Far East Journal of Mathematical Sciences (FJMS)

Published Online: January 2014
Available online at http://pphmj.com/journals/fjms.htm
Volume 82, Number 2, 2013, Pages 157-182

NEW NOTIONS VIA $^*gp$-CLOSED SETS

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Abstract

In this paper, we introduce and study the notion of a new class of functions, namely, star generalized precontinuous, star generalized preclosed and strongly star generalized preclosed functions.

1. Introduction

Kumar [28] introduced and investigated the notion of $^*g$-continuous functions in topological spaces. Also, Kumar [26, 27], Rose [22] and

Received: July 30, 2013; Revised: November 17, 2013; Accepted: November 29, 2013
2010 Mathematics Subject Classification: 54A05, 54C10, 54D10, 54C55.
Keywords and phrases: $\hat{g}$-continuous, $^*g$-continuous, $\alpha\hat{g}$-continuous, star generalized precontinuous, star generalized preclosed and strongly star generalized preclosed functions.

Mashhour et al. [14] introduced the notion of $g^{*}p$-continuous functions, $\hat{g}$-continuous functions, $\alpha\hat{g}$-continuous functions and precontinuous functions in topological spaces. Levine [10] introduced the notion of strongly continuous functions and studied some of their properties. The notion of perfectly continuous functions in topological spaces was introduced in [19]. Reilly and Vamanamurthy [20] and Kumar [25] introduced the notion of preirresolute and $\hat{g}$-irresolute functions. El-Maghrabi and Al-Ahmadi [7] had introduced star generalized preclosed sets and studied its properties using $\hat{g}$-open and obtained some of its properties and results.

In the present paper, we introduce and study the notion of a new class of functions, namely, star generalized precontinuous, star generalized preclosed, strongly star generalized precontinuous and strongly star generalized preclosed functions and studied some of their basic properties.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset *A* of a space (X, τ), $cl(A)$, $int(A)$ and A^{c} or $X \setminus A$ denoted the closure of *A*, the interior of *A* and the complement of *A* in *X*, respectively.

Let us recall the following definitions which are useful in the sequel.

Definition 2.1. A subset *A* of a space (X, τ) is called:

(i) a *semi-open set* [11] if $A \subseteq cl(int(A))$ and a *semi-closed set* if $int(cl(A)) \subseteq A$,

(ii) a *preopen set* [14] if $A \subseteq int(cl(A))$ and a *preclosed set* if $cl(int(A)) \subseteq A$,

(iii) an α-*open set* [18] if $A \subseteq int(cl(int(A)))$ and an α-*closed set* if $cl(int(cl(A))) \subseteq A$,

(iv) a *regular open set* [24] if $A = int(cl(A))$ and a *regular closed set* if $cl(int(A)) = A$.

The semi closure (resp. preclosure, α-closure) of a subset A of a space (X, τ) is the intersection of all semi-closed (resp. preclosed, α-closed) sets containing A and is denoted by $scl(A)$ (resp. $pcl(A)$, $\alpha cl(A)$).

Definition 2.2. A subset A of a topological space (X, τ) is called:

(i) a *generalized closed* (briefly, *g-closed*) *set* [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ),

(ii) a $\hat{g}$*-closed set* [25] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ),

(iii) a *generalized semi-preclosed* (briefly, *gsp-closed*) *set* [6] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ),

(iv) a g^{*}*-closed set* [24] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ),

(v) a g^{*}*-preclosed* (briefly, $g^{*}p$*-closed*) [25] (or *strongly generalized preclosed* (briefly, *strongly gp-closed*)) [4] set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ),

(vi) an $\alpha\hat{g}$*-closed set* [22] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ),

(vii) a ${}^{*}g$*-closed set* [28] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ),

(viii) a ${}^{*}gp$*-closed set* [7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).

The complement of a g-closed (resp. $\hat{g}$-closed, gsp-closed, g^{*}-closed,

$g^* p$-closed, $\alpha\hat{g}$-closed, ${}^*g$-closed, ${}^*gp$-closed) set is called *g-open* (resp. $\hat{g}$*-open*, *gsp-open*, g^**-open*, $g^* p$*-open*, $\alpha\hat{g}$*-open*, ${}^*g$*-open*, ${}^*gp$*-open*).

The intersection (resp. the union) of all ${}^*gp$-closed (resp. ${}^*gp$-open) [9] sets containing (resp. contained in) A is called the ${}^*gp$*-closure* (resp. the ${}^*gp$*-interior*) of A and will be denoted by ${}^*gp\text{-}cl(A)$ (resp. ${}^*gp\text{-}int(A)$).

Proposition 2.1 [7]**.** *For a space* (X, τ), *we have*:

(i) *every closed* (*resp.* ${}^*g$*-closed*, *preclosed*, α*-closed*) *set is* ${}^*gp$*-closed*,

(ii) *every* g^**-closed* (*resp.* $g^* p$*-closed*, $\alpha\hat{g}$*-closed*) *set is* ${}^*gp$*-closed*,

(iii) *every* ${}^*gp$*-closed set is gsp-closed*.

Definition 2.3. A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) *precontinuous* [14] if the inverse image of each closed set of (Y, σ) is preclosed in (X, τ),

(ii) α*-continuous* [13] if the inverse image of each closed set of (Y, σ) is α-closed in (X, τ),

(iii) *g-continuous* [2] if the inverse image of each closed set of (Y, σ) is *g*-closed in (X, τ),

(iv) $\hat{g}$*-continuous* [27] if the inverse image of each closed set of (Y, σ) is $\hat{g}$-closed in (X, τ),

(v) *gsp-continuous* [6] if the inverse image of each closed set of (Y, σ) is *gsp*-closed in (X, τ),

(vi) g^**-continuous* [24] if the inverse image of each closed set of (Y, σ) is g^*-closed in (X, τ),

(vii) $g^{*}p$*-continuous* [26] (or *strongly generalized precontinuous* (briefly, *strongly gp-continuous*)) [4] if the inverse image of each closed set of (Y, σ) is $g^{*}p$-closed (or strongly *gp*-closed) in (X, τ),

(viii) ${}^{*}g$*-continuous* [28] if the inverse image of each closed set of (Y, σ) is ${}^{*}g$-closed in (X, τ),

(ix) $\alpha\hat{g}$*-continuous* [22] if the inverse image of each closed set of (Y, σ) is $\alpha\hat{g}$-closed in (X, τ),

(x) *strongly continuous* [10] if the inverse image of each subset of (Y, σ) is clopen in (X, τ),

(xi) *perfectly continuous* [19] if the inverse image of each open set of (Y, σ) is clopen in (X, τ).

Definition 2.4. A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) *preirresolute* [20] if the inverse image of each preopen set of (Y, σ) is preopen in (X, τ),

(ii) $\hat{g}$*-irresolute* [25] if the inverse image of each $\hat{g}$-closed set of (Y, σ) is $\hat{g}$-closed in (X, τ),

(iii) $g^{*}p$*-irresolute* [26] (or *strongly generalized preirresolute* (briefly, *strongly gp-irresolute*)) [4] if the inverse image of each $g^{*}p$-closed (or strongly *gp*-closed) set of (Y, σ) is $g^{*}p$-closed (or strongly *gp*-closed) in (X, τ).

Definition 2.5. A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) *preclosed* [14] *map* if the image of each closed set of (X, τ) is preclosed set in (Y, σ),

(ii) α-*closed* [16] *map* if the image of each closed set of (X, τ) is α-closed set in (Y, σ),

(iii) *gsp-closed* [5] *map* if the image of each closed set of (X, τ) is *gsp*-closed set in (Y, σ),

(iv) g^*-*closed map* if the image of each closed set of (X, τ) is g^*-closed set in (Y, σ),

(v) g^*p-*closed* (or *strongly generalized preclosed* (briefly, *strongly gp-closed*)) [4] *map* if the image of each closed set of (X, τ) is g^*p-closed (or strongly *gp*-closed) set in (Y, σ),

(vi) ${}^*g$-*closed map* if the image of each closed set of (X, τ) is ${}^*g$-closed set in (Y, σ),

(vii) $\alpha\hat{g}$-*closed* [22] *map* if the image of each closed set of (X, τ) is $\alpha\hat{g}$-closed set in (Y, σ).

Definition 2.6 [15]**.** A function $f : (X, \tau) \to (Y, \sigma)$ is called *M-preclosed* if $f(U)$ is a preclosed set in (Y, σ), for each preclosed set U in (X, τ).

Theorem 2.1 [7]**.** *If A is an open and B is a* ${}^*gp$*-open subset of a space* (X, τ), *then* $A \cap B$ *is* ${}^*gp$*-open*.

Definition 2.7 [7]**.** A subset N_x of a space (X, τ) is called a ${}^*gp$-*neighbourhood* (briefly, ${}^*gp$-*nbd*) of point $x \in X$ if there exists a ${}^*gp$-open set G such that $x \in G \subseteq N$.

Proposition 2.2 [7]**.** *For a space* (X, τ) *and* $A \subseteq X$, *the following statements hold*:

(i) *if* $A \subseteq F$, F *is a* ${}^*gp$*-closed set, then* $A \subseteq {}^*gp\text{-}cl(A) \subseteq F$,

(ii) *if* $G \subseteq A$, G *is a* ${}^*gp$*-open set, then* $G \subseteq {}^*gp\text{-}int(A) \subseteq A$.

Proposition 2.3 [7]. *For a topological space* (X, τ) *and* $A \subseteq X$, *the following statements hold*:

(i) *if* A *is a* ${}^{*}gp$*-closed* (*resp.* ${}^{*}gp$*-open*) *set, then* $A = {}^{*}gp\text{-}cl(A)$ (*resp.* $A = {}^{*}gp\text{-}int(A)$),

(ii) $int(A) \subseteq {}^{*}gp\text{-}int(A)$ *and* ${}^{*}gp\text{-}cl(A) \subseteq cl(A)$,

(iii) ${}^{*}gp\text{-}cl(X - A) = X - {}^{*}gp\text{-}int(A)$,

(iv) ${}^{*}gp\text{-}int(X - A) = X - {}^{*}gp\text{-}cl(A)$.

Definition 2.8 [7]. A topological space (X, τ) is called:

(i) ${}_{*gp}T$*-space* if every ${}^{*}gp$-closed set is closed,

(ii) ${}_{*gp}T_{1/2}$*-space* if every ${}^{*}gp$-closed set is preclosed.

Proposition 2.4 [3, 17]. *For subset A of X and subset B of Y, we have* $pcl(A \times B) = pcl(A) \times pcl(B)$ *and* $pint(A \times B) = pint(A) \times pint(B)$.

3. ${}^{*}gp$-continuous Functions

In this section, we give and study the concept of a ${}^{*}gp$-continuous function. Also, some of their properties and relations between them and other types of functions are presented. Further, we introduce some applications such as: ${}_{*gp}T$-space and ${}_{*gp}T_{1/2}$-spaces.

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called ${}^{*}gp$*-continuous* if the inverse image of each closed set of (Y, σ) is ${}^{*}gp$-closed in (X, τ).

Theorem 3.1. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a mapping, then*:

(i) *every continuous* (*resp.* ${}^{*}g$*-continuous, precontinuous,* α*-continuous*) *map is* ${}^{*}gp$*-continuous*,

(ii) *every* g^**-continuous* (*resp.* g^*p*-continuous*, $\alpha\hat{g}$*-continuous*) *map is* ${}^*gp$*-continuous*,

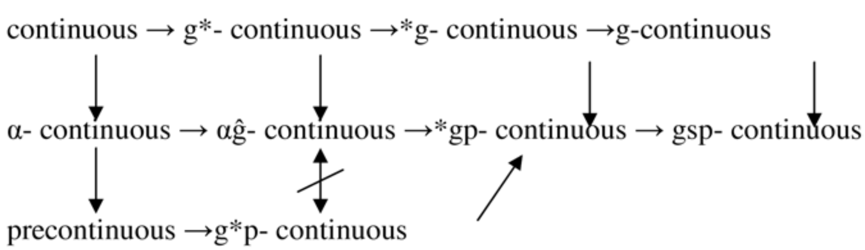
(iii) *every* ${}^*gp$*-continuous map is gsp-continuous*.

Proof. (i) Let $f : (X, \tau) \to (Y, \sigma)$ be a continuous (resp. ${}^*g$-continuous, precontinuous, α-continuous) map and V be a closed set of (Y, σ). Then $f^{-1}(V)$ is closed (resp. ${}^*g$-closed, preclosed, α-closed) in (X, τ), hence by Proposition 2.1, $f^{-1}(V)$ is a ${}^*gp$-closed in (X, τ). Therefore, (X, τ) is ${}^*gp$ -continuous.

(ii) Let $f : (X, \tau) \to (Y, \sigma)$ be a g^*-continuous (resp. g^*p-continuous, $\alpha\hat{g}$-continuous) map and V be a closed set of (Y, σ). Then $f^{-1}(V)$ is g^*-closed (resp. g^*p-closed, $\alpha\hat{g}$-closed) in (X, τ). So, by Proposition 2.1(ii), $f^{-1}(V)$ is ${}^*gp$-closed in (X, τ). Hence, (X, τ) is ${}^*gp$-continuous.

(iii) Let $f : (X, \tau) \to (Y, \sigma)$ be a ${}^*gp$-continuous map and V be a closed set of (Y, σ). Then $f^{-1}(V)$ is ${}^*gp$-closed in (X, τ), hence, by Proposition 2.1(iii), $f^{-1}(V)$ is *gsp*-closed in (X, τ). So, (X, τ) is *gsp*-continuous.

According to the above definition, we give the implications between these functions and other types of functions by the following diagram:



The converse of these implications need not be true in [20, 23, 25, 26] and the following examples.

Example 3.1. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}\}$ be topologies on X, Y, respectively. Define a map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$, then f is ${}^{*}gp$-continuous but not continuous (resp. precontinuous, α-continuous), since $\{a, c\}$ is a closed set of (Y, σ) and $f^{-1}(\{a, c\}) = \{a, b\}$ is not closed (resp. preclosed, α-closed) in (X, τ).

Example 3.2. Let $X = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{b, c\}\}$. Define a map $g : (X, \tau) \to (X, \tau)$ by $g(a) = b$, $g(b) = a$ and $g(c) = c$, hence g is ${}^{*}gp$-continuous but not ${}^{*}g$-continuous (resp. g^{*}-continuous, $\alpha\hat{g}$-continuous), since $\{a\}$ is a closed set of (Y, σ) and $f^{-1}(\{a\}) = \{b\}$ is not ${}^{*}g$-closed (resp. g^{*}-closed, $\alpha\hat{g}$-closed) in (X, τ).

Example 3.3. Let $X = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}\}$. Define a map $h : (X, \tau) \to (X, \tau)$ by $h(a) = b$, $h(b) = c$ and $h(c) = a$, then h is ${}^{*}gp$-continuous, but not $g^{*}p$-continuous, since $\{b, c\}$ is a closed set of (Y, σ) and $f^{-1}(\{b, c\}) = \{a, b\}$ is not $g^{*}p$-closed in (X, τ).

Example 3.4. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ be two topologies on X, Y, respectively. Define a map $\theta : (X, \tau) \to (Y, \sigma)$ by $\theta(a) = b$, $\theta(b) = a$ and $\theta(c) = c$, hence θ is *gsp*-continuous but not ${}^{*}gp$-continuous, since $\{a\}$ is a closed set of (Y, σ) and $f^{-1}(\{a\}) = \{b\}$ is not ${}^{*}gp$-closed in (X, τ).

Remark 3.1. The notions of ${}^{*}gp$-continuous and $\hat{g}$-continuous function are independent of each other. In Example 3.1, f is ${}^{*}gp$-continuous but not a $\hat{g}$-continuous map, since $\{a, c\}$ is a closed set of (Y, σ) and $f^{-1}(\{a, c\}) = \{a, b\}$ is not $\hat{g}$-closed in (X, τ).

Theorem 3.2. *A map* $f:(X, \tau) \to (Y, \sigma)$ *is* $^*gp$*-continuous if and only if the inverse image of each open set of* (Y, σ) *is* $^*gp$*-open in* (X, τ).

Theorem 3.3. *Let* $f:(X, \tau) \to (Y, \sigma)$ *be a* $^*gp$*-continuous mapping. Then the following statements hold*:

(i) *for each* $x \in X$ *and each nbd N of* $f(x)$ *in Y, there exists* $^*gp$*-nbd W of x in X such that* $f(W) \subseteq N$,

(ii) $f(^*gp\text{-}cl(A)) \subseteq cl(f(A))$ *for each* $A \subseteq X$,

(iii) $^*gp\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ *for each* $B \subseteq Y$,

(iv) *if f is a bijective, then* $int(f(A)) \subseteq f(^*gp\text{-}int(A))$ *for each* $A \subseteq X$,

(v) *if f is a bijective, then* $f^{-1}(int(B)) \subseteq {}^*gp\text{-}int(f^{-1}(B))$ *for each* $B \subseteq Y$.

Proof. (i) Suppose that $x \in X$. Then $f(x) \in Y$, since N is a nbd of $f(x)$ in Y, there exists an open set $G \subseteq Y$ such that $f(x) \in G \subseteq N$. Therefore, $x \in f^{-1}(G) \subseteq f^{-1}(N)$. But, f is $^*gp$-continuous, then $f^{-1}(G)$ is a $^*gp$-open set of X. Thus, by Definition 2.7, $f^{-1}(N)$ is $^*gp$*-nbd* of x in X. Setting $W = f^{-1}(N)$ this implies that $f(W) \subseteq N$.

(ii) For each $A \subseteq X$, then $f(A) \subseteq Y$, but $f(A) \subseteq cl(f(A))$ which is a closed set of Y. Since f is $^*gp$-continuous, $f^{-1}(cl(f(A)))$ is a $^*gp$-closed set of X. Hence, by Proposition 2.3(i), $^*gp\text{-}cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$, but

$$\begin{aligned} {}^*gp\text{-}cl(A) &\subseteq {}^*gp\text{-}cl(f^{-1}(f(A))) \subseteq {}^*gp\text{-}cl(f^{-1}(cl(f(A)))) \\ &= f^{-1}(cl(f(A))), \end{aligned}$$

then $f(^*gp\text{-}cl(A)) \subseteq f(f^{-1}(cl(f(A)))) \subseteq cl(f(A))$ for each $A \subseteq X$.

(iii) Let $B \subseteq Y$. Since $f^{-1}(cl(B))$ is a ${}^{*}gp$ -closed set of X, by Proposition 2.2(i), $f^{-1}(B) \subseteq {}^{*}gp\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each $B \subseteq Y$.

(iv) For each $A \subseteq X$, then $f(A) \subseteq Y$, hence $int(f(A))$ is an open set of Y. But f is ${}^{*}gp$ -continuous, then by Theorem 3.2, $f^{-1}(int(f(A)))$ is ${}^{*}gp$ -open set in X. Hence, by Proposition 2.3(i), $f^{-1}(int(f(A))) = {}^{*}gp\text{-}int(f^{-1}(int(f(A)))) \subseteq {}^{*}gp\text{-}int(f^{-1}(f(A)))$. Since f is bijective, $f^{-1}(int(f(A))) \subseteq {}^{*}gp\text{-}int(A)$, hence $f(f^{-1}(int(f(A)))) \subseteq f({}^{*}gp\text{-}int(A))$. Therefore, $int(f(A)) \subseteq f({}^{*}gp\text{-}int(A))$ for each $A \subseteq X$.

(v) Since $B \subseteq Y$ and f is a bijective ${}^{*}gp$ -continuous map, $f^{-1}(B) \subseteq X$ and by (iv), we have $int(f(f^{-1}(B))) \subseteq f({}^{*}gp\text{-}int(f^{-1}(B)))$. Thus, $f^{-1}(int(B)) \subseteq {}^{*}gp\text{-}int(f^{-1}(B))$ for every $B \subseteq Y$.

The following examples show that the converse of the above Theorem 3.3((iv), (v)) is not true.

Example 3.5. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{a, b\}\}$ be two topologies on X, Y, respectively. Define a map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c) = b$, hence (iv) and (v) are achieved but f is not bijective.

Remark 3.2. The composition of two ${}^{*}gp$ -continuous mappings needs not be ${}^{*}gp$ -continuous. Let $X = Y = Z = \{a, b, c\}$ with topologies $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\sigma = \{Y, \varphi, \{a\}, \{a, c\}\}$ and $\eta = \{Z, \varphi, \{a, b\}\}$ on X, Y and Z, respectively. Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$ and $g : (Y, \sigma) \to (Z, \eta)$ be a mapping defined by $g(a) = b$, $g(b) = a$ and $g(c) = c$. Then f and g are ${}^{*}gp$ -continuous but $g \circ f$ is not ${}^{*}gp$ -continuous, since $\{c\}$ is a closed set of (Z, η) and $(g \circ f)^{-1}(\{c\}) = \{a\}$ is not ${}^{*}gp$ -closed in (X, τ).

Theorem 3.4. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a* $^*gp$*-continuous and* $g : (Y, \sigma) \to (Z, \eta)$ *is a continuous map, then their composition* $g \circ f : (X, \tau) \to (Z, \eta)$ *is* $^*gp$*-continuous.*

Proof. Let G be a closed set of (Z, η). Then $g^{-1}(G)$ is closed in (Y, σ). But f is $^*gp$-continuous, hence $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is $^*gp$-closed in (X, τ). Therefore, $g \circ f$ is $^*gp$-continuous.

Proposition 3.1. *The product of two* $^*gp$*-open sets of spaces is a* $^*gp$*-open set in the product space.*

Proof. Let A and B be two $^*gp$-open sets of spaces (X, τ) and (Y, σ), respectively. Also, let $W = A \times B \subseteq X \times Y$ and $F \subseteq W$ be a $\hat{g}$-closed set in $X \times Y$. Then there exist two $\hat{g}$-closed sets $F_1 \subseteq A$ and $F_2 \subseteq B$, such that $F_1 \subseteq p\text{-}int(A)$ and $F_2 \subseteq p\text{-}int(B)$. Therefore, $F_1 \times F_2 \subseteq A \times B$ and by Proposition 2.4, $F_1 \times F_2 \subseteq p\text{-}int(A) \times p\text{-}int(B) = p\text{-}int(A \times B)$. So, $A \times B$ is $^*gp$-open in $X \times Y$.

Theorem 3.5. *Let* $f_i : X_i \to Y_i$ *be* $^*gp$*-continuous maps for each* $i \in \{1, 2\}$ *and let* $f : X_1 \times X_2 \to Y_1 \times Y_2$ *be defined by* $f((x_1, x_2)) = (f(x_1), f(x_2))$. *Then f is* $^*gp$*-continuous.*

Proof. Let V_1 and V_2 be two open sets in Y_1 and Y_2, respectively. Since $f_i : X_i \to Y_i$ are $^*gp$-continuous maps for each $i \in \{1, 2\}$, $f_1^{-1}(V_1)$ and $f_2^{-1}(V_2)$ are $^*gp$-open sets in X_1 and X_2, respectively, hence by Proposition 3.1, $f_1^{-1}(V_1) \times f_2^{-1}(V_2)$ is $^*gp$-open in $X_1 \times X_2$. Therefore, f is $^*gp$-continuous.

Theorem 3.6. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a* $^*gp$*-continuous map. Then* *the following hold*:

(i) *f is continuous if* (X, τ) *is* ${}_{*gp}T$ *-space*,

(ii) *f is precontinuous if* (X, τ) *is* ${}_{*gp}T_{1/2}$ *-space*.

Proof. (i) Let *V* be a closed set of (Y, σ) and $f : (X, \tau) \to (Y, \sigma)$ be a ${}^{*}gp$ -continuous map. Then $f^{-1}(V)$ is ${}^{*}gp$ -closed in (X, τ). But (X, τ) is ${}_{*gp}T$ -space, hence $f^{-1}(V)$ is closed in (X, τ). Therefore, *f* is continuous.

(ii) Let *V* be a closed set of (Y, σ) and $f : (X, \tau) \to (Y, \sigma)$ be a ${}^{*}gp$ -continuous map. Then $f^{-1}(V)$ is ${}^{*}gp$ -closed in (X, τ). But (X, τ) is ${}_{*gp}T_{1/2}$ -space, hence $f^{-1}(V)$ is preclosed in (X, τ). So, *f* is precontinuous.

Theorem 3.7. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be preirresolute and* $g : (Y, \sigma) \to (Z, \eta)$ *be* ${}^{*}gp$ *-continuous maps. Then* $g \circ f : (X, \tau) \to (Z, \eta)$ *is precontinuous if* (Y, σ) *is a* ${}_{*gp}T_{1/2}$ *-space*.

Proof. Let *V* be a closed set of (Z, η) and *g* be a ${}^{*}gp$ -continuous map. Then $g^{-1}(V)$ is ${}^{*}gp$ -closed in (Y, σ). But (Y, σ) is a ${}_{*gp}T_{1/2}$ -space, then $g^{-1}(V)$ is preclosed in (Y, σ). Since *f* is preirresolute, hence $f^{-1}(g^{-1}(V))$ is preclosed in (X, τ). Therefore, $(g \circ f)^{-1}(V)$ is preclosed in (X, τ). Hence, $g \circ f$ is precontinuous.

4. ${}^{*}gp$ -closed and ${}^{*}gp$ -open Mappings

This section is devoted to introduce and study the concepts of ${}^{*}gp$ -closed and ${}^{*}gp$ -open mappings. Also, some of their characterizations are studied.

Definition 4.1. A mapping $f:(X, \tau) \to (Y, \sigma)$ is said to be ${}^*gp$-*closed* (resp. ${}^*gp$-*open*) if the image of each closed (resp. open) set of (X, τ) is ${}^*gp$-closed (resp. ${}^*gp$-open) in (Y, σ).

Theorem 4.1. *If* $f:(X, \tau) \to (Y, \sigma)$ *is a mapping, then*:

(i) *every closed* (*resp. a* ${}^*g$-*closed, a preclosed, an* α-*closed*) *map is* ${}^*gp$-*closed,*

(ii) *every* g^*-*closed* (*resp. a* g^*p-*closed* (*or strongly gp-closed*), *an* $\alpha\hat{g}$-*closed*) *map is* ${}^*gp$-*closed,*

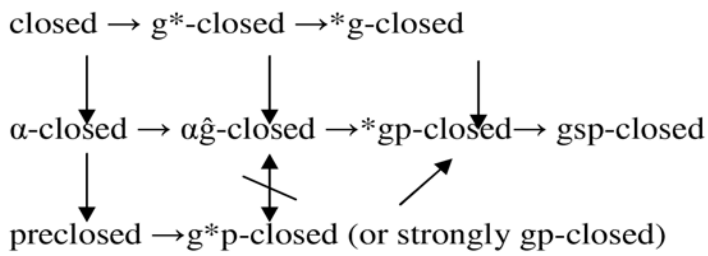
(iii) *every* ${}^*gp$-*closed map is gsp-closed.*

Proof. (i) Let $f:(X, \tau) \to (Y, \sigma)$ be a closed (resp. a ${}^*g$-closed, a preclosed, an α-closed) map and V be a closed set of (X, τ). Then $f(V)$ is closed (resp. ${}^*g$-closed, preclosed, α-closed) in (Y, σ), hence $f(V)$ is a ${}^*gp$-closed set of (Y, σ). Therefore, f is ${}^*gp$-closed.

(ii) Let $f:(X, \tau) \to (Y, \sigma)$ be a g^*-closed (resp. a g^*p-closed (or strongly *gp*-closed), an $\alpha\hat{g}$-closed) map and V be a closed set of (X, τ). Hence, $f(V)$ is g^*-closed (resp. g^*p-closed (or strongly *gp*-closed), $\alpha\hat{g}$-closed) in (Y, σ), then $f(V)$ is a ${}^*gp$-closed set of (Y, σ). So, f is ${}^*gp$-closed.

(iii) Let $f:(X, \tau) \to (Y, \sigma)$ be a ${}^*gp$-closed map and V be a closed set of (X, τ). Then $f(V)$ is ${}^*gp$-closed in (Y, σ). Hence, $f(V)$ is a *gsp*-closed set of (Y, σ). Therefore, f is *gsp*-closed.

By the following diagram, we give the implications between the concepts of ${}^*gp$-closed and other types of mappings:



The converse of these implications need not to be true in [5, 25] and by the following examples.

Example 4.1. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{b, c\}\}$, $\sigma = \{Y, \varphi, \{b\}\}$ be two topologies on X, Y, respectively, and $f : (X, \tau) \to (Y, \sigma)$ be a mapping which is defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is $^{*}gp$ -closed but not closed (resp. preclosed, α-closed, g^{*} -closed, $g^{*}p$ -closed (or strongly gp-closed)). Since $\{b, c\}$ is a closed set of X and $f(\{b, c\}) = \{a, b\}$ is $^{*}gp$ -closed but not closed (resp. preclosed, α-closed, g^{*} -closed, $g^{*}p$ -closed (or strongly gp-closed)) in Y.

Example 4.2. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{b, c\}\}$, $\sigma = \{Y, \varphi, \{a\}, \{a\ c\}\}$ be two topologies on X, Y, respectively, and $g : (X, \tau) \to (Y, \sigma)$ be a mapping which is defined by $g(a) = c$, $g(b) = a$ and $g(c) = b$. Then g is $^{*}gp$ -closed but not $^{*}g$ -closed, since $\{a\}$ is a closed set of X and $f(\{a\}) = \{c\}$ is $^{*}gp$ -closed but not $^{*}g$ -closed in Y. Further, the map g in Example 3.2, which is defined by $g(a) = b$, $g(b) = a$ and $g(c) = c$ is $^{*}gp$ -closed but not $\alpha\hat{g}$ -closed, since $\{a\}$ is a closed set of X and $g(\{a\}) = \{b\}$ is $^{*}gp$ -closed but not $\alpha\hat{g}$ -closed in Y.

Example 4.3. Let $X = Y = \{a, b, c, d\}$ with two topologies $\tau = \{X, \varphi, \{a, b\}, \{c, d\}\}$, $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$ and a mapping $h : (X, \tau) \to (Y, \sigma)$ which is defined by $h(a) = a$, $h(b) = c$,

$h(c) = d$ and $h(d) = b$. Then h is *gsp*-closed but not $^*gp$-closed, since $\{a, b\}$ is a closed set of X and $f(\{a, b\}) = \{a, c\}$ is *gsp*-closed but not $^*gp$-closed in Y.

Remark 4.1. The notions of $^*gp$-closed and $^*gp$-continuous maps are independent of each other. This is shown by the following examples.

Example 4.4. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{X, \varphi, \{a\}\}$, $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$. Then a mapping $f : (X, \tau) \to (Y, \sigma)$ which is defined by $f(a) = a$, $f(b) = b$ and $f(c) = c$ is $^*gp$-closed but not $^*gp$-continuous, since $\{a\}$ is a closed set of Y and $f^{-1}(\{a\}) = \{a\}$ is not $^*gp$-closed in X.

Example 4.5. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\sigma = \{Y, \varphi, \{a, b\}\}$. Hence a mapping $g : (X, \tau) \to (Y, \sigma)$ which is defined by $g(a) = a$, $g(b) = b$ and $g(c) = c$ is $^*gp$-continuous but not $^*gp$-closed, since $\{a, c\}$ is a closed set of X and $f(\{a, c\}) = \{a, b\}$ is not $^*gp$-closed in Y.

Theorem 4.2. *A bijective mapping* $f : (X, \tau) \to (Y, \sigma)$ *is a* $^*gp$*-closed* (*resp.* $^*gp$*-open*) *if and only if for any subset* A *of* (Y, σ) *and any open* (*resp. a closed*) *set* U *of* (X, τ) *containing* $f^{-1}(A)$, *there exists a* $^*gp$*-open* (*resp. a* $^*gp$*-closed*) *subset* B *of* (Y, σ) *containing* A *such that* $f^{-1}(B) \subseteq U$.

Proof. Suppose that U is an open set of X containing $f^{-1}(A)$. Then $X - U$ is closed. But f is a $^*gp$-closed map, then $f(X - U)$ is $^*gp$-closed in Y and hence $B = Y - f(X - U)$ is $^*gp$-open in Y. But $X - U \subseteq X - f^{-1}(A)$, then $A \subseteq Y - f(X - U) = B$. Therefore, $f^{-1}(B) \subseteq U$.

Conversely, assume that F is a closed set of X. Then $X - F$ is an open set of X and $f^{-1}(Y - f(F)) \subseteq X - F$ and hence by hypothesis, there exists a ${}^{*}gp$-open set B of Y containing $Y - f(F)$ such that $f^{-1}(B) \subseteq X - F$ this implies that $B \subseteq Y - f(F)$ and so $B = Y - f(F)$ which is ${}^{*}gp$-open in Y. Therefore, $f(F)$ is a ${}^{*}gp$-closed set of Y. It follows that f is ${}^{*}gp$-closed.

Theorem 4.3. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a bijective* ${}^{*}gp$*-open mapping and* $A = f^{-1}(B)$, *for any open subset* B *of* Y, *then the restriction* $f_A : (A, \tau_A) \to (Y, \sigma)$ *is* ${}^{*}gp$*-open*.

Proof. Suppose that H is an open subset of A. Then there exists an open set U of X such that $H = A \cap U$ and therefore by Theorem 2.1, $f(H) = f_A(H) = B \cap f(U)$ which is ${}^{*}gp$-open in Y and hence $f_A(H)$ is ${}^{*}gp$-open.

Theorem 4.4. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a* ${}^{*}gp$*-closed mapping*. *Then the following statements hold*:

(i) ${}^{*}gp\text{-}cl(f(A)) \subseteq f(cl(A))$ *for every subset* $A \subseteq X$,

(ii) *if f is a bijective, then* $f^{-1}({}^{*}gp\text{-}cl(B)) \subseteq cl(f^{-1}(B))$ *for every subset B of Y*.

Proof. (i) Since $A \subseteq cl(A) \subseteq X$ is closed and f is a ${}^{*}gp$-closed map, hence $f(cl(A))$ is a ${}^{*}gp$-closed set of (Y, σ). Therefore, ${}^{*}gp\text{-}cl(f(A)) \subseteq {}^{*}gp\text{-}cl(f(cl(A))) = f(cl(A))$ for every $A \subseteq X$.

(ii) Since f is a ${}^{*}gp$-closed mapping and $B \subseteq Y$, $f^{-1}(B) \subseteq X$. Hence, from (i), ${}^{*}gp\text{-}cl(f(f^{-1}(B))) \subseteq f(cl(f^{-1}(B)))$ and therefore,

$$f^{-1}({}^{*}gp\text{-}cl(B)) \subseteq cl(f^{-1}(B))$$

for every $B \subseteq Y$.

Theorem 4.5. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a* $^*gp$*-open mapping, then the following statements hold*:

(i) $f(int(A)) \subseteq {}^*gp\text{-}int(f(A))$ *for every* $A \subseteq X$,

(ii) $int(f^{-1}(B)) \subseteq f^{-1}({}^*gp\text{-}int(B))$ *for every* $B \subseteq Y$.

Proof. (i) Since $int(A) \subseteq A \subseteq X$ is an open set of X and f is a $^*gp$-open map, $f(int(A))$ is a $^*gp$-open set of Y and therefore $f(int(A)) = {}^*gp\text{-}int(f(int(A))) \subseteq {}^*gp\text{-}int(f(A))$ for every $A \subseteq X$.

(ii) Since f is a $^*gp$-open mapping and $B \subseteq Y$, $f^{-1}(B) \subseteq X$. Hence, from (i), $f(int(f^{-1}(B))) \subseteq {}^*gp\text{-}int(B)$ and hence

$$int(f^{-1}(B)) \subseteq f^{-1}({}^*gp\text{-}int(B))$$

for each $B \subseteq Y$.

Remark 4.2. The composition of two $^*gp$-closed mappings need not to be $^*gp$-closed. Let $X = Y = Z = \{a, b, c\}$ with the topologies $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$, $\sigma = \{Y, \varphi, \{b, c\}\}$ and $\eta = \{Z, \varphi, \{a\}, \{a\ b\}, \{a, c\}\}$ on X, Y and Z, respectively. Then a mapping $f : (X, \tau) \to (Y, \sigma)$ which is defined by $f(a) = c$, $f(b) = b$, $f(c) = a$ and $g : (Y, \sigma) \to (Z, \eta)$ which is defined by $g(a) = b$, $g(b) = a$, $g(c) = c$ are $^*gp$-closed but $g \circ f$ is not $^*gp$-closed, since $\{b, c\}$ is a closed set of X but $(g \circ f)(\{b, c\}) = \{a, b\}$ is not $^*gp$-closed in Z.

Theorem 4.6. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a closed map and* $g : (Y, \sigma) \to (Z, \eta)$ *is a* $^*gp$*-closed map, then the composition mapping* $g \circ f : (X, \tau) \to (Z, \eta)$ *is* $^*gp$*-closed.*

Proof. Let V be a closed subset of X and f be a closed map. Then $f(V)$ is closed in Y. But g is a ${}^*gp$-closed map, then $g(f(V))$ is ${}^*gp$-closed in Z. Hence, $g \circ f$ is a ${}^*gp$-closed map.

Theorem 4.7. *Let* $f : (X, \tau) \to (Y, \sigma)$ *and* $g : (Y, \sigma) \to (Z, \eta)$ *be two mappings. Then the following statements hold*:

(i) *if f is a surjective continuous map and* $g \circ f$ *is* ${}^*gp$*-closed* (*resp.* ${}^*gp$*-open*), *then g is* ${}^*gp$*-closed* (*resp.* ${}^*gp$*-open*),

(ii) *if g is an injective* ${}^*gp$*-continuous map and* $g \circ f$ *is closed* (*resp. open*), *then f is* ${}^*gp$*-closed* (*resp.* ${}^*gp$*-open*).

Proof. (i) We prove the theorem for the case of ${}^*gp$-closed map. Let $A \subseteq Y$ be a closed set. Since f is continuous, $f^{-1}(A) \subseteq X$ is closed. But $g \circ f$ is a ${}^*gp$-closed map, hence $(g \circ f)(f^{-1}(A))$ is ${}^*gp$-closed in Z. But f is surjective, hence $g(A)$ is ${}^*gp$-closed in Z and hence g is ${}^*gp$-closed.

(ii) Assume that $G \subseteq X$ is a closed set. But $g \circ f$ is a closed map, then $(g \circ f)(G) \subseteq Z$ is closed. Since g is an injective ${}^*gp$-continuous map, $f(G)$ is ${}^*gp$-closed in Y. Hence, f is ${}^*gp$-closed.

Theorem 4.8. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a bijective mapping. Then the following statements are equivalent*:

(i) *f is* ${}^*gp$*-open*,

(ii) *f is* ${}^*gp$*-closed*,

(iii) $f^{-1} : (Y, \sigma) \to (X, \tau)$ *is* ${}^*gp$*-continuous*.

Definition 4.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called *strongly* ${}^*gp$-

closed (resp. *strongly* $^*gp$*-open*) if the image of every $^*gp$-closed (resp. $^*gp$-open) set of (X, τ) is $^*gp$-closed (resp. $^*gp$-open) in (Y, σ).

Proposition 4.1. *Every strongly* $^*gp$*-closed map is* $^*gp$*-closed. In Remark* 4.2, *a mapping* $g : (Y, \sigma) \to (Z, \eta)$ *is* $^*gp$*-closed but not strongly* $^*gp$*-closed, since* $\{b\}$ *is a* $^*gp$*-closed set of Y, but* $g(\{b\}) = \{a\}$ *is not* $^*gp$*-closed in Z.*

Theorem 4.9. *A bijective map* $f : (X, \tau) \to (Y, \sigma)$ *is strongly* $^*gp$*-closed if and only if for every subset B of Y and for each* $^*gp$*-open set U of X containing* $f^{-1}(B)$, *there exists a* $^*gp$*-open set V of* (Y, σ) *such that* $B \subseteq V$ *and* $f^{-1}(V) \subseteq U$.

Proof. Suppose that U is a $^*gp$-open set in X containing $f^{-1}(B)$. Then $X - U$ is $^*gp$-closed in X. But f is strongly $^*gp$-closed, then $f(X - U)$ is $^*gp$-closed in Y and hence $V = Y - f(X - U)$ is $^*gp$-open in Y. Since $X - U \subseteq X - f^{-1}(B)$, $B \subseteq Y - f(X - U) = V$. Therefore, $f^{-1}(V) \subseteq U$.

Conversely, assume that F is a $^*gp$-closed set of X. Then $X - F$ is $^*gp$-open and $f^{-1}(Y - f(F)) \subseteq X - F$ and hence by hypothesis there exists a $^*gp$-open set V of Y containing $Y - f(F)$ such that $f^{-1}(V) \subseteq X - F$ this implies that $V \subseteq Y - f(F)$ and so $V = Y - f(F)$ which is $^*gp$-open in Y. Therefore, $f(F)$ is a $^*gp$-closed set of Y. Hence, f is strongly $^*gp$-closed.

Theorem 4.10. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a bijective strongly* $^*gp$*-closed mapping, then the following statements hold*:

(i) $f^{-1}({}^*gp\text{-}cl(B)) \subseteq p\text{-}cl(f^{-1}(B))$ *for every subset B of Y*,

(ii) ${}^*gp\text{-}cl(f(A)) \subseteq f(p\text{-}cl(A))$ *for every subset A of X.*

Proof. (i) Let $B \subseteq Y$. Then $f^{-1}(B) \subseteq X$. But $f^{-1}(B) \subseteq p\text{-}cl(f^{-1}(B))$ and $p\text{-}cl(f^{-1}(B))$ is a ${}^*gp$ -closed set of (X, τ), hence by hypothesis $f(p\text{-}cl(f^{-1}(B))$ is ${}^*gp$ -closed in (Y, σ). Since $B \subseteq f(p\text{-}cl(f^{-1}(B))$, ${}^*gp\text{-}cl(B) \subseteq {}^*gp\text{-}cl(f(p\text{-}cl(f^{-1}(B)))) = f(p\text{-}cl(f^{-1}(B)))$. Therefore,

$$f^{-1}({}^*gp\text{-}cl(B)) \subseteq p\text{-}cl(f^{-1}(B)).$$

(ii) Since f is a bijective strongly ${}^*gp$ -closed mapping and $A \subseteq X$, $f(A) \subseteq Y$. Hence, by using (i), $f^{-1}({}^*gp\text{-}cl(f(A))) \subseteq p\text{-}cl(f^{-1}(f(A)))$ and hence ${}^*gp\text{-}cl(f(A)) \subseteq f(p\text{-}cl(A))$.

Theorem 4.11. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a strongly* ${}^*gp$ *-open mapping, then the following statements hold*:

(i) *for each* $x \in X$ *and each* ${}^*gp$ *-open set U of X containing x, then there exists a* ${}^*gp$ *-open set W of Y containing* $f(x)$ *such that* $f(U) \subseteq W$,

(ii) $p\text{-}int(f^{-1}(B)) \subseteq f^{-1}({}^*gp\text{-}int(B))$ *for every subset B of Y,*

(iii) $f(p\text{-}int(A)) \subseteq {}^*gp\text{-}int(f(A))$ *for every subset A of X.*

Proof. (i) Let U be a ${}^*gp$ -open set of X containing x. Since f is a strongly ${}^*gp$ -open map, $f(U)$ is a ${}^*gp$ -open set of Y containing $f(x)$. Put $f(U) = W$, therefore, W is ${}^*gp$ -open in Y and $f(x) \in W$. Then $f(U) \subseteq W$.

(ii) Let $B \subseteq Y$. Then $f^{-1}(B) \subseteq X$. But $p\text{-}int(f^{-1}(B)) \subseteq f^{-1}(B)$ and $p\text{-}int(f^{-1}(B))$ is a ${}^*gp$ -open set of (X, τ), hence, by hypothesis $f(p\text{-}int(f^{-1}(B)))$ is ${}^*gp$ -open in Y. Also, $f(p\text{-}int(f^{-1}(B))) \subseteq B$, then

$f(p\text{-}int(f^{-1}(B))) = {}^*gp\text{-}int(f(p\text{-}int(f^{-1}(B)))) \subseteq {}^*gp\text{-}int(B)$. Therefore, $p\text{-}int(f^{-1}(B)) = f^{-1}({}^*gp\text{-}int(B))$ for every subset B of Y.

(iii) Since f is a strongly ${}^*gp$-closed mapping and $A \subseteq X$, $f(A) \subseteq Y$ and by using (ii), $p\text{-}int(f^{-1}(f(A))) \subseteq f^{-1}({}^*gp\text{-}int(f(A)))$ and hence $f(p\text{-}int(A)) \subseteq {}^*gp\text{-}int(f(A))$.

Theorem 4.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be $\hat{g}$-irresolute and M-preclosed map. Then f is strongly ${}^*gp$-closed.*

Proof. Let A be a ${}^*gp$-closed set of (X, τ) and U be a $\hat{g}$-open set of (Y, σ) such that $f(A) \subseteq U$. Then $A \subseteq f^{-1}(U)$. But f is a $\hat{g}$-irresolute map, then $f^{-1}(U)$ is a $\hat{g}$-open set of (X, τ). Since A is a ${}^*gp$-closed set of (X, τ), $pcl(A) \subseteq f^{-1}(U)$ and hence $f(A) \subseteq f(p\text{-}cl(A)) \subseteq f(f^{-1}(U)) \subseteq U$. But f is an M-preclosed map and $p\text{-}cl(A)$ is a preclosed set of (X, τ), then $f(p\text{-}cl(A))$ is preclosed in (Y, σ) and hence $p\text{-}cl(f(A)) \subseteq p\text{-}cl(f(p\text{-}cl(A)))$ $= f(p\text{-}cl(A)) \subseteq U$. This shows that $f(A)$ is ${}^*gp$-closed in (Y, σ). Hence, f is strongly ${}^*gp$-closed.

Proposition 4.2. *The composition of two strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open) maps is strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open).*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open) maps and F be a ${}^*gp$-closed (resp. ${}^*gp$-open) set of (X, τ). Then $f(F)$ is ${}^*gp$-closed (resp. ${}^*gp$-open) in (Y, σ). But g is a strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open) map, hence $g(f(F))$ is ${}^*gp$-closed (resp. ${}^*gp$-open) in (Z, η), that is, $(g \circ f)(F)$ is

${}^*gp$-closed (resp. ${}^*gp$-open) in (Z, η). Therefore, $g \circ f$ is strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open).

Theorem 4.13. *The following hold for the mappings* $f : (X, \tau) \to (Y, \sigma)$ *and* $g : (Y, \sigma) \to (Z, \eta)$:

(i) $g \circ f$ *is a strongly* ${}^*gp$*-closed* (*resp. strongly* ${}^*gp$*-open*) *map if both f and g are strongly* ${}^*gp$*-closed* (*resp. strongly* ${}^*gp$*-open*),

(ii) $g \circ f$ *is a* ${}^*gp$*-closed* (*resp.* ${}^*gp$*-open*) *map if f is* ${}^*gp$*-closed* (*resp.* ${}^*gp$*-open*) *and g is strongly* ${}^*gp$*-closed* (*resp. strongly* ${}^*gp$*-open*),

(iii) *g is a* ${}^*gp$*-closed* (*resp.* ${}^*gp$*-open*) *map if f is a surjective* ${}^*gp$*-continuous and* $g \circ f$ *is a strongly* ${}^*gp$*-closed* (*resp. strongly* ${}^*gp$*-open*) *mapping*,

(iv) $g \circ f$ *is a* ${}^*gp$*-closed map if f is a closed map and g is* $\hat{g}$*-irresolute and M-preclosed map*,

(v) $g \circ f$ *is a* ${}^*gp$*-closed if f is a closed and g is a strongly* ${}^*gp$*-closed map*.

Proof. (i) Let *V* be a ${}^*gp$-closed (resp. ${}^*gp$-open) set of *X*. Then $f(V)$ is ${}^*gp$-closed (resp. ${}^*gp$-open) in *Y*. But *g* is strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open), then $g(f(V))$ is a ${}^*gp$-closed (resp. ${}^*gp$-open) set of *Z*. Hence $g \circ f$ is strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open).

(ii) Let *V* be a closed (resp. open) set of *X* and *f* be a ${}^*gp$-closed map. Then $f(V)$ is ${}^*gp$-closed (resp. ${}^*gp$-open) in *Y*. But *g* is a strongly ${}^*gp$-closed (resp. strongly ${}^*gp$-open) map, then $g(f(V))$ is ${}^*gp$-closed (resp. ${}^*gp$-open) in *Z*. Hence $g \circ f$ is ${}^*gp$-closed (resp. ${}^*gp$-open).

(iii) Let V be a closed (resp. open) set of Y and f be a $^*gp$-continuous map. Then $f^{-1}(V)$ is a $^*gp$-closed (resp. $^*gp$-open) set of X. But $g \circ f$ is strongly $^*gp$-closed (resp. strongly $^*gp$-open), then $g \circ f(f^{-1}(V))$ is $^*gp$-closed (resp. $^*gp$-open) in Z. Hence $g(f(f^{-1}(V))) = g(V)$ is a $^*gp$-closed (resp. $^*gp$-open) in Z. Therefore, g is $^*gp$-closed (resp. $^*gp$-open).

(iv) Let V be a closed set of X and f be a closed map. Then $f(V)$ is a closed set of Y and hence $f(V)$ is $^*gp$-closed in Y. Then, by Theorem 4.12, g is a strongly $^*gp$-closed map. Then $g(f(V))$ is a $^*gp$-closed set of Z. Hence $g \circ f$ is $^*gp$-closed (resp. $^*gp$-open).

(v) Let V be a closed set of X and f be a closed map. Then $f(V)$ is closed in Y and hence $f(V)$ is $^*gp$-closed in Y. But g is a strongly $^*gp$-closed map, then $g(f(V))$ is a $^*gp$-closed set of Z. Hence, $g \circ f$ is $^*gp$-closed.

Conclusion

Maps have always been of tremendous importance in all branches of mathematics and the whole science. On the other hand, topology plays a significant role in quantum physics, high energy and superstring theory [8, 9]. Thus, we have obtained a new class of mappings called $^*gp$-*continuous* which may have possible application in quantum physics, high energy and superstring theory.

Acknowledgement

The authors are grateful for the support of the Deanship of Scientific Research Taibah University who financially supported this work, under contracted research project (434/3098).

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