



# **EFFECT OF THE HEAT SOURCE, RADIATION ABSORPTION AND CHEMICAL REACTION ON THE UNSTEADY MHD FREE CONVECTIVE FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL MOVING PLATE**

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## **Abstract**

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An analysis has been carried out to investigate the effects of chemical

Received: April 6, 2013; Accepted: August 1, 2013

2010 Mathematics Subject Classification: 76W05, 80A20.

Keywords and phrases: MHD, heat and mass transfer, radiation absorption, heat source, chemical reaction, accelerated, vertical plate.

reaction and radiation absorption on unsteady MHD flow with heat and mass transfer of an incompressible, viscous, electrically conducting fluid over an exponentially accelerated vertical moving plate with heat source. An exact solution for the flow problem has been obtained by solving the governing equations using Laplace-transform technique. At time  $t' > 0$ , the plate is exponentially accelerated with a velocity  $u = u_0 \exp(at')$  in its own plane. At the same time, the plate temperature and concentration levels near the plate are raised to  $T'_w$  and  $C'_w$ , respectively. Graphical results for velocity, temperature and concentration profiles are presented and discussed.

### Nomenclature

$a^*$	Absorption coefficient
$a$	Accelerated parameter
$B_0$	Magnetic induction
$C'$	Species concentration
$C'_w$	Concentration of the plate
$C'_\infty$	Concentration of the fluid far away from the plate
$C$	Dimensionless concentration
$C_p$	Specific heat at constant pressure
$g$	Acceleration due to gravity
$Gr$	Thermal Grashof number
$Gm$	Mass Grashof number
$M$	Magnetic field parameter
$D$	Mass diffusivity
$Nu$	Nusselt number
$Pr$	Prandtl number

$Q_1$	Radiation absorption parameter
$Q_0$	Dimensional heat absorption coefficient
$Q_1'$	Coefficient of proportionality for the absorption of radiation
$Sc$	Schmidt number
$K_l$	Chemical reaction parameter
$k$	Chemical reaction parameter
$T'$	Dimensional temperature
$T_W'$	Temperature of the fluid at the wall
$T_\infty'$	Temperature of the fluid far away from the plate
$t'$	Dimensional time
$t$	Dimensionless time
$u'$	Velocity of the fluid in the $x'$ -direction
$u_0$	Velocity of the plate
$u$	Dimensionless velocity
$y'$	Co-ordinate axis normal to the plate
$y$	Dimensionless co-ordinate axis normal to the plate
$erf$	Error function
$erfc$	Complementary error function

**Greek symbols**

$\alpha$	Thermal diffusivity
$\beta$	Volumetric coefficient of thermal expansion
$\beta^*$	Volumetric coefficient of expansion with concentration
$\phi$	Heat absorption parameter

$\mu$	Coefficient of viscosity
$\kappa$	Thermal conductivity of the fluid
$\nu$	Kinematic viscosity
$\rho$	Density of the fluid
$\sigma$	Electric conductivity
$\theta$	Dimensionless temperature

**Subscripts**

$w$	Conditions on the wall
$\infty$	Free stream conditions

**Introduction**

The study of first order chemical reaction with combined heat and mass transfer is attracted by many researchers and received a considerable amount of attention in recent years. In many processes such as energy transfer in a wet cooling tower in a desert cooler flow, evaporation at the surface of a water body and in heat and mass transfer occur simultaneously. Some applications of this type of flow can be found in many industries such as in power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions dissociating fluids. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effects of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing radiation. Hence, heat transfer by thermal radiation is becoming of greater importance when we concerned with space applications and higher operating temperatures.

The study of magneto hydro-dynamics with heat and mass transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, radio propagation through the ionosphere, etc. In engineering, we find its applications like in MHD pumps, MHD bearings, etc. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable on the solar surface. In free convection flow, the study of effects of magnetic field plays a major role in liquid metals, electrolytes and ionized gases. In power engineering, the thermal physics of hydro magnetic problems with mass transfer have enormous applications. Radiative flows are encountered in many industrial and environment processes, e.g., heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry.

Bestman [1] investigated the natural convection boundary layer with suction and mass transfer in a porous medium. He found that suction stabilizes the boundary layer and affords the most efficient method in boundary layer yet known. The unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate studied by Abdus Sattar and Hamid Kalim [2]. Makinde [3] examined free convection flow with thermal radiation and mass transfer past a moving vertical plate. Ibrahim et al. [4] have studied nonclassical thermal effects in Stokes second problem for micropolar fluids using perturbation method. Muthucumaraswamy and Ganesan [5] reported the effect of chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal vertical plate.

Deka et al. [6] studied the effect of the first-order chemical homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. MHD flow of a stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction was studied by Chamkha [7]. Soundalgekar and Patti [8] examined the flow past an impulsively started isothermal infinite vertical

plate with mass transfer effects. Gebhart and Pera [9] investigated a problem of nature of vertical convection flow resulting from the combined buoyancy effects of thermal and mass diffusion. Chamkha [10] studied unsteady MHD with heat and mass transfer past a semi-infinite vertical moving plate embedded in a porous medium in the presence of heat source or sink. Raptis [11] examined the effect of radiation on steady flow of a viscous incompressible fluid through a porous medium bounded by a porous plate with constant suction velocity. Raptis and Perdikis [12] analyzed free convection flow of water near 4°C along vertical moving porous plate in a boundary layer. Recently, Ibrahim et al. [13] studied the effects of chemical reaction and radiation absorption on the unsteady MHD natural convection flow past a semi-infinite vertical permeable moving plate with heat source and suction.

The aim of the present work is to investigate the effects of radiation absorption, chemical reaction, mass diffusion and heat source parameter of heat generating fluid on unsteady MHD natural convection flow with heat and mass transfer over an exponentially accelerated vertical plate with constant temperature in the presence of transverse applied magnetic field. The dimensionless governing equations are solved using the Laplace-transform technique. The results for velocity, temperature and concentration are obtained in terms of exponential and complementary error functions.

### Mathematical Analysis

In this problem, we consider an unsteady two-dimensional heat and mass transfer flow of a laminar, viscous, incompressible, electrically conducting and radiation absorption fluid past an exponentially accelerated vertical moving plate with constant heat source in the presence of uniform transverse magnetic field and a first-order chemical reaction. Initially, it is assumed that the plate and fluid are at the same temperature  $T'_\infty$  in the stationary condition with concentration level  $C'_\infty$  at all the points. The  $x'$ -axis is taken along the plate in vertical upward direction and  $y'$ -axis is taken normal to it. At time  $t' > 0$ , the plate is exponentially accelerated with a velocity  $u_0 \exp(at')$  in

its own plane. And at the same time, the temperature from the plate is raised to  $T'_w$  and the concentration level near the plate is also raised to  $C'_w$ . A transverse magnetic field of uniform strength  $B_0$  is assumed to be applied normal to the plate. It is also assumed that (i) the fluid properties are constant except for the density variation that induces the buoyancy force. (ii) The induced magnetic field is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small. (iii) The viscous dissipation is neglected in the energy equation. (iv) The effects of variation in density ( $\rho$ ) (with temperature) and species concentration are considered only on the body force term, in accordance with usual Boussinesq approximation. (v) The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. (vi) Since the flow of the fluid is assumed to be in the direction of  $x'$ -axis, so the physical quantities are functions of the space co-ordinates  $y'$  and  $t'$  only. Under these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma\beta_0^2 u'}{\rho}, \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c_p} (T' - T'_\infty) + Q_1'(C' - C'_\infty), \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_l(C' - C'_\infty) \quad (3)$$

with the following initial and boundary conditions:

$$\begin{aligned} t' \leq 0 : u' &= 0, \quad T' = T'_\infty, \quad C' = C'_\infty \text{ for all } y', \\ t' > 0 : u' &= u_0 \exp(a't'), \quad T' = T'_w, \quad C' = C'_w \text{ at } y' = 0, \\ \text{and } u' &= 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty. \end{aligned} \quad (4)$$

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$\begin{aligned}
C &= \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad G_r = \frac{g\beta v(T'_w - T'_\infty)}{u_0^3}, \\
G_m &= \frac{g\beta^* v(C'_w - C'_\infty)}{u_0^3}, \quad Pr = \frac{\mu C_p}{\kappa}, \quad a = \frac{a'v}{u_0^2}, \\
k &= \frac{vK_l}{u_0^2}, \quad Sc = \frac{v}{D}, \quad M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \\
\phi &= \frac{Q_0 v}{\rho c_p u_0^2}, \quad Q_1 = \frac{v Q'_1 (C'_w - C'_\infty)}{(T'_w - T'_\infty) u_0^2},
\end{aligned} \tag{5}$$

we get the following governing equations which are dimensionless:

$$\frac{\partial u}{\partial t} = G_r \theta + G_m C + \frac{\partial^2 u}{\partial y^2} - Mu, \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \phi \theta + Q_1 C, \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - kC. \tag{8}$$

The initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned}
t' \leq 0 : u &= 0, \quad \theta = 0, \quad C = 0 \text{ for all } y, \\
t > 0 : u &= \exp(at), \quad \theta = 1, \quad C = 1 \text{ at } y = 0, \\
t > 0 : u &\rightarrow 0, \quad \theta \rightarrow 0, \quad c \rightarrow 0 \text{ as } y \rightarrow \infty.
\end{aligned} \tag{9}$$

All the appeared physical parameters are defined in the Nomenclature. The dimensionless governing equations from (6) to (8), subject to the boundary conditions (9), are solved by usual Laplace transform technique and the solutions for velocity, temperature and concentration fields are obtained as follows in terms of exponential and complementary error functions:



$$C(y, t) = \frac{1}{2} \left[ \begin{aligned} &\exp(y\sqrt{kSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kt}\right) \\ &+ \exp(-y\sqrt{kSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kt}\right) \end{aligned} \right], \quad (10)$$

$$\begin{aligned} \theta(y, t) = & \frac{1}{2} \left(1 - \frac{b}{c}\right) \left[ \begin{aligned} &\exp(y\sqrt{\phi Pr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{\phi t}\right) \\ &+ \exp(-y\sqrt{\phi Pr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{\phi t}\right) \end{aligned} \right] \\ & + \frac{b}{2c} \exp(ct) \left[ \begin{aligned} &\exp(y\sqrt{\phi Pr + cPr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{(\phi + c)t}\right) \\ &+ \exp(-y\sqrt{\phi Pr + cPr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{(\phi + c)t}\right) \end{aligned} \right] \\ & + \frac{b}{2c} \left[ \begin{aligned} &\exp(y\sqrt{kSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kt}\right) \\ &+ \exp(-y\sqrt{kSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kt}\right) \end{aligned} \right] \\ & - \frac{b}{2c} \exp(ct) \left[ \begin{aligned} &\exp(y\sqrt{kSc + cSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(k + c)t}\right) \\ &+ \exp(-y\sqrt{kSc + cSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(k + c)t}\right) \end{aligned} \right], \quad (11) \end{aligned}$$

$$\begin{aligned} u(y, t) = & \frac{\exp(at)}{2} \left[ \begin{aligned} &\exp(y\sqrt{M + a}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(M + a)t}\right) \\ &+ \exp(-y\sqrt{M + a}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(M + a)t}\right) \end{aligned} \right] \\ & - \left(\frac{A_1 + A_2}{2}\right) \left[ \begin{aligned} &\exp(y\sqrt{M}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{Mt}\right) \\ &+ \exp(-y\sqrt{M}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{Mt}\right) \end{aligned} \right] \\ & + \frac{A_3 \exp(et)}{2} \left[ \begin{aligned} &\exp(y\sqrt{M + e}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(M + e)t}\right) \\ &+ \exp(-y\sqrt{M + e}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(M + e)t}\right) \end{aligned} \right] \end{aligned}$$

$$\begin{aligned}
& - \frac{(A_5 - A_6) \exp(ct)}{2} \left[ \exp(y\sqrt{M+c}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(M+c)t}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{M+c}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(M+c)t}\right) \right] \\
& + \frac{A_4 \exp(nt)}{2} \left[ \exp(y\sqrt{M+n}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(M+n)t}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{M+n}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(M+n)t}\right) \right] \\
& + \frac{A_1}{2} \left[ \exp(y\sqrt{\phi Pr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{\phi t}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{\phi Pr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{\phi t}\right) \right] \\
& - \frac{A_6 \exp(ct)}{2} \left[ \exp(y\sqrt{\phi Pr + cPr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{(\phi+c)t}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{\phi Pr + cPr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{(\phi+c)t}\right) \right] \\
& - \frac{A_3 \exp(et)}{2} \left[ \exp(y\sqrt{\phi Pr + ePr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{(\phi+e)t}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{\phi Pr + ePr}) \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{(\phi+e)t}\right) \right] \\
& + \frac{A_2}{2} \left[ \exp(y\sqrt{kSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kt}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{kSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kt}\right) \right] \\
& - \frac{A_5 \exp(ct)}{2} \left[ \exp(y\sqrt{\phi Sc + cSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(k+c)t}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{\phi Sc + cSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(k+c)t}\right) \right] \\
& - \frac{A_4 \exp(nt)}{2} \left[ \exp(y\sqrt{kSc + nSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(k+n)t}\right) \right. \\
& \quad \left. + \exp(-y\sqrt{kSc + nSc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(k+n)t}\right) \right], \quad (12)
\end{aligned}$$

where

$$\begin{aligned}
 b &= \frac{Q_l Pr}{Sc - Pr}, \quad c = \frac{\phi Pr - kSc}{Sc - Pr}, \quad d = \frac{Gr}{Pr - 1}, \quad e = \frac{M - \phi Pr}{Pr - 1}, \\
 l &= \frac{Gr}{Sc - 1}, \quad n = \frac{M - kSc}{Sc - 1}, \quad r = \frac{Gm}{Sc - 1}, \\
 A_1 &= \frac{\phi Gr Pr - kSc Gr - Q_l Pr Gr}{(M - \phi Pr)(\phi Pr - kSc)}, \\
 A_2 &= \frac{\phi Gm Pr - kGm Sc + Q_l Pr Gr}{(M - kSc)(\phi Pr - kSc)}, \\
 A_3 &= \frac{Gr[(\phi Pr - kSc)(Pr - 1) - (M - \phi Pr)(Sc - Pr) - Q_l Pr(Pr - 1)]}{(M - \phi Pr)[(\phi Pr - kSc)(Pr - 1) - (M - \phi Pr)(Sc - Pr)]}, \\
 A_4 &= \frac{Gm[(\phi Pr - kSc)(Sc - 1) - (M - kSc)(Sc - Pr)] + Q_l Pr Gr(Sc - 1)}{(M - kSc)[(\phi Pr - kSc)(Sc - 1) - (M - kSc)(Sc - Pr)]}, \\
 A_5 &= \frac{Q_l Pr Gr(Sc - Pr)}{(\phi Pr - kSc)[(\phi Pr - kSc)(Sc - 1) - (M - kSc)(Sc - Pr)]}, \\
 A_6 &= \frac{Q_l Gr Pr(Sc - Pr)}{(\phi Pr - kSc)[(\phi Pr - kSc)(Pr - 1) - (M - \phi Pr)(Sc - Pr)]}.
 \end{aligned}$$

### Nusselt Number

From temperature field, now, we study the Nusselt number which is given in non-dimensional form as follows:

$$Nu = -\left[\frac{d\theta}{dy}\right]_{y=0}. \quad (13)$$

From equations (11) and (13), we get Nusselt number as follows:

$$\begin{aligned}
 Nu &= \left(1 - \frac{b}{c}\right) \left[ \sqrt{\frac{Pr}{\pi t}} \exp(-\phi t) + \sqrt{\phi Pr} \operatorname{erf} \sqrt{\phi t} \right] \\
 &\quad + \frac{b}{c} \left[ \sqrt{\frac{Pr}{\pi t}} \exp(-\phi t) + \exp(ct) \sqrt{\phi Pr + cPr} \operatorname{erf} \sqrt{(\phi + c)t} \right]
 \end{aligned}$$

$$\begin{aligned}
& + \frac{b}{c} \left[ \sqrt{\frac{Sc}{\pi t}} \exp(-kt) + \sqrt{kSc} \operatorname{erf} \sqrt{kt} \right] \\
& - \frac{b}{c} \left[ \sqrt{\frac{Sc}{\pi t}} \exp(-kt) + \exp(ct) \sqrt{kSc + cSc} \operatorname{erf} \sqrt{(k+c)t} \right].
\end{aligned}$$

### Sherwood Number

From concentration field, now, we study Sherwood number which is given in non-dimensional form as follows:

$$Sh = - \left[ \frac{dC}{dy} \right]_{y=0}. \quad (14)$$

From equations (10) and (14), we get Sherwood number as follows:

$$Sh = \sqrt{\frac{Sc}{\pi t}} \exp(-kt) + \sqrt{kSc} \operatorname{erf} \sqrt{kt}.$$

### Results and Discussion

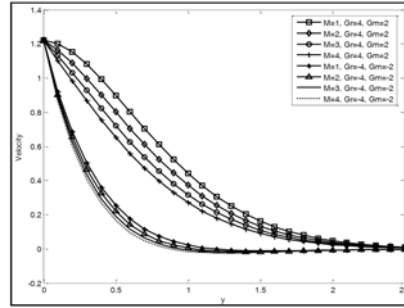
To get a physical insight into the problem, the numerical evaluation of the analytical results reported in the previous section was performed and a set of results is reported graphically in Figures 1-15 for the cases of heating ( $Gr < 0$ ,  $Gm < 0$ ) and cooling ( $Gr > 0$ ,  $Gm > 0$ ) of the plate. The heating and cooling take place by setting up free-convection current due to temperature and concentration gradient. These results are obtained to illustrate the effects of various physical parameters like magnetic parameter  $M$ , absorption radiation parameter  $Q_1$ , chemical reaction parameter  $k$ , Schmidt parameter  $Sc$ , coefficient of heat absorption  $\phi$ , thermal Grashof number  $Gr$  and mass Grashof number  $Gm$  on the velocity, temperature and the concentration profiles.

Figure 1 reveals the effect of magnetic field parameter on fluid velocity. It is observed that an increase in magnetic parameter  $M$ , the velocity decreases in case of cooling and heating of the plate for  $Pr = 0.71$ . It is due to the fact that the application of transverse magnetic field will result a

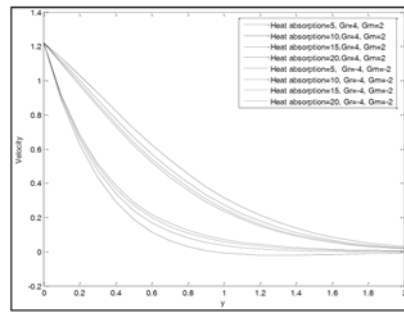
resistive type force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. It is observed from Figures 2 and 3 and Tables 1 and 2 that there is a fall in velocity with increase of heat absorption parameter  $\phi$  or chemical reaction parameter  $k$  or Schmidt number  $Sc$  in case of cooling of the plate while it increases in the case of heating of the plate. Figures 4 and 5 show the effects of  $Q_1$  (radiation absorption parameter),  $Gr$  (thermal Grashof number) and  $Gm$  (mass Grashof number) on the velocity field  $u$ . From these figures, it is found that the velocity  $u$  increases as  $Q_1$  or  $Gr$  or  $Gm$  increases in case of cooling of the plate. It is because that increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow transport and a reverse effect is identified in case of heating of the plate.

Figure 6 reveals the velocity variation with time  $t$  for the cases of both cooling and heating. It is observed that the velocity increases as time  $t$  increases for cooling case whereas in the heating, the velocity increases up to certain  $y$  value and then decreases slowly moving away from the plate. It is seen from Figure 7 that the velocity increases with increase of single acceleration in cases of cooling and heating of the plate.

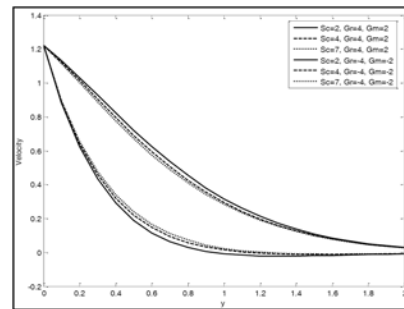
The influence of various flow parameters on the fluid temperature are illustrated in Figures 8-11. From these figures, it is seen that the fluid temperature decreases with an increase in heat absorption parameter  $\phi$  while it increases with increase of chemical reaction parameter  $k$  or radiation absorption parameter  $Q_1$  or time  $t$ . The concentration profiles for different values of  $Sc$  (Schmidt number), chemical reaction parameter  $k$  and time  $t$  are presented through Figures 12 and 14. From these figures, it is observed that the concentration decreases with an increase in  $Sc$  or  $k$  while it increases with time  $t$ . Figure 15 reveals the Sherwood number against time  $t$ . It is found that Sherwood number increases with increasing values of  $Sc$  (Schmidt number) or  $k$  (chemical reaction parameter). Finally, from Table 1 it is seen that Nusselt number increases with an increase in  $Pr$  or  $\phi$  or  $Sc$  or  $k$  and decreases with an increase in  $Q_1$  or  $t$ .



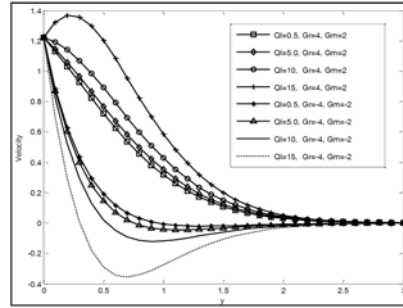
**Figure 1.** Velocity profiles against coordinate  $y$  for different values of magnetic parameter  $M$  with  $Pr = 0.71$ ,  $k = 0.5$ ,  $Sc = 2.01$ ,  $Q_1 = 0.5$ ,  $\phi = 5$ ,  $a = 0.5$  and  $t = 0.4$ .



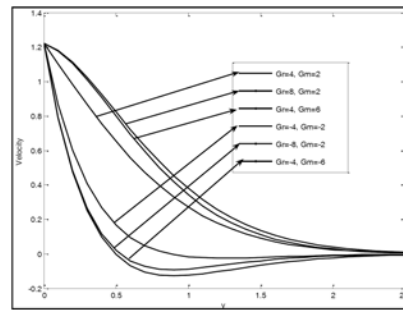
**Figure 2.** Velocity profiles against coordinate  $y$  for different values of heat absorption parameter  $\phi$  with  $Pr = 0.71$ ,  $M = 3$ ,  $k = 0.5$ ,  $Sc = 2.01$ ,  $Q_1 = 0.5$ ,  $a = 0.5$  and  $t = 0.4$ .



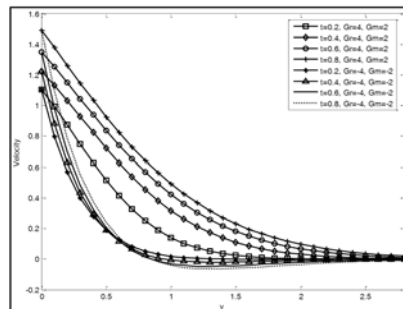
**Figure 3.** Velocity profiles against coordinate  $y$  for different values of Schmidt number  $Sc$  with  $Pr = 0.71$ ,  $k = 0.5$ ,  $M = 3$ ,  $Q_1 = 0.5$ ,  $\phi = 5$ ,  $a = 0.5$  and  $t = 0.4$ .



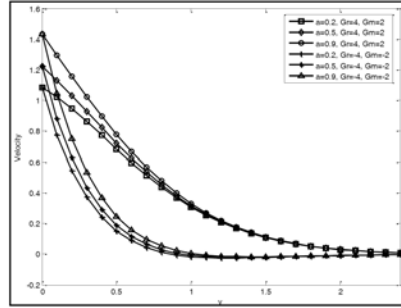
**Figure 4.** Velocity profiles against coordinate  $y$  for different values of radiation absorption parameter  $Q_1$  with  $Pr = 0.71$ ,  $k = 0.5$ ,  $Sc = 2.01$ ,  $M = 3$ ,  $\phi = 5$ ,  $a = 0.5$  and  $t = 0.4$ .



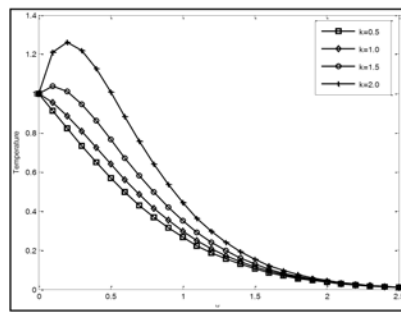
**Figure 5.** Velocity profiles against coordinate  $y$  for different values of  $Gr$  and  $Gm$  with  $Pr = 0.71$ ,  $k = 0.5$ ,  $Sc = 2.01$ ,  $M = 3$ ,  $Q_1 = 0.5$ ,  $\phi = 5$ ,  $a = 0.5$  and  $t = 0.4$ .



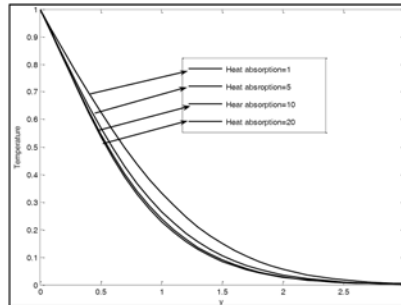
**Figure 6.** Velocity profiles against coordinate  $y$  for different values of time  $t$  with  $Pr = 0.71$ ,  $k = 0.5$ ,  $Sc = 2.01$ ,  $Q_1 = 0.5$ ,  $\phi = 5$ ,  $a = 0.5$ ,  $M = 3$ .



**Figure 7.** Velocity profiles against coordinate  $y$  for different values of accelerated parameter  $a$  with  $Pr = 0.71$ ,  $k = 0.5$ ,  $Sc = 2.01$ ,  $Q_1 = 0.5$ ,  $\phi = 5$ ,  $a = 0.5$  and  $t = 0.4$ .

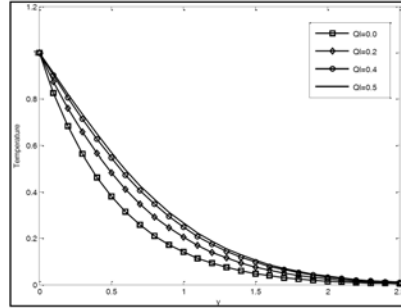


**Figure 8.** Temperature profiles against coordinate  $y$  for different values of chemical reaction parameter with  $Pr = 0.71$ ,  $Sc = 0.6$ ,  $Q_1 = 0.5$ ,  $\phi = 5$  and  $t = 0.4$ .

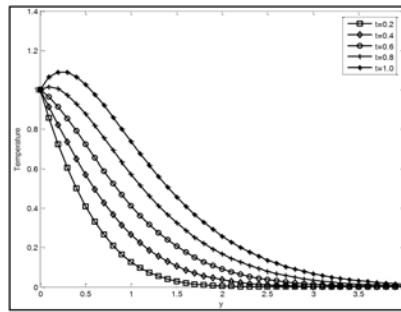


**Figure 9.** Temperature profiles against coordinate  $y$  for different values of heat absorption parameter with  $Pr = 0.71$ ,  $Sc = 0.6$ ,  $Q_1 = 0.5$ ,  $k = 0.5$  and  $t = 0.4$ .

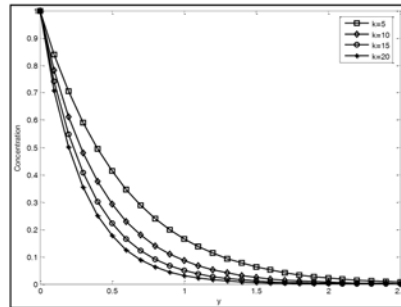




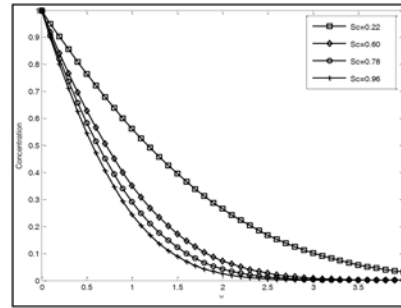
**Figure 10.** Temperature profiles against coordinate  $y$  for different values of chemical reaction parameter with  $Pr = 0.71$ ,  $Sc = 0.6$ ,  $k = 0.5$ ,  $\phi = 5$  and  $t = 0.4$ .



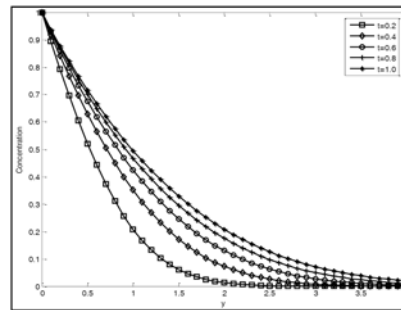
**Figure 11.** Temperature profiles against coordinate  $y$  for different values of time  $t$  with  $Pr = 0.71$ ,  $Sc = 0.6$ ,  $Q_l = 0.5$ ,  $k = 0.5$ ,  $\phi = 5$ .



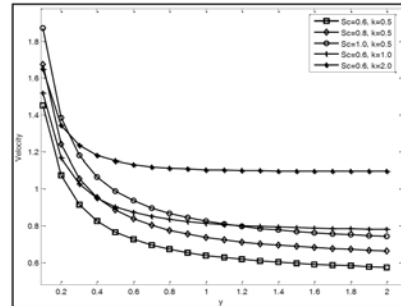
**Figure 12.** Concentration profiles against coordinate  $y$  for different values of chemical reaction parameter.



**Figure 13.** Concentration profiles against coordinate  $y$  for different values of Schmidt number.



**Figure 14.** Concentration profiles against coordinate  $y$  for different values of time  $t$ .



**Figure 15.** Sherwood number.

**Table 1.** Nusselt number

$Pr$	$Q$	$\phi$	$Sc$	$k$	$t$	Nusselt number
0.71	0.5	5	2.01	0.5	0.4	1.55756231125684
0.1	0.5	5	2.01	0.5	0.4	0.66654830455982
0.71	1.0	5	2.01	0.5	0.4	0.45394978493532
0.71	0.5	10	2.01	0.5	0.4	2.38811529402225
0.71	0.5	5	2.01	0.5	0.2	1.84050726903451
0.71	0.5	5	4	0.5	0.4	1.80224032247801
0.71	0.5	5	2.01	0.2	0.4	0.07553216104178

**Table 2.** Velocity for different  $k$  when  $M = 3$ ,  $Q_t = 0.5$ ,  $Sc = 2.01$ ,  $\phi = 5$ ,  $a = 0.5$ ,  $Gr = 4$ ,  $Gm = 2$  and  $t = 0.4$ 

$Pr$	$y$	$k = 0.0$	$k = 0.2$	$k = 0.4$
0.71	0	1.2214	1.2214	1.2214
	0.2000	1.0354	1.0340	1.0327
	0.4000	0.8281	0.8260	0.8240
	0.6000	0.6302	0.6280	0.6259
	0.8000	0.4586	0.4567	0.4549
	1.0000	0.3200	0.3187	0.3173
	1.2000	0.2147	0.2138	0.2129
	1.4000	0.1387	0.1382	0.1377
	1.6000	0.0865	0.0862	0.0859
	1.8000	0.0520	0.0519	0.0517
	2.0000	0.0303	0.0302	0.0301

**Table 3.** Velocity for different  $k$  when  $M = 3$ ,  $Q_l = 0.5$ ,  $Sc = 2.01$ ,  $\phi = 5$ ,  $a = 0.5$ ,  $Gr = -4$ ,  $Gm = -2$  and  $t = 0.4$ 

$Pr$	$y$	$k = 0.0$	$k = 0.2$	$k = 0.4$
0.71	0	1.2214	1.2214	1.2214
	0.2000	0.6240	0.6254	0.6267
	0.4000	0.2881	0.2902	0.2923
	0.6000	0.1107	0.1129	0.1150
	0.8000	0.0249	0.0268	0.0286
	1.0000	-0.0111	-0.0097	-0.0084
	1.2000	-0.0221	-0.0212	-0.0203
	1.4000	-0.0219	-0.0214	-0.0209
	1.6000	-0.0178	-0.0175	-0.0172
	1.8000	-0.0130	-0.0129	-0.0127
	2.0000	-0.0089	-0.0088	-0.0088

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