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THE IMPACT OF INVESTOR'S VIEW ON HEDGING EFFECTIVENESS

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Abstract

Investors vary in how they locate a portfolio with a futures position, but the optimal hedging ratio typically relies on the equilibrium parameters. In contrast to optimal risk-return hedging, we employ two popular approaches - the Bayesian framework and a conditional regression - to examine the impact of investor's views on hedging effectiveness. In terms of hedging effectiveness, this study suggests a feasible measure to determine whether the Bayesian learning compares favorably with the conditional regression updating.

1. Introduction

Risk-return hedging is one of the most important hedging strategies in the futures markets. The determination of the optimal hedging ratio and its hedging effectiveness measure are of both academic and practical interest. Starting with Howard and D'Antonio [5], a number of studies have fitted various risk-return relative measures to the hedging ratio and hedging effectiveness. Kuo and Chen [7] suggested a simplification to benefit

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empirical studies. Lien [8] indicated that the risk-return approach produces a biased result. Chiu [3] presented a simple test for risk-return hedging effectiveness. The ostensible purpose of each of these portfolio selection processes is to generate an effective hedging strategy for investors. However, these risk-return analyses address optimal hedging policies, which are theoretically based on the equilibrium expected return vector and covariance matrix of a portfolio. Of course, the risk-return hedging also inherits the sensitivities of mean-variance analysis due to the changes in the assets means or covariance (see, e.g., Best and Grauer [1]). While using historical data to replace the equilibrium, previous studies nonetheless do not investigate the impact of the investor's view on hedging effectiveness via a Bayesian learning process.

To reduce the operational risk of a portfolio, an active investor is likely to impose his subjective view on the estimation of risk parameters. This process is intuitive for several reasons. First, investors may disagree over just what expected return vector and covariance matrix represent the equilibrium. As previously noted by Black and Litterman [2], combining historical data with "expert knowledge" (which is generally performed in practice) can lead to more reasonable results than relying on pure portfolio optimization. Second, a traditional way to estimate the equilibrium parameters is to substitute the sample counterpart (generally a long horizon time series) into the model parameters. However, investors may be interested in recent volatility rather than the long run equilibrium parameters. Lastly, the primary reason for using the sample counterpart is that we cannot reasonably expect to access the true equilibrium parameters. It is important to emphasize that, whenever we have information on these parameters, we will use it rather than the sample counterpart.

To incorporate the investor's view into the equilibrium parameters, a systematic adjusting mechanism of the parameters should explain the highly correlated relationship between spot and futures. This study will employ the appealing Black-Litterman [2] Bayesian model, which is one of the most important updating process in the finance literatures, to produce a forecast view. For comparison, Qian and Gorman's [9] simplification of Black-Litterman framework is also used to obtain a conditional distribution of the mean vector and covariance matrix under normal assumptions. Our objective is to compare the hedging performance of the Black-Litterman model and the Qian-Gorman conditional distribution when the investor imposes his views on the futures' return and its volatility. We also derive the conditions for the order of hedging effectiveness in these three approaches.

The article proceeds as follows: Section 2 reformulates the Howard-D'Antonio relative measure in a simple form. Section 3 examines the use of this measure to resolve the hedging performance in both the Black-Litterman framework and the Qian-Gorman conditional distribution. A measure of the marginal loss of the learning process and an example are given to illustrate our derivations. Section 4 presents the conclusions.

2. Hedging Effectiveness Measure

It is widely known in futures hedging that Ederington's minimum-variance approach [4] is based only on the viewpoint of risk reduction. Howard and D'Antonio [5] extended Ederington's minimum-variance hedging policy to the risk-return framework, in which the investor determines the hedging ratio by maximizing the Sharpe ratio [10]. Consider an active investor who is endowed with a spot hedged by futures positions. Then, the Howard-D'Antonio approach is a straightforward result of solving a hedged portfolio allocation "$x = (x_s, x_f)$" for the risk-return optimization:

$$\max_{\ell' x = 1} \left(\frac{x'\mu}{\sqrt{x'\Sigma x}} \right), \tag{1}$$

where the excess return of a hedged portfolio is $x'r = x_s r_s + x_f r_f$ with the expectation $x'\mu = x_s\mu_s + x_f\mu_f$ and the variance $x'\Sigma x = x_s^2\sigma_s^2 + 2x_s x_f \sigma_{sf} + x_f^2\sigma_f^2$. Also, the analytical solution employs the following notation:

r = The 2×1 random vector of the returns of spot and futures.

μ = The 2×1 equilibrium returns vector, $(\mu_s, \mu_f)'$.

$\overline{\mu}$ = The 2×1 returns vector of a prior distribution, $(\overline{\mu}_s, \overline{\mu}_f)'$.

Σ = The 2×2 equilibrium covariance matrix of r.

$\sigma_s(\sigma_f)$ = The volatility (standard deviation) of spot (futures).

ρ = The correlation coefficient between spot and futures.

x = The 2×1 portfolio weight vector of spot and futures positions.

ℓ = The 2×1 vector of ones such that the total wealth is invested.

v = The 2×1 returns vector due to investor's view, $(v_s, v_f)'$.

Ω = The 2×2 residuals' covariance matrix due to investor's view.

$\omega_s(\omega_f)$ = The residuals' standard deviation of spot (futures).

ω_{sf} = The residuals' covariance between spot and futures.

P = The 2×2 transformation matrix of investor's view.

With respect to the mean vector and the covariance matrix, (μ, Σ), it is well known that the solution of model (1) is described as the maximum Sharpe ratio allocation, denoted by x_{TP},

$$x_{TP} = \frac{\Sigma^{-1}\mu}{\ell'\Sigma^{-1}\mu} \tag{2}$$

and the maximum Sharpe ratio relative to x_{TP} is computed as

$$\theta_{TP} = \frac{\mu_{TP}}{\sigma_{TP}} = \sqrt{\mu'\Sigma^{-1}\mu}. \tag{3}$$

To evaluate the hedging potential of a portfolio, we reformulate the Howard-D'Antonio hedging effectiveness relative measure as

$$H_{TP} = \frac{\theta_{TP}}{\theta_S} = \frac{\sqrt{\mu' \Sigma^{-1} \mu}}{\theta_S} = \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda^2}, \tag{4}$$

where θ_S denotes the Sharpe ratio of the spot position and $\lambda = \left(\frac{\mu_f}{\sigma_f} \Big/ \frac{\mu_s}{\sigma_s} \right)$ is the relative risk-return of the futures against the risk-return ratio of the spot position. There is an important reason for employing Howard-D'Antonio relative measure as the comparison criterion. This relative measure resolves itself exclusively into two components: the correlation between spot and futures, and the risk-return relative of futures against the risk-return ratio of the spot position, which greatly simplifies the comparison among several approaches.

3. Main Results

Because the model parameters (μ, Σ) are not directly observed, in practice, investors must instead use the equilibrium parameters based on historical measurements of asset return and volatility (see, e.g., Jobson and Korkie [6]). To reduce the operational risk of a portfolio, an active investor is likely to impose his subjective viewpoints on the estimation of the risk parameters, such as the expected return, volatility, and risk-return ratio, rather than the equilibrium parameters. Therefore, incorporating the investor's view, market equilibrium, and sample information into estimates is desirable in practice. In this section, we solve a hedged strategy affected by the investor's view. Based on the Howard-D'Antonio criterion, we employ two popular approaches to adjusting risk factors (including the futures' return and the futures' volatility). The corresponding formulas for the maximum hedging effectiveness are derived in each model. These approaches are described below:

1. Black and Litterman [2] pioneered a Bayesian model to obtain a weighted mean vector of the equilibrium return vector and the investor's view, in which they set a prior distribution to the unknown distribution of the expected return vector. We may construe the Black-Litterman framework as a non-standard Bayesian model, as the weighted mean vector does not contain the sample information.

2. Qian and Gorman [9] introduced a practical simplification of the Black-Litterman model. The Qian-Gorman method obtains not only an updating mean vector but also a conditional covariance based on the investor's view regarding certain assets. This method is actually a regression-based framework to adjust both the mean and the covariance matrix.

3.1. Qian-Gorman conditional distribution (method A)

For simplicity, we use Qian and Gorman's conditional distribution as the reference model to evaluate the alternative model. Under a normal distribution, Qian and Gorman directly assign a conditional distribution to the asset returns even though (r_s, r_f) deviates from its equilibrium distribution. Given a hedged portfolio with a spot asset and futures position, the conditional distribution of r_s given r_f can be formulated as

$$r_s \mid r_f \sim N\left(\mu + \frac{\rho\sigma_s}{\sigma_f}(r_f - \mu_f),\ \sigma_s^2(1-\rho^2)\right). \tag{5}$$

Let the investor forecast the expected return and risk of the futures position (μ_f, σ_f) as $(\tilde{\mu}_f, \tilde{\sigma}_f)$. The concept behind deriving the adjusted moments is to take the conditional expectation, variance, and covariance of (5). The adjusted returns vector and covariance matrix follow from the Qian-Gorman two-asset model as

$$\mu_A = \begin{bmatrix} \tilde{\mu}_s \\ \tilde{\mu}_f \end{bmatrix} = \begin{bmatrix} \mu + \frac{\rho\sigma_s}{\sigma_f}(\tilde{\mu}_f - \mu_f) \\ \tilde{\mu}_f \end{bmatrix}, \tag{6}$$

and

$$\Sigma_A = \begin{bmatrix} \tilde{\sigma}_s^2 & \tilde{\sigma}_{sf} \\ \tilde{\sigma}_{sf} & \tilde{\sigma}_f^2 \end{bmatrix} = \begin{bmatrix} \frac{\rho^2\sigma_s^2}{\sigma_f^2}\tilde{\sigma}_f^2 + \sigma_s^2(1-\rho^2) & \frac{\rho\sigma_s}{\sigma_f}\tilde{\sigma}_f^2 \\ \frac{\rho\sigma_s}{\sigma_f}\tilde{\sigma}_f^2 & \tilde{\sigma}_f^2 \end{bmatrix}. \tag{7}$$

According to equations (4), (6), and (7), we now proceed to derive Qian and Gorman's risk-return hedging effectiveness and its properties as follows:

Theorem 1. *The hedging effectiveness relative to the Qian-Gorman procedures is given by*

$$H_A = \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda_A^2}, \tag{8}$$

where $\lambda_A = \left(\frac{\tilde{\mu}_f}{\tilde{\sigma}_f} \Big/ \frac{\mu_s}{\sigma_s}\right)$ *is Qian and Gorman's risk-return relationship of the futures against the risk-return ratio of the spot position.*

Proof. With respect to the adjusted mean vector and the covariance matrix (μ_A, Σ_A), the square of the maximum Sharpe ratio follows equations (6) and (7):

$$\mu_A'\Sigma_A^{-1}\mu_A = \frac{\tilde{\sigma}_f^2\left[\mu - \frac{\rho\sigma_s}{\sigma_f}\mu_f\right]^2 + \tilde{\mu}_f^2\sigma_s^2(1-\rho^2)}{\sigma_s^2\tilde{\sigma}_f^2(1-\rho^2)}.$$

The adjusted Qian-Gorman hedging effectiveness can therefore be reduced to

$$H_A = \frac{\sqrt{\mu_A'\Sigma_A^{-1}\mu_A}}{\frac{\mu_s}{\sigma_s}} = \sqrt{\frac{\left(1-\rho\frac{\frac{\mu_f}{\sigma_f}}{\frac{\mu_s}{\sigma_s}}\right)^2}{(1-\rho^2)} + \left(\frac{\frac{\tilde{\mu}_f}{\tilde{\sigma}_f}}{\frac{\mu_s}{\sigma_s}}\right)^2} = \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda_A^2}.$$

Note that the expression for H_A has the same form as equation (4), except that the equilibrium hedging effectiveness relationship λ^2 is replaced by Qian and Gorman's adjusted hedging effectiveness relationship λ_A^2. $\square$

3.2. Black-Litterman learning (method B)

We use a similar set of notations to the Black-Litterman model [2]. Under a bivariate normal distribution, we assign the mean vector μ a prior distribution expressed as follows:

$$\mu = \begin{bmatrix} \mu_s \\ \mu_f \end{bmatrix} = \begin{bmatrix} \bar{\mu}_s \\ \bar{\mu}_f \end{bmatrix} + \begin{bmatrix} \varepsilon_s \\ \varepsilon_f \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \varepsilon_s \\ \varepsilon_f \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} d\sigma_s^2 & d\sigma_{sf} \\ d\sigma_{sf} & d\sigma_f^2 \end{bmatrix} \right),$$

where d is the constant that reveals investor's confidence that the mean vector μ is close to the prior equilibrium value $\bar{\mu}$; and the residual vector ε measures the deviation of the random vector μ from $\bar{\mu}$ with a zero mean and a covariance matrix $d\Sigma$. Black and Litterman show that the investor's views on the relative performance of the spot and futures can be using in a matrix operator given by

$$P\mu = v + \varepsilon_v \quad \text{and} \quad \varepsilon_v \sim N(0, \Omega),$$

where the residual vector ε_v measures the deviation of the investor's views vector from v with a zero mean and the residual covariance matrix Ω. In our discussion, the investor expresses his view only on the futures' return, and the spot return is then updated via a Bayesian scheme. We therefore may design the transformation matrix and the residual covariance matrix as follows:

$$P = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \Omega = \begin{bmatrix} \omega_s^2 & \omega_{sf} \\ \omega_{sf} & \omega_f^2 \end{bmatrix}.$$

Black and Litterman derive the Bayesian updated returns vector and covariance matrix of the model as follows:

$$\begin{aligned} \mu_B &= [(d\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(d\Sigma)^{-1}\mu + P'\Omega^{-1}v], \\ \Sigma_B &= \Sigma + [(d\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}. \end{aligned} \tag{9}$$

According to equation (9), we compute the updates to the expected return and covariance matrix as

$$\mu_B = \begin{bmatrix} \hat{\mu}_s \\ \hat{\mu}_f \end{bmatrix} = \begin{bmatrix} \mu_s + \dfrac{\gamma_1}{\gamma_2}\dfrac{\sigma_{sf}}{\omega_f^2}(v_f - \mu_f) \\ \mu_f + \dfrac{\gamma_1}{\gamma_2}\dfrac{\sigma_f^2}{\omega_f^2}(v_f - \mu_f) \end{bmatrix} \tag{10}$$

and

$$\Sigma_B = \begin{bmatrix} \hat{\sigma}_s^2 & \hat{\sigma}_{sf} \\ \hat{\sigma}_{sf} & \hat{\sigma}_f^2 \end{bmatrix} = \begin{bmatrix} \sigma_s^2\left(1+\dfrac{\gamma_1}{\gamma_2}\right)+\dfrac{\gamma_1^2}{\gamma_2}\gamma_3 & \sigma_{sf}\left(1+\dfrac{\gamma_1}{\gamma_2}\right) \\ \sigma_{sf}\left(1+\dfrac{\gamma_1}{\gamma_2}\right) & \sigma_f^2\left(1+\dfrac{\gamma_1}{\gamma_2}\right) \end{bmatrix}, \tag{11}$$

where

$$\gamma_1 = d(\sigma_s^2\sigma_f^2 - \sigma_{sf}^2), \quad \gamma_3 = \frac{\omega_s^2}{\omega_s^2\omega_f^2 - \omega_{sf}^2}, \quad \gamma_2 = \sigma_s^2\sigma_f^2 - \sigma_{sf}^2 + \gamma_1\gamma_3\sigma_f^2.$$

Similar to Qian and Gorman's approach, we now derive the Black-Litterman risk-return hedging effectiveness and its properties as follows:

Theorem 2. *If both the Black-Litterman updating and Qian-Gorman updating have the same views on the futures' return and volatility, then the hedging effectiveness of the Black-Litterman model is given by*

$$H_B = \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda_B^2} = \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda_A^2 k} \tag{12}$$

and

$$k = \frac{\tilde{\sigma}_f^2}{\sigma_f^2} + \left(\frac{\tilde{\sigma}_f^2}{\sigma_f^2} - 1\right) d\sigma_f^2 \left(\frac{\omega_s^2}{\omega_s^2\omega_f^2 - \omega_{sf}^2}\right), \tag{13}$$

where k is an adjusted scalar compared to the Qian-Gorman procedures.

Proof. For comparison on the same basis, without loss of generality, assume that $\tilde{\mu}_f = \hat{\mu}_f$ and $\tilde{\sigma}_f^2 = \hat{\sigma}_f^2$ such that

$$\tilde{\mu}_f = \mu_f + \frac{\gamma_1}{\gamma_2}\frac{\sigma_f^2}{\omega_f^2}(v_f - \mu_f) \quad \text{and} \quad \tilde{\sigma}_f^2 = \sigma_f^2\left(1 + \frac{\gamma_1}{\gamma_2}\right).$$

The updated mean (10) and covariance matrix (11) are thus transformed into

$$\Sigma_B = \begin{bmatrix} \frac{\rho^2(\sigma_s\sqrt{k})^2}{(\sigma_f\sqrt{k})^2}\tilde{\sigma}_f^2 + (1-\rho^2)(\sigma_s\sqrt{k})^2 & \frac{\rho(\sigma_s\sqrt{k})}{(\sigma_f\sqrt{k})}\tilde{\sigma}_f^2 \\ \frac{\rho(\sigma_s\sqrt{k})}{(\sigma_f\sqrt{k})}\tilde{\sigma}_f^2 & \tilde{\sigma}_f^2 \end{bmatrix},$$

$$\mu_B = \begin{bmatrix} \mu + \frac{\rho(\sigma_s\sqrt{k})}{(\sigma_f\sqrt{k})}(\tilde{\mu}_f - \mu_f) \\ \tilde{\mu}_f \end{bmatrix},$$

$$k = \frac{\tilde{\sigma}_f^2}{\sigma_f^2} + \left(\frac{\tilde{\sigma}_f^2}{\sigma_f^2} - 1\right) d\sigma_f^2 \left(\frac{\omega_s^2}{\omega_s^2\omega_f^2 - \omega_{sf}^2}\right). \tag{14}$$

Given the algebraic relationship between (6), (7), and (14), Black and Litterman's hedging effectiveness is therefore reduced to the following expression:

$$H_B = \sqrt{\frac{\left(1 - \rho\frac{\frac{\mu_f}{\sigma_f}}{\frac{\mu_s}{\sigma_s}}\right)^2}{(1-\rho^2)} + \left(\frac{\frac{\tilde{\mu}_f}{\tilde{\sigma}_f}\sqrt{k}}{\frac{\mu_s}{\sigma_s}}\right)^2} = \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda_B^2}.$$

Note that the constant k is the adjusted constant compared to the Qian-Gorman procedures:

$$\lambda_B^2 = \left(\frac{\tilde{\mu}_f}{\tilde{\sigma}_f}\sqrt{k} \middle/ \frac{\mu_s}{\sigma_s}\right)^2 = \lambda_A^2 k.$$

This completes the proof. $\square$

3.3. The relationship between H_A, H_B, and H_{TP}

Equipped with Theorem 1 and Theorem 2, here we compare the relationship of the hedging effectiveness between H_{TP}, H_A, and H_B. Our goal is to reach a relative measure for assessing the hedging effectiveness, but the relationship in the below theorem still serves a useful purpose: to ascertain whether a subjective view results in overestimating or underestimating the true equilibrium hedging effectiveness.

Theorem 3. *If both the Black-Litterman learning and Qian-Gorman updating have the same views on the futures' return and volatility, the relationship of the hedging effectiveness between the Black-Litterman tangency portfolio, the Qian-Gorman tangency portfolio, and the true tangency portfolio is summarized as follows*:

1. *If* $\sigma_f < \tilde{\sigma}_f$ *and* $\frac{\mu_f}{\sigma_f} < \frac{\tilde{\mu}_f}{\tilde{\sigma}_f}$, *then* $H_{TP} < H_A < H_B$.

2. *If* $\sigma_f > \tilde{\sigma}_f$ *and* $\frac{\mu_f}{\sigma_f} > \frac{\tilde{\mu}_f}{\tilde{\sigma}_f}$, *then* $H_{TP} > H_A > H_B$.

3. *If* $\sigma_f > \tilde{\sigma}_f$ *and* $\frac{\mu_f}{\sigma_f} < \frac{\tilde{\mu}_f}{\tilde{\sigma}_f}$, *then* $H_A > H_{TP} > H_B$.

4. *If* $\sigma_f < \tilde{\sigma}_f$ *and* $\frac{\mu_f}{\sigma_f} > \frac{\tilde{\mu}_f}{\tilde{\sigma}_f}$, *then* $H_B > H_{TP} > H_A$.

5. *If* $\sigma_f = \tilde{\sigma}_f$, *then* $H_{TP} = H_A = H_B$.

Proof. Excluding the trivial case where the investor's view coincides with the true equilibrium $H_{TP} = H_A = H_B$, there are five possibilities.

First, it is straightforward to compare the difference between the hedging effectiveness of the Black-Litterman tangency portfolio and of the Qian-Gorman tangency portfolio:

$$H_B - H_A = \frac{\tilde{\lambda}^2\left[\left(\frac{\tilde{\sigma}_f^2}{\sigma_f^2} - 1\right)\left(\frac{d\sigma_f^2}{\omega_f^2} + 1\right)\right]}{\sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \tilde{\lambda}^2 k} + \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \tilde{\lambda}^2}}.$$

In the case that the investor's view has a larger futures volatility and Sharpe ratio than the true futures volatility and Sharpe ratio, that is, $\sigma_f < \tilde{\sigma}_f$ and $\frac{\mu_f}{\sigma_f} < \frac{\tilde{\mu}_f}{\tilde{\sigma}_f}$, the Black-Litterman hedging will overestimate the hedging effectiveness compared to the Qian-Gorman hedging because

$$\sigma_f < \tilde{\sigma}_f \Rightarrow H_B - H_A > 0 \Rightarrow H_B > H_A. \tag{15}$$

In this case, we also compare the difference between the hedging effectiveness of the Qian-Gorman tangency portfolio and the true tangency portfolio

$$H_{TP} - H_A = \frac{\lambda^2 - \lambda_A^2}{\sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda^2} + \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda_A^2}}.$$

This comparison also implies that the Qian-Gorman hedging overestimates the true hedging effectiveness because

$$\frac{\mu_f}{\sigma_f} < \frac{\tilde{\mu}_f}{\tilde{\sigma}_f} \Rightarrow \lambda_A^2 > \lambda^2 \Rightarrow H_A > H_{TP}. \tag{16}$$

Combining equations (15) and (16), we obtain the following relationship:

$$H_{TP} < H_A < H_B.$$

A series of similar arguments verify the remaining cases. □

Because both H_A and H_B may overestimate (or underestimate) H_{TP}, or one of them may overestimate H_{TP} while the other underestimates H_{TP}, it is difficult to compare the Qian-Gorman updating with Black-Litterman learning. Basically, Theorem 3 should be viewed as a necessary but not sufficient procedure for controlling risk. It must be supplemented by limits and controls in addition to the risk-return hedging effectiveness.

Note that an important property of H_B (or H_A) is that the larger the investor's view on the futures volatility, the higher the $|H_B|$ (or $|H_A|$) will be. It would then appear that if one wishes to achieve a substantial hedging effectiveness in a risk-return hedging, one must merely employ a higher futures volatility. However, we do not take this "advice" too seriously, because these updating results of H_B (or H_A) do not take into account the marginal learning effect. Thus, what we need is an intuitive measure of the model misspecification that is defined as the deviation from the equilibrium in the learning process. Such a measure may be considered as:

$$\text{The absolute marginal loss of the hedging effectiveness} = \left| \frac{\Delta H}{\Delta \lambda} \right|, \tag{17}$$

where $\Delta H_A = H_{TP} - H_A$ (or $\Delta H_B = H_{TP} - H_B$) is the amount of the loss of hedging effectiveness due to the Black-Litterman learning or Qian-Gorman updating; and $\Delta\lambda_A = \lambda - \lambda_A$ (or $\Delta\lambda_B = \lambda - \lambda_B$) is the difference between the investor's view and the equilibrium state. Overall, investors prefer the more conservative $\left| \frac{\Delta H}{\Delta \lambda} \right|$ when applying updating procedures. That is, we suggest that the investor should employ the Qian-Gorman updating if

$$\left| \frac{\Delta H_A}{\Delta \lambda_A} \right| < \left| \frac{\Delta H_B}{\Delta \lambda_B} \right| \tag{18}$$

because it is relatively close to the equilibrium state. Otherwise, this study recommends the Black-Litterman learning.

3.4. Illustration

Based on the above discussions, we assume that the true mean vector and covariance matrix, which may be obtained according to the sample estimation using a long horizon of returns series, are given as follows:

$$\mu = \begin{bmatrix} 0.060 \\ 0.065 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 0.040 & 0.054 \\ 0.054 & 0.090 \end{bmatrix}.$$

We can calculate the hedging effectiveness for the true tangency portfolio as

$$\lambda = 0.72222 \quad \text{and} \quad H_{TP} = \sqrt{\frac{(1-\rho\lambda)^2}{(1-\rho^2)} + \lambda^2} = 1.07997.$$

First, assume that the investor predicts a higher futures return associated with a higher volatility in the coming transactions. Let the futures return and its volatility be forecasted as $(\tilde{\mu}_f, \tilde{\sigma}_f) = (0.08, 0.10)$. The hedging effectiveness and he marginal loss of the hedging effectiveness of the Qian-Gorman updating are computed as

$$\lambda_A = 0.84327, \; H_A = 1.16441,$$

$$\left| \frac{\Delta H_A}{\Delta \lambda_A} \right| = \frac{1.16441 - 1.07997}{0.84327 - 0.72222} = 1.43366.$$

We next assume that the investor chooses his belief degree $d = 0.5$ and the residual's covariance matrix as follows:

$$\Omega = \begin{bmatrix} \omega_s^2 & \omega_{sf} \\ \omega_{sf} & \omega_f^2 \end{bmatrix} = \begin{bmatrix} 0.01000 & 0.00866 \\ 0.00866 & 0.03000 \end{bmatrix}.$$

The hedging effectiveness and the marginal loss of the hedging effectiveness of the Black-Litterman learning are computed as

$$\lambda_B = 1.04550, \; H_B = 1.31826,$$

$$\left| \frac{\Delta H_B}{\Delta \lambda_B} \right| = \frac{1.31826 - 1.07997}{1.04550 - 0.72222} = 0.73709.$$

For the investor, the Qian-Gorman updating does not deviate far from the true equilibrium hedging effectiveness. However, the absolute marginal loss of the hedging effectiveness indicates that

$$\left|\frac{\Delta H_A}{\Delta\lambda_A}\right| > \left|\frac{\Delta H_B}{\Delta\lambda_B}\right|.$$

In our illustration, this result may suggest that the Black-Litterman learning compares favorably with the Qian-Gorman updating.

4. Conclusion

This article is to compare the hedging performance of the Black-Litterman model and the Qian-Gorman conditional distribution when the investor imposes his views on the futures' return and its volatility. We apply the Howard-D'Antonio relative measure to evaluate the hedging performance of these approaches. Finally, we provide a simple and feasible decision rule in the choice between two hedging models. This measure may also be used with other learning procedures.

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