.73%
LLB	469.86	30.75	60.11%	460.77	27.44	46.00%	458.91	25.71	19.67%
LLJ	522.49	43.74	112.41%	512.58	40.04	95.23%	510.57	38.12	30.49%

Table 7. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of three auxiliary variables for model 3

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	524.75	38.45	111.04%	516.44	35.36	96.41%	514.77	33.74	35.10%
LLR	376.08	29.26	77.89%	368.87	26.59	65.53%	367.39	25.19	20.21%
LLB	349.85	22.90	44.76%	343.08	20.44	34.26%	341.71	19.14	17.82%
LLJ	364.97	30.55	78.48%	358.05	27.97	66.53%	356.64	26.62	21.36%
$h = n^{-1/5}$									
CLR	500.27	36.66	105.86%	492.35	33.71	91.91%	490.74	32.16	33.41%
LLR	358.53	27.90	74.16%	351.65	25.35	62.39%	350.26	24.01	19.23%
LLB	333.53	21.83	42.64%	327.07	19.48	32.70%	325.76	18.25	18.45%
LLJ	347.93	29.12	74.81%	341.34	26.66	63.42%	340.00	25.38	20.33%
$h = n^{-1/7}$									
CLR	538.74	39.49	113.96%	530.21	36.29	98.89%	528.49	34.64	36.00%
LLR	386.11	30.04	79.95%	378.70	27.30	67.20%	377.19	25.87	20.70%
LLB	359.18	23.51	45.96%	352.23	20.98	35.18%	350.82	19.65	16.91%
LLJ	374.71	31.36	80.55%	367.59	28.71	68.22%	366.15	27.34	21.93%

Table 8. MSE, MAE and MAPE of the total estimation under different methods with different sample sizes and bandwidths in the case of three auxiliary variables for model 4

$h = n^{-1/3}$									
Method	$n = 25$			$n = 50$			$n = 100$		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
CLR	607.20	46.15	133.20%	619.74	42.42	115.65%	617.72	40.49	42.08%
LLR	431.68	35.11	93.39%	442.64	31.90	78.58%	440.88	30.23	24.23%
LLB	401.41	27.48	53.69%	411.70	24.52	41.08%	410.05	22.97	18.10%
LLJ	419.14	36.66	94.19%	429.66	33.57	79.80%	427.97	31.95	25.60%
$h = n^{-1/5}$									
CLR	587.72	43.08	124.31%	578.42	39.60	107.89%	576.54	37.79	39.26%
LLR	421.21	32.78	87.21%	413.13	29.78	73.38%	411.49	28.21	22.56%
LLB	391.84	25.65	50.10%	384.25	22.89	38.41%	382.71	21.43	17.55%
LLJ	408.77	34.21	87.89%	401.02	31.33	74.43%	399.44	29.82	23.90%
$h = n^{-1/7}$									
CLR	636.69	46.67	134.78%	626.63	42.90	116.89%	624.58	40.94	42.64%
LLR	456.31	35.50	94.47%	447.55	32.27	79.46%	445.78	30.57	24.43%
LLB	433.42	36.28	93.21%	425.20	33.21	78.93%	423.53	31.62	25.33%
LLJ	433.71	28.39	55.52%	425.31	25.33	42.44%	423.61	23.72	18.37%

Abbreviation. CLR: classical linear regression, LLR: local linear regression, LLB: local linear regression with bootstrap and LLJ: local linear regression with jackknife.

References

[1] F. J. Breidt, G. Claeskens and J. D. Opsomer, Model-assisted estimation for complex surveys using penalized splines, Biometrika 92 (2005), 831-846.

[2] F. J. Breidt and J. D. Opsomer, Local polynomial regression estimators in survey sampling, Ann. Statist. 28 (2000), 1026-1053.

[3] G. Casella, S. Fienberg and I. Olkin, All of Nonparametric Statistics, Springer, New York, 2006.

[4] R. L. Chambers, A. H. Dorfman and T. E. Wehrly, Bias robust estimation in finite populations using nonparametric calibration, J Amer. Statist. Assoc. 88 (1993), 268-277.

[5] A. H. Dorfman, Nonparametric regression for estimating totals in finite populations, Proceedings of the Section on Survey Research Methods, American Statistical Association, 1992, pp. 622-625.

[6] A. H. Dorfman and P. Hall, Estimators of the finite population distribution function using nonparametric regression, Ann. Statist. 21(3) (1993), 1452-1475.

[7] A. H. Dorfman, A comparison of design-based and model-based estimators of the finite population distribution function, Aust. J. Stat. 35 (1993), 29-41.

[8] B. Efron, Bootstrap methods: another look at the jackknife, Ann. Statist. 7 (1979), 1-26.

[9] H. Friedl and E. Stampfer, Jackknife resampling, Encyclopedia of Environmetrics 2 (2002), 1089-1098.

[10] J. Y. Kim, F. J. Breidt and J. D. Opsomer, Nonparametric regression estimation of finite population totals under two-stage sampling, Technical Report, Department of Statistics, Colorado State University, 2009.

[11] L. Kuo, Classical and prediction approaches to estimating distribution functions from survey data, Proceedings of the Section on Survey Research Methods, Amer. Statist. Assoc., Alexandria, VA, 1988, pp. 280-285.

[12] M. Rueda and A. Arcos, On estimating the median from survey data using multiple auxiliary information, Metrika 54 (2001), 59-76.

[13] D. Ruppert and M. P. Wand, Multivariate locally weighted least squares regression, Ann. Statist. 22(3) (1994), 1346-1370.

[14] S. Sahinler and D. Topuz, Bootstrap and jackknife resampling algorithms for estimation of regression parameters, Journal of Applied Quantitative Methods 2(2) (2007), 188-199.

[15] R. Stine, Bootstrap prediction intervals for regression, J. Amer. Statist. Assoc. 80 (1985), 1026-1031.

[16] R. Stine, Modern Methods of Data Analysis, John Fox, ed., Scotland, 1990, pp. 325-373.

[17] C. F. J. Wu, Jackknife, bootstrap and other resampling methods in regression analysis, Ann. Statist. 14 (1986), 1343-1350.

[18] A. Ye, R. J. Hyndman and Z. Li, Local linear multivariate regression with variable bandwidth in the presence of heteroscedasticity, Working Paper 8/06, Department of Econometrics and Business Statistics, Monash University, 2006.

[19] D. Ziedan, M. S. Khder, S. Elattar and S. Mohamed, Estimation of total for finite population by using nonparametric methods, JP Journal of Fundamental and Applied Statistics 3 (2012), 75-91.