**Far East Journal of Applied Mathematics**

Published Online: January 2014
Available online at http://pphmj.com/journals/fjam.htm
Volume 84, Number 1, 2013, Pages 1-10

# MAXIMUM OVERLAP DISCRETE WAVELET METHODS IN MODELING BANKING DATA

**S. Al Wadi[a], Faisal Ababneh[a], Hazem Alwadi[b] and Mohd Tahir Ismail[c]**

[a]Department of Mathematics
Faculty of Science
Al Hussien bin Talal University
Ma'an, Jordan
e-mail: sadam_alwadi@yahoo.co.uk
fmababneh@gmail.com

[b]Department of Social Sciences
Faculty of Science
Jordan University of Science and Technology
Irbid, Jordan
e-mail: hazemalwadi@yahoo.com

[c]School of Mathematical Science
Universiti Sains Malaysia
11800 Minden, Penang, Malaysia
e-mail: mtahir@cs.usm.my

## Abstract

Recently, maximum overlap discrete wavelet transform (MODWT) has gained very high attention in many fields and applications such as finance, engineering, signal processing, applied mathematics and statistics. In this paper, we present the advantages of MODWT in analyzing financial time series data and extend the working done by

Received: July 5, 2013; Revised: September 19, 2013; Accepted: September 25, 2013
2010 Mathematics Subject Classification: 42C40.
Keywords and phrases: fluctuations, WT, MODWT, banking data, ASE.

[13]. Amman stock exchange (ASE) from Jordan was selected as a tool to show the ability of MODWT in detecting the fluctuations in the banking sector in ASE. Finally, the claim is that MODWT is better than wavelet transform (WT) in the analysis processes has approved in content of ASE.

## 1. Introduction

During the last three decades, the banking sector was very stable in Jordan, very few number of banks have exited from the market, since this sector has a very nice environment for growing, well capitalized, liquid and profitable and privately owned, open to external investors. Moreover, this sector has comprehensive banking services: retail, corporate, Islamic, and e-banking and loans, Payment System: Real Time Gross Settlements and Electronic Cheque Clearing System. Generally, this sector has regional banks: 137 branches outside Jordan.

These reasons motivate the researchers to focus in analyzing the banking sector in Jordan and study their fluctuations since the analysis of stock market data specially the banking data is one of the key issues to obtain insight into the data; to identify patterns, trends or correlations; to detect the fluctuations in the data and to understand the past and current behaviors of the stock market. In addition, identifying the distribution of the analyzed data is very important and necessary for understanding the market dynamics as well as to determine when particular events occurred.

Usually time series are not deterministic series. In fact, in many cases, the researchers considered the series to be stationary time series. One way to model any time series is to consider it as a deterministic function plus white noise. The white noise in any time series process can be minimized by some procedures which are called the *de-noising*. Then a better model can be obtained. Consequently, to obtain a good de-noising, there are some mathematical models that can be applied such as Fourier transform and WT.

In this regard, WT has used as an effective method to analyze or decompose stock market data compared to fast Fourier transform (FFT), for more details, refer to [11, 12]. Whereas the following sections of this paper will support this contention by proving the effectiveness and efficiency of MODWT in analyzing stock market data and study real world problems in the Jordan banking.

This paper is organized as follows: The next section describes the mathematical literature review. Section 3 provides a description of data set. Section 4 describes the methodology. The experimental results are presented to demonstrate the effectiveness of WT in using banking data will be presented in Section 5. In Section 6, we summarize our contributions and mention the conclusion.

## 2. Literature Review

WT is a mathematical model that transforms the original signal (especially with time domain) into a different domain for analysis and processing [8]. This model is very suitable with the non-stationary data, i.e., mean and autocorrelation of the signal are not constant over time that is well known, most of the financial time series data are non-stationary that is why we applied WT.

In mathematical literature, FFT decomposed the original signal into a linear combination as a sine and cosine function whereas by WT, the signal is decomposed as a sum of a more flexible function called *wavelet* that is localized in both time and frequency. The WT was used to adopt a wavelet prototype function (mother wavelet). Temporal analysis is constructed with a contracted, high-frequency version of prototype wavelet whereas frequency analysis is performed with a dilated, low frequency version of the prototype wavelet. Because the function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data decompositions can be constructed by just using the corresponding wavelet coefficients. There are several types of WT. Depending on the applications, regarding the continuous input signal, the time and scale parameters can be continuous, leading to the continuous WT (CWT). On the other hand, the discrete WT (DWT) can also be used for discrete time signals.

In the WTs case, consider that the time domain is the original domain. Although, WTs are the transformation processes from time domain to time scale domain, these processes are known as signal decomposition because a given signal is decomposed into several other signals with different levels of resolution. These processes allow recovering the original time domain signal without losing any information. WT has reverse process which is called the *inverse WT* or *signal reconstruction* [4, 7].

As a critical review about the model used in this paper, MODWT has some more advantage than DWT which motivates us to focus in its application and compare it with DWT, such as; MODWT is not orthonormal (in fact, MODWT is highly redundant), MODWT can be defined naturally for any sample sizes (i.e., $N$ need not be a multiple of a power of two) and the analysis of variance (ANOVA) can be applied on MODWT wavelet coefficients. Therefore, in this paper, we will focus on the most famous types of DWTs which are HWT (Haar wavelet transform) and dWT (Daubechies wavelet transform), then compare them with MOWDT dynamic using an insurance time series data in content of ASE. After intensive research in the literature, many researchers in the area of finance have focused on the application of WT in their research since WT is a well known method for handling data with high degree of uncertainties. The application of WT in analyzing stock market data includes the following areas: forecasting, understanding the dynamics of the stock markets, volatility and decomposition of stock market data. Whereas for the past decade, no researcher has focused on the application of WT to solve financial issues, such as improving the forecasting accuracy in the content of ASE, then making a comparative study between the wavelet functions such as DWT and MODWT [3, 6, 7]. Moreover, after intensive research in the literature, the researchers notice that:

First, according to the past decade, no researcher has paying attention on the application of MODWT to solve financial issues such as study the banking data in ASE, then comparing HWT, dWT with MODWT.

Second, most of the applications of the WT were as; for the last 10 years, a number of comparative studies using different methodologies have been carried out using various WT functions alone as well as in combination with other WT models. More specifically, in the method used to select the values for the variables, CWT has no constraint on the selection of the possible values; while in DWT, the selection of possible values is restricted. One of the most important families of the DWT is OWT. The rationale for choosing the DWT in this study is explained as follows [7]:

First, researchers experienced difficulty describing stock market data accurately using a combination of sine and cosine terms (e.g., FFT). Consequently, researchers introduced WT which successfully solved the problem. In order to corroborate this claim and prove it experimentally in the stock market area, this study will conduct a comparison between the FFT and OWT.

Second, OWT in general, are extremely fast algorithms and widely used for analyzing stock market data [10, 6, 7]. Orthogonality property provides independency for the detail coefficients and therefore allows the addition of one or more of the detail coefficients of different levels to the approximation coefficient of the first level in different combinations for purposes of various analyses. Most importantly, OWT has a “compact support” property. The significance of having a compact support property is that when signals are fitted, they provide localized results rather than global results [10, 6, 7], the mathematical models can be introduced briefly as follows:

**Definition** [2, 5]**.** Discrete WT can be defined by the following function:

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^{j}t - k),$$

$$j,\, k \in Z;\; z = \{0, 1, 2, ...\},$$

where $\psi$ is a real valued function having compactly supported, and $\int_{-\infty}^{\infty}\psi(t)dt = 0.$ Generally, the WT was evaluated by using dilation equations, given as:

$$\phi(t) = \sqrt{2}\sum_{k} l_k \phi(2t - k),$$

$$\psi(t) = \sqrt{2}\sum_{k} h_k \phi(2t - k).$$

Father and mother wavelets were defined by the last two equations, where $\phi(2t - k)$ represents the father wavelet, and $\psi(t)$ represents the mother wavelet. For more details about the mathematical definitions of HWT, DWT and MOWDT, father wavelet and mother wavelet refer to [3, 6, 7, 11-14].

## 3. Data Description

In order to illustrate the effectiveness of MODWT, the ASE banking data sets are selected for discussion. In order to apply DWT, we consider a daily return data for the time period from April 1993 (the days when stock exchanges were open) until December 2009 with a total of 4096 observations. The total number of observations for mathematical convenience is suggested to be divisible by $2^j$, for more details, refer to [5, 9]. Whereas in order to apply MODWT, the data can be used with unlimited observations without any conditions.

## 4. Methodology

The WT and MODWT convert the row data series into two sets; approximation series (CA1 (n)) and detail series (DA1 (n)). These two series present a better behavior, i.e., more stable in variance and no outliers than the original price series, then they can be predicted more accurately. The reason for the better behavior of these two series is the filtering effect of the WT. In this paper, the approximation series has been used since this series behaves as the main component of the transform, while the detail series provides "small" adjustments. The procedure explained in this paper is as follows:

First, decompose through the WT and MODWT the available historical banking data. Second, comparing the figures which are produced from MODWT and DWT in order to decide the best model.

## 5. Experimental Results

In this section, the analysis of the data using HWT and dWT was discussed in [13]. Therefore, the results using HWT will briefly be presented in this section and then compare with MODWT results.

The application of the WT was decomposed as in [13] to the historical data, then the results were as follows: the historical data decomposed into a variety of resolution levels that expose their essential structure and it generates detail coefficients at each one of the three decomposition levels. According to WT mechanism, the three levels of decomposition can be carried out by the WT using the following equation: $S = a_3 + d_3 + d_2 + d_1,$ where $S$ refers to the original signal which is represented in the top part of Figure 1. Then the next part consists of one approximation level $(a_3)$ which shows the plot of the approximation coefficients for the transformed data using WT. The following parts of $d_1,$ $d_2$ and $d_3$ represent the detail levels, whereby $d_1$ is the plot of the first level of the detail coefficients, $d_2$ is the plot of the second level of the detail coefficients and $d_3$ is the plot of the third level of the detail coefficients. Any of these three levels $(d_1,$ $d_2$ and $d_3)$ can be adopted for explaining the data.

Starting with $d_1$ which is the first detail level (see Figure 1), the transformed data is filtered from $d_1$ until $d_3$ through the detail levels. As can be seen the data becomes smoother in $d_2,$ since the amount of data will be reduced automatically in the hope of obtaining a suitable level for detecting the stock market behavior. In this regard, we notice that at level $d_3,$ most of the fluctuations and high frequencies appear after the observation number 3000, which is from 2004 onwards.

Whereas the fluctuations can be shown in a better way using MODWT as presented in Figure 2, this figure almost gives similar to dWT but in more details for the fluctuation specially after the observation number 2500 which means the year 2000, while these DWTs concentrate for the observations after number 3000 which means the year number 2003. Moreover, Figure 2 gives the target smoothly than Figure 1.

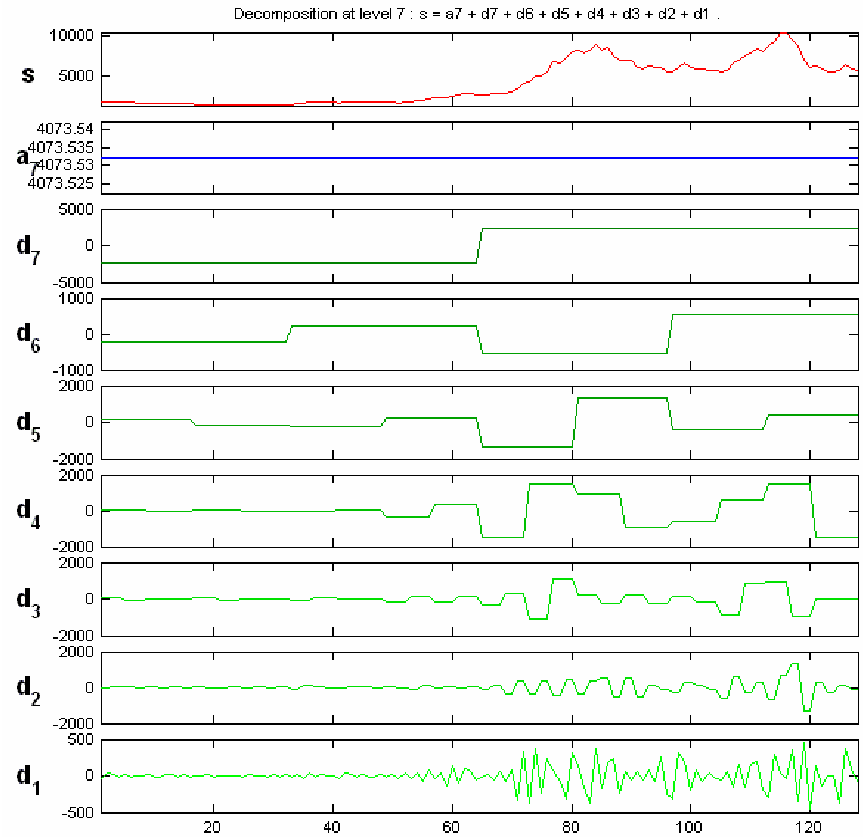


**Figure 1.** Decomposition levels using dWT.

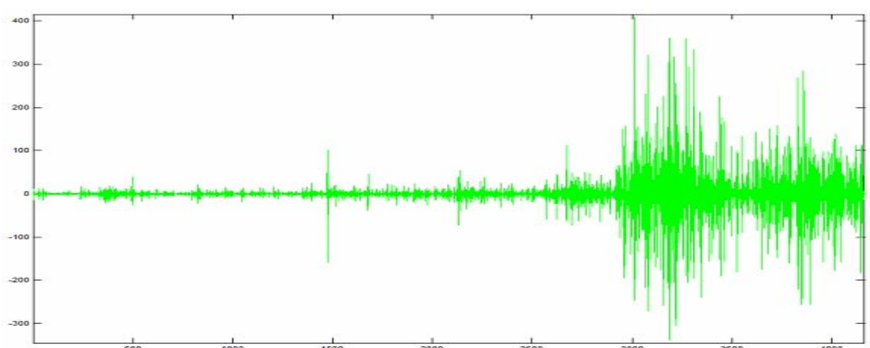

**Figure 2.** MODWT distribution.

## 6. Conclusion

This study implemented MODWT and DWT on ASE banking data. The success application of this study is in removal the outliers and irregular data. Therefore, in this empirical study, the sample data set was experimentally tested in terms of decomposition levels. The purpose of doing so was to find out, MODWT is a suitable model and show a better understanding for the data set comparing with the DWT. Moreover, this study can be extended to implement for forecasting in order to improve the forecasting accuracy.

## References

[1] A. J. Rocha Reis and A. P. Alves da Silva, Feature extraction via multiresolution analysis for short term load forecasting, IEEE Trans. Power Systems 20 (2005), 189-198.

[2] Chang Chiann and Pedro A. Moretin, A wavelet analysis for time series, Nonparametric Statistics 10 (1998), 1-46.

[3] I. Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, PA, 1992.

[4] In-Keun Yu, Chang-Il Kim and Y. H. Song, A novel short-term load forecasting technique using WT analysis, Electric Machines and Power Systems 24 (2000), 537-549.

[5] Philippe Masset, Analysis of financial time-series using Fourier and wavelet methods, Faculty of Economics and Social Science, University of Fribourg, Switzerland, 2008.

[6] P. Manchanda, J. Kumar and A. H. Siddiqi, Mathematical methods for modeling price fluctuations of financial time series, J. Franklin Inst. 344 (2007), 613-636.

[7] R. Gencay, F. Seluk and B. Whitcher, An Introduction to Wavelets and Other Filtering Methods in Finance and Economics, Academic Press, New York, 2002.

[8] S. Skander, On the use of the wavelet decomposition for time series prediction, Neurocomputing 48(1-4) (2002), 267-277.

[9] Todd Wittman, Time-series clustering and association analysis of financial data, CS 8980 Project, 2002.

[10] M. Misiti, Y. Misiti, G. Oppenhiem and J. Poggi, Wavelets and their Applications, ISTE Ltd., Great Britain, United States, 2007.

[11] Mohd Tahir Ismail, Samsul Ariffin Abdul Kareem and S. Al Wadi, A study of structural breaks in Malaysian stock market, African J. Business Management 5(6) (2011), 2418-2425.

[12] S. Al Wadi, Mohd Tahir Ismail and Samsul Ariffin Abdul Karim, A comparison between the Daubechies wavelet transformation and the fast Fourier transformation in analyzing insurance time series data, Far East J. Appl. Math. 45(1) (2010), 53- 63.

[13] Faisal Ababneh, S. Al Wadi and Mohd Tahir Ismail, Haar and Daubechies wavelet methods in modeling banking sector, Int. Math. Forum 8(12) (2013), 551-566.

[14] D. Percival and A. Walden, Wavelet Methods for Time Series Analysis, Reprint, Cambridge University Press, Cambridge, 2002.