Far East Journal of Applied Mathematics

Published Online: February 2014
Available online at http://pphmj.com/journals/fjam.htm
Volume 84, Number 2, 2013, Pages 89-94

A NEW Q-S SYNCHRONIZATION SCHEME FOR DISCRETE CHAOTIC SYSTEMS

Adel Ouannas

LAMIS Laboratory
Department of Mathematics and Computer Science
University of Tebessa
12002, Algeria
e-mail: ouannas_adel@yahoo.fr

Abstract

In this paper, a new control scheme is developed to investigate the Q-S synchronization for different dimensional chaotic dynamical systems in discrete-time. Numerical simulation is used to verify the effectiveness of the proposed scheme.

1. Introduction

Since the pioneering work of Pecora and Carroll [2], chaos synchronization has played significant roles because of its potential applications in secure communication [16, 17]. Various methods have been developed to design the controllers in the continuous-time dynamical systems such as OGY method [3], active and adaptive control technique [9, 15], backstepping design [12] and sliding mode control [10], etc. The design methods have been applied to investigate chaos synchronization of some continuous-time chaotic systems.

Recently, more and more attentions were paid to the chaos

Received: December 9, 2013; Accepted: January 10, 2014
2010 Mathematics Subject Classification: 37D45.
Keywords and phrases: Q-S synchronization, discrete-time, chaos systems, new scheme.

synchronization in the discrete-time dynamical systems [18, 19]. In [6], Yang defined a generalized-type synchronization (called *Q-S synchronization*) in the continuous-time systems. Recently, Zhenya presented in [18] the definition of Q-S synchronization in the discrete-time dynamical systems:

Definition 1. For two discrete-time dynamical systems described:

$$\begin{cases} \text{(i)} \quad X(k+1) = f(X(k)) \\ \text{and} \\ \text{(ii)} \quad Y(k+1) = g(Y(k)) + U(X(k), Y(k)), \end{cases} \tag{1}$$

where $(X(k), Y(k)) \in \mathbb{R}^{n+m}$. Let $Q(X(k)) = [Q_1(X(k)), ..., Q_d(X(k))]$ and $S(X(k)) = [S_1(Y(k)), ..., S_d(Y(k))]$ be boundary vector functions, it is said that systems (i) and (ii) are *Q-S synchronized*, if there exists a proper $U(X(k), Y(k))$ such that

$$\lim_{k \to +\infty} \| Q(X(k)) - S(Y(k)) \| \to 0. \tag{2}$$

In this paper, based on new design method, we would like to present a constructive scheme to investigate Q-S synchronization between n-D and m-D chaotic systems in discrete-time, the synchronization errors are given in d-D.

The rest of this paper is arranged as follows: In Section 2, we introduce the new Q-S synchronization scheme. In Section 3, we investigate Q-S synchronization between the Hénon map and the 3D generalized Hénon map. Finally, we give conclusion in Section 4.

2. A New Q-S Synchronization Controller

We consider the master chaotic system in the following form:

$$X(k+1) = F(X(k)), \tag{3}$$

where $X(k) = (x_1(k), x_2(k), ..., x_n(k))^T \in \mathbb{R}^n$ and $F : \mathbb{R}^n \to \mathbb{R}^n$.

The slave chaotic system is described by

$$Y(k+1) = G(Y(k)) + U, \tag{4}$$

where $Y(k) = (y_1(k), y_2(k), \ldots, y_m(k))^T \in \mathbb{R}^m$, $G : \mathbb{R}^m \to \mathbb{R}^m$ and U is the vector controller to be determined.

We define the synchronization error by

$$e(k) = Q(X(k)) - S(Y(k)), \tag{5}$$

where $Q : \mathbb{R}^n \to \mathbb{R}^d$ and $S : \mathbb{R}^m \to \mathbb{R}^d$, are arbitrary boundary vector functions.

The synchronization error between the master system (3) and the slave system (4), can be derived as follows:

$$e(k+1) = Q[F(X(k))] - S[G(Y(k)) + U]. \tag{6}$$

To achieve Q-S synchronization between systems (3) and (4), we can choose the vector controller U as follows:

$$U = -G(Y(k)) + S^{-1}[Q(F(X(k))) + L[S(Y(k)) - Q(X(k))]], \tag{7}$$

where $S^{-1} : \mathbb{R}^d \to \mathbb{R}^m$ is the inverse function of S and $L = diag(l_1, l_2, \ldots, l_d) \in \mathbb{R}^{d\times d}$, such that

$$| l_i | < 1, \quad i = 1, 2, \ldots, d. \tag{8}$$

Hence, we have the following result.

Theorem 2. *The master system* (3) *and the slave system* (4) *can be globally Q-S synchronized under the controller law* (7).

Proof. By substituting Equation (7) into Equation (6), the synchronization error can be written as:

$$e(k+1) = Le(k). \tag{9}$$

Consider the quadratic Lyapunov function

$$V(e(k)) = \sum_{i=1}^{d} e_i^2(k). \tag{10}$$

We obtain $\Delta V(e(k)) = \sum_{i=1}^{d}(e_i^2(k+1) - e_i^2(k)) = \sum_{i=1}^{d}(l_i^2 - 1)e_i^2(k)$. By using (8), we get $\Delta V(e(k)) < 0$. Thus, by Lyapunov stability it is immediate that $\lim_{k\to\infty} e_i(k) \to 0$, $(1 \leq i \leq d)$. Hence, we have proved the above result. □

3. Numerical Simulation

To validate the proposed synchronization method, an example of chaotic system is considered in this paper.

The master system is the 3D generalized Hénon map [20] described by:

$$X(k+1) = F(X(k)) = \begin{pmatrix} 1 + x_3(k) - \alpha x_2^2(k) \\ 1 + \beta x_2(k) - \alpha x_1^2(k) \\ \beta x_2(k) \end{pmatrix}, \tag{11}$$

where $\alpha = 1.4$, $\beta = 0.2$.

The slave system is the Hénon map [5] described by:

$$Y(k+1) = G(Y(k)) + U = \begin{pmatrix} y_2(k) + 1 - a y_1^2(k) \\ b y_1(k) \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \tag{12}$$

where $a = 1.4$, $b = 0.3$ and $(u_1, u_2)^T$ is the vector controller. According to (7), the vector controller can be derived as

$$U = -G(Y(k)) + S^{-1}[Q(F(X(k))) + L[S(Y(k)) - Q(X(k))]], \tag{13}$$

where

$$S(Y(k)) = (y_1(k), y_2(k), 3y_1(k), y_2(k) + 4)^T, \tag{14}$$

$$Q(X(k)) = (x_1(k), x_2(k), x_3(k), 3x_3(k) + 4)^T, \tag{15}$$

and $L = diag(0.9, -0.25, -0.76, 0.13)$.

Finally, with the help of MATLAB, we get the numeric results that are shown in Figure 1.

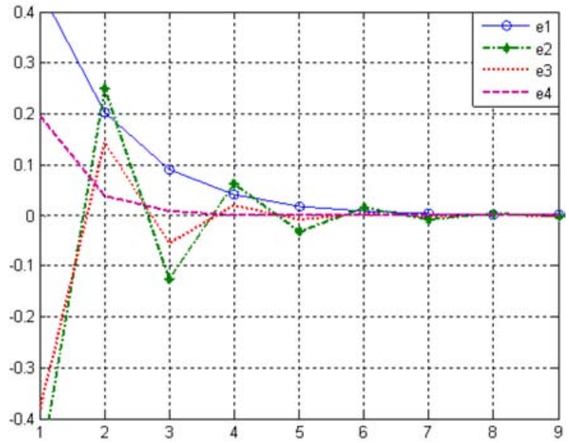

Figure 1. Time evolution of synchronization errors e1, e2, e3 and e4.

4. Conclusion

In this paper, we have developed a systematic and powerful synchronization scheme, which was used to study Q-S synchronization between two n-D and m-D chaotic dynamical systems in discrete-time. Numerical simulation was used to verify the effectiveness of the proposed scheme.

References

[1] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares and C. S. Zhou, The synchronization of chaotic systems, Phys. Rep. 366 (2002), 1-101.

[2] L. M. Pecora and T. L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. 64 (1990), 821-824.

[3] E. Ott, C. Grebogi and J. A. Yorke, Controlling chaos, Phys. Rev. Lett. 64 (1990), 1196-1199.

[4] K. Pyragas, Continuous control of chaos by self-controlling feedback, Phys. Lett. A 170 (1992), 421-428.

[5] M. Hénon, Commun. Math. Phys. 50 (1976), 69.

[6] X. S. Yang, Phys. Lett. A 260 (1999), 340.

[7] M. C. Ho and Y. C. Hung, Synchronization of two different chaotic systems using generalized active control, Phys. Lett. A 301 (2002), 424-428.

[8] J. Lu, X. Wu, X. Han and J. Lu, Adaptive feedback synchronization of unified chaotic systems, Phys. Lett. A 329 (2004), 327-333.

[9] S. H. Chen and J. Lu, Synchronization of an uncertain unified systems via adaptive control, Chaos Solitons Fractals 14 (2002), 643-647.

[10] H. T. Yau, Design of adaptive sliding mode controller for chaos synchronization with uncertainties, Chaos Solitons Fractals 22 (2004), 341-347.

[11] J. Qiang, Projective synchronization of a new hyperchaotic Lorenz system chaotic systems, Phys. Lett. A 370 (2007), 40-45.

[12] X. Tan, J. Zhang and Y. Yang, Synchronizing chaotic systems using backstepping design, Chaos Solitons Fractals 16 (2003), 37-45.

[13] X. Li, Generalized projective synchronization using nonlinear control method, Int. J. Nonlinear Sci. 8 (2009), 79-85.

[14] Y. W. Wang and Z. H. Guan, Generalized synchronization of continuous chaotic systems, Chaos Solitons Fractals 27 (2006), 97-101.

[15] X. Zhang and H. Zhu, Anti-synchronization of two different hyperchaotic systems via active and adaptive control, Int. J. Nonlinear Sci. 6 (2008), 216-223.

[16] L. Kocarev and U. Parlitz, General approach for chaotic synchronization with applications to communication, Phys. Rev. Lett. 74 (1995), 5028-5030.

[17] Y. Tao, Chaotic secure communication systems - history and new results, Telecomm. Review 9 (1999), 597-634.

[18] Yan Zhenya, Q-S synchronization in 3D Hénon-like map and generalized Hénon map via a scalar controller, Phys. Lett. A 342(4) (2005), 309-317.

[19] M. Itoh, T. Yang and L. O. Chua, Internat. J. Bifur. Chaos Appl. Sci. Eng. 11 (2001), 551.

[20] D. L. Hitzl and F. Zele, Physica D 14 (1985), 305.

[21] J. Zhao and J. Lu, Parameter identification and backstepping control of uncertain Lu system, Chaos Solitons Fractals 17 (2003), 721-729.