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SELF CENTERED AND SELF MEDIAN INTUITIONISTIC FUZZY GRAPHS

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Abstract

In this paper, the concept of $\mu\nu$-status, minimum and maximum status of an IFG is defined. The definition of a self median intuitionistic fuzzy graph and the necessary and sufficient conditions for an IFG to be self median are given. Also, we discussed the relationship between self median IFG, self centered IFG and constant IFG. We analyzed the conditions for the IFGs, $G_1 \cup G_2$, $G_1 + G_2$ and $G_1 \circ G_2$ are to be complete.

1. Introduction

Atanassov and Shannon introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs) in [2, 3, 16, 17]. Intuitionistic fuzzy sets have been applied in a wide variety of fields

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including computer science, engineering, mathematics, medicine, chemistry and economics. Karunambigai et al. introduced the concept of minmax IFG, operations of IFG, complements of IFG, constant IFG, totally constant IFG, isomorphism on IFG, isomorphism on strong IFG, self centered IFG and discussed all its properties in [11-15]. These concepts lead us to define status, total status, self median, union and intersection of IFGs and also conditions for the IFGs $G_1 \cup G_2$, $G_1 + G_2$, $G_1 \circ G_2$ to be complete IFG.

A graph G^* is an ordered pair $G^* = (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or lines, which are 2-element subsets of V. Distance-balanced graphs have first been defined by Handa in [10]. Distance-balanced graphs are also called *self median graphs*. An unweighted, connected graph is distance-balanced (also called *self median*) if there exists a number d such that, for any vertex v, the sum of the distances from v to all other vertices is d. A graph is self median if its median is the whole vertex set. Thus, a graph G is self median if and only if the value $d_{G(v)}$ is constant over all vertices v of G. Balakrishnan et al. [5] noticed that a connected graph G is distance-balanced if and only if it is self median. Thus, the concepts of distance-balanced and self median are the same. Distance-balanced graphs are relevant in the area of facility location problems because the median of a graph comprises of vertices that have a minimal sum of distances to all other vertices [4]. They are also useful in mathematical chemistry.

For a scenario, the service facilities like bank, hospital and fire-station should be located in centralized location in a district or an area. When deciding where to locate a service facility, we have to minimize the average distance that a person travelled to the service facility. This is equivalent to minimizing the total distance travelled by all people within the district. For such situations, the concept of median is described.

The status of a vertex v_i is denoted by $S(v_i)$ and is defined as $S(v_i) = \sum_{\forall v_j \in V} \delta(v_i, v_j)$. The total status of a fuzzy graph G is denoted by $t[S(G)]$

and is defined as $t[S(G)] = \sum_{\forall v_i \in V} S(v_i)$. The median of a fuzzy graph *G* is the set of nodes with minimum status. A graph *G* is distance-balanced if and only if *G* is transmission-regular. Let *G* and *H* be nontrivial and regular graphs. Then the symmetric difference $G \oplus H$ is distance-balanced. A fuzzy graph *G* is said to be *self median* if all the vertices have the same status. Every self median fuzzy graph is a self centered fuzzy graph. Every cube Q_n is self median fuzzy graph. The notion of the eccentric digraph of a graph was introduced by Buckley [7]. This construction was refined and extended by others, including Boland and Miller [6], to any digraph. This has led to the study of the behavior of the iterated application of this operator (see Gimbert et al. [9]) (the notion of the eccentric graph of a graph was introduced by Akiyama et al. [1]). The center of a graph is the set of vertices with minimum eccentricity. Graphs in which all vertices are central are called *self centered graphs*. Let *G* be a graph. Then the eccentric digraph $ED(G)$ is symmetric if and only if *G* is self centered. The notion of self centered fuzzy graphs was introduced in [18]. Interconnection networks are universal in today's society, for example, telecommunications networks, flight routes and social networks. The topology of the above network is usually built or designed using directed/undirected graph depending upon application. In all cases, there are some common fundamental characteristics of networks such as the number of nodes, number of connections at each node, total number of connections, clustering of nodes, etc. Many of the most important basic properties, underpinning the functionalities of a network, are related to the distance between the nodes in a network, eccentricities of the nodes, the radius of the network and the diameter of the network (see [8]).

In this paper, we shall survey the results on operations on IFG and present several new results. In the next section, we have given the basic definitions of an intuitionistic fuzzy graph theory, in Section 3, we provide the characterization of self median intuitionistic fuzzy graphs, when the crisp graph is cycle and also the relationship between self median IFG, self centered IFG and constant IFG. In Section 4, we analyzed the conditions for the IFGs $G_1 \cup G_2$, $G_1 + G_2$, $G_1 \circ G_2$ to be complete IFG.

Notations

Throughout this paper, we consider $G : \langle \mu, \nu, V, E \rangle$ as minmax IFG and all the properties are analyzed only for minmax IFG.

2. Basic Definitions

In this section, we give the basic definitions and state the theorems which are used to construct the forthcoming theorems.

Theorem 2.1 [11]. *Every complete intuitionistic fuzzy graph* $G : (V, E)$ *is a self centered IFG and* $r_\mu(G) = \dfrac{1}{\mu_{1i}}$, $r_\nu(G) = \dfrac{1}{\nu_{1i}}$, *where* μ_{1i} *is the least and* ν_{1i} *is the greatest*.

Theorem 2.2 [12]. *Let* $G : \langle \mu, \nu, V, E \rangle$ *be an IFG where crisp graph G is an odd cycle. Then G is a constant IFG if and only if* (μ_2, ν_2) *is a constant function*.

Theorem 2.3 [12]. *Let* $G : \langle \mu, \nu, V, E \rangle$ *be an IFG where crisp graph G is an even cycle. Then G is a constant IFG if and only if either* (μ_2, ν_2) *is a constant function or alternate edges have same membership values and non-membership values*.

Theorem 2.4 [11]. *Let* $G : \langle \mu, \nu, V, E \rangle$ *be a connected IFG with path covers* P_1 *and* P_2 *of G. Then the necessary and sufficient condition for an IFG to be self centered IFG is*

$$\delta_\mu(v_i, v_j) = d_\mu(G), \forall (v_i, v_j) \in P_1 \text{ and}$$

$$\delta_\nu(v_i, v_j) = r_\nu(G), \forall (v_i, v_j) \in P_2. \tag{1}$$

Definition 2.1 [14]. An intuitionistic fuzzy graph (IFG) is of the form $G : \langle \mu, \nu, V, E \rangle$ is said to be a *minmax IFG* if

(i) $V = \{v_0, v_1, ..., v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\nu_1 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$, for every $v_i \in V$ $(i = 1, 2, ..., n)$,

(ii) $E \subseteq V \times V$, where $\mu_2 : V \times V \to [0, 1]$ and $\nu_2 : V \times V \to [0, 1]$ are such that

$$\mu_2(v_i, v_j) \le \min(\mu_1(v_i), \mu_1(v_j)),$$

$$\nu_2(v_i, v_j) \le \max(\nu_1(v_i), \nu_1(v_j))$$

denote the degree of membership and non-membership of an edge $(v_i, v_j) \in E$, respectively, where $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E$.

Definition 2.2 [15]**.** Let $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ and $G_2 : \langle \mu, \nu, V_2, E_2 \rangle$ be two IFGs. Then the join of G_1 and G_2 is an IFG, denoted by $G_1 + G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \cup E' \rangle$ and is defined as

$$(\mu_1 + \mu_1')(v_i) = (\mu_1 \cup \mu_1')(v_i) \text{ if } v_i \in V_1 \cup V_2,$$

$$(\nu_1 + \nu_1')(v_i) = (\nu_1 \cup \nu_1')(v_i) \text{ if } v_i \in V_1 \cup V_2,$$

$$(\mu_2 + \mu_2')(v_i, v_j) = (\mu_2 \cup \mu_2')(v_i, v_j) \text{ if } (v_i, v_j) \in E_1 \cup E_2,$$

$$(\mu_2 + \mu_2')(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)) \text{ if } (v_i, v_j) \in E',$$

$$(\nu_2 + \nu_2')(v_i, v_j) = (\nu_2 \cup \nu_2')(v_i, v_j) \text{ if } (v_i, v_j) \in E_1 \cup E_2,$$

$$(\nu_2 + \nu_2')(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j)) \text{ if } (v_i, v_j) \in E',$$

where E' is the set of all edges joining the vertices of V_1 and V_2.

Definition 2.3 [15]. Let $G = G_1 \times G_2 = \langle \mu, \nu, V, E'' \rangle$ be the Cartesian product of two graphs G_1 and G_2 where $V = V_1 \times V_2$ and

$$E'' = \{(u_1, u_2)(u_1, v_2) : u_1 \in V_1, \forall (u_2, v_2) \in E_2\}$$

$$\cup \{(u_1, v_2)(v_1, v_2) : v_2 \in V_2, \forall (u_1, v_1) \in E_1\}.$$

Then the Cartesian product of G_1 and G_2 is an IFG, denoted by $G_1 \times G_2$ and is defined as $G_1 \times G_2 = \langle \mu, \nu, V, E'' \rangle$, where

(i) $(\mu_1 \times \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2)), \forall (u_1, u_2) \in V_1 \times V_2$ and

$$(\nu_1 \times \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2)), \forall (u_1, u_2) \in V_1 \times V_2.$$

(ii) $(\mu_2 \times \mu_2')(u_1, u_2)(u_1, v_2) = \min(\mu_1(u_1), \mu_2'(u_2, v_2)), \forall u_1 \in V_1$ and $(u_2, v_2) \in E_2$ and $(\nu_2 \times \nu_2')(u_1, u_2)(u_1, v_2) = \max(\nu_1(u_1), \nu_2'(u_2, v_2)),$ $\forall u_1 \in V_1$ and $(u_2, v_2) \in E_2$.

(iii) $(\mu_2 \times \mu_2')(u_1, u_2)(v_1, u_2) = \min(\mu_1'(u_2), \mu_2(u_1, v_1)), \forall u_2 \in V_2$ and $(u_1, v_1) \in E_1$ and $(\nu_2 \times \nu_2')(u_1, u_2)(v_1, u_2) = \max(\nu_1'(u_2), \nu_2(u_1, v_1)), \forall u_2 \in V_2$ and $(u_1, v_1) \in E_1$.

Definition 2.4 [15]. Let $G = G_1 \circ G_2 = \langle \mu, \nu, V, E \rangle$ be the composition of two graphs G_1 and G_2 where $V = V_1 \circ V_2$ and

$$E = \{(u_1, u_2)(u_1, v_2) : u_1 \in V_1, \forall (u_2, v_2) \in E_2\}$$

$$\cup \{(u_1, v_2)(v_1, v_2) : v_2 \in V_2, \forall (u_1, v_1) \in E_1\}$$

$$\cup \{(u_1, u_2)(v_1, v_2) : (u_1, v_1) \in E_1, \forall u_2 \neq v_2\}.$$

Then the composition of G_1 and G_2 is an IFG, denoted by $G_1 \circ G_2$ and is defined as

(i) $(\mu_1 \circ \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2)), \forall (u_1, u_2) \in V_1 \times V_2$ and

$$(\nu_1 \circ \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2)), \forall (u_1, u_2) \in V_1 \times V_2.$$

(ii) $(\mu_2 \circ \mu'_2)(u_1, u_2)(u_1, v_2) = \min(\mu_1(u_1), \mu'_2(u_2, v_2)), \forall u_1 \in V_1$ and $(u_2, v_2) \in E_2$ and $(\nu_2 \circ \nu'_2)(u_1, u_2)(u_1, v_2) = \max(\nu_1(u_1), \nu'_2(u_2, v_2))$, $\forall u_1 \in V_1$ and $(u_2, v_2) \in E_2$.

(iii) $(\mu_2 \circ \mu'_2)(u_1, u_2)(v_1, u_2) = \min(\mu'_1(u_2), \mu_2(u_1, v_1)), \forall u_2 \in V_2$ and $(u_1, v_1) \in E_1$ and $(\nu_2 \circ \nu'_2)(u_1, u_2)(v_1, u_2) = \max(\nu'_1(u_2), \nu_2(u_1, v_1)), \forall u_2 \in V_2$ and $(u_1, v_1) \in E_1$ and $(\mu_2 \circ \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu'_1(u_2), \mu'_1(v_2), \mu_2(u_1, v_1)), \forall (u_1, u_2)(v_1, v_2) \in E - E''$ and

$$(\nu_2 \circ \nu'_2)(u_1, u_2)(v_1, v_2) = \max(\nu'_1(u_2), \nu'_1(v_2), \nu_2(u_1, v_1)),$$

$$\forall (u_1, u_2)(v_1, v_2) \in E - E'',$$

where

$$E'' = \{(u_1, u_2)(u_1, v_2) : u_1 \in V_1, \forall (u_2, v_2) \in E_2\}$$
$$\cup \{(u_1, v_2)(v_1, v_2) : v_2 \in V_2, \forall (u_1, v_1) \in E_1\}.$$

Definition 2.5 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the μ-length of a path $P : v_1, v_2, ..., v_n$ in G, $l_\mu(P)$, is defined as $l_\mu(P) = \sum_{i=1}^{n-1} \frac{1}{\mu_2(v_i, v_{i+1})}$.

Definition 2.6 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the ν-length of a path $P : v_1, v_2, ..., v_n$ in G, $l_\nu(P)$, is defined as $l_\nu(P) = \sum_{i=1}^{n-1} \frac{1}{\nu_2(v_i, v_{i+1})}$.

Definition 2.7 [11]**.** The $\mu\nu$-length of a path $P : v_1, v_2, ..., v_n$ in G, $l_{\mu\nu}(P)$, is defined as $l_{\mu\nu}(P) = (l_\mu, l_\nu)$.

Definition 2.8 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the μ-distance, $\delta_\mu(v_i, v_j)$, is the smallest μ-length of any $v_i - v_j$ path P in G, where $v_i, v_j \in V$. That is, $\delta_\mu(v_i, v_j) = \min(l_\mu(P))$.

Definition 2.9 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the ν-distance, $\delta_\nu(v_i, v_j)$, is the smallest ν-length of any $v_i - v_j$ path P in G, where $v_i, v_j \in V$. That is, $\delta_\nu(v_i, v_j) = \min(l_\nu(P))$.

Definition 2.10 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the distance, $\delta(v_i, v_j)$, is defined as $\delta(v_i, v_j) = (\delta_\mu(v_i, v_j), \delta_\nu(v_i, v_j))$.

Definition 2.11 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. For each $v_i \in V$, the μ-eccentricity of v_i, denoted by $e_\mu(v_i)$ and is defined as $e_\mu(v_i) = \max\{\delta_\mu(v_i, v_j) : v_i \in V, v_i \neq v_j\}$.

Definition 2.12 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. For each $v_i \in V$, the ν-eccentricity of v_i, denoted by $e_\nu(v_i)$ and is defined as $e_\nu(v_i) = \min\{\delta_\nu(v_i, v_j) : v_i \in V, v_i \neq v_j\}$.

Definition 2.13 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. For each $v_i \in V$, the eccentricity of v_i, denoted by $e(v_i)$ and is defined as $e(v_i) = (e_\mu(v_i), e_\nu(v_i))$.

Definition 2.14 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the μ-radius of G is denoted by $r_\mu(G)$ and is defined as $r_\mu(G) = \min\{e_\mu(v_i) : v_i \in V\}$.

Definition 2.15 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the ν-radius of G is denoted by $r_\nu(G)$ and is defined as $r_\nu(G) = \min\{e_\nu(v_i) : v_i \in V\}$.

Definition 2.16 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the radius of G is denoted by $r(G)$ and is defined as $r(G) = (r_\mu(G), r_\nu(G))$.

Definition 2.17 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the μ-diameter of G is denoted by $d_\mu(G)$ and is defined as $d_\mu(G) = \max\{e_\mu(v_i) : v_i \in V\}$.

Definition 2.18 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the ν-diameter of G is denoted by $d_\nu(G)$ and is defined as $d_\nu(G) = \max\{e_\nu(v_i) : v_i \in V\}$.

Definition 2.19 [11]**.** Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the diameter of G is denoted by $d(G)$ and is defined as $d(G) = (d_\mu(G), d_\nu(G))$.

Definition 2.20 [11]**.** A vertex $v_i \in V$ is called a *central vertex* of a connected IFG $G : \langle \mu, \nu, V, E \rangle$, if $r_\mu(G) = e_\mu(v_i)$ and $r_\nu(G) = e_\nu(v_i)$ and the set of all central vertices of an IFG is denoted by $C(G)$.

Definition 2.21 [11]**.** $\langle C(G) \rangle = H : \langle \mu', \nu', V', E' \rangle$ is an IF subgraph of $G : \langle \mu, \nu, V, E \rangle$ induced by the central vertices of G, is called the *center* of G.

Definition 2.22 [11]**.** A connected IFG $G : \langle \mu, \nu, V, E \rangle$ is a self centered graph, if every vertex of G is a central vertex, that is, $r_\mu(G) = e_\mu(v_i)$ and $r_\nu(G) = e_\nu(v_i)$, $\forall v_i \in V$.

3. Self Median Intuitionistic Fuzzy Graphs

In this section, we study briefly about the self median IFG. First, let us understand the basic terminologies which are needed and then we start with the condition for an IFG to be self median. Also, we give the relationship between self median, self centered IFG and constant IFG.

Definition 3.1. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the μ-status of a vertex v_i is denoted by $S_\mu(v_i)$ and is defined as $S_\mu(v_i) = \sum_{\forall v_j \in V} \delta_\mu(v_i, v_j)$.

Definition 3.2. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the ν-status of a vertex v_i is denoted by $S_\nu(v_i)$ and is defined as $S_\nu(v_i) = \sum_{\forall v_j \in V} \delta_\nu(v_i, v_j)$.

Definition 3.3. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the $\mu\nu$-status of a vertex v_i is denoted by $S_{\mu\nu}(v_i)$ and is defined as $S_{\mu\nu}(v_i) = (S_\mu(v_i), S_\nu(v_i))$.

Example 3.1. In the following Figure 3.1, the IFG is $G : \langle \mu, \nu, V, E \rangle$ such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$.

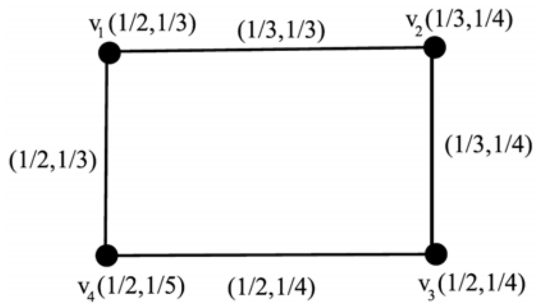


Figure 3.1. Intuitionistic fuzzy graph.

By routine calculations, we have

$\delta_\mu(v_1, v_2) = 3$, $\delta_\mu(v_1, v_3) = 4$, $\delta_\mu(v_1, v_4) = 2$, $\delta_\mu(v_2, v_3) = 3$,

$\delta_\mu(v_2, v_4) = 5$, $\delta_\mu(v_3, v_4) = 2$, $\delta_\nu(v_1, v_2) = 3$, $\delta_\nu(v_1, v_3) = 7$,

$\delta_\nu(v_1, v_4) = 3$, $\delta_\nu(v_2, v_3) = 4$, $\delta_\nu(v_2, v_4) = 6$, $\delta_\nu(v_3, v_4) = 4$,

$S_\mu(v_1) = 9$, $S_\mu(v_2) = 11$, $S_\mu(v_3) = 9$, $S_\mu(v_4) = 9$, $S_\nu(v_1) = 13$,

$S_\nu(v_2) = 13$, $S_\nu(v_3) = 15$, $S_\nu(v_4) = 13$.

Therefore, $S_{\mu\nu}(v_1) = (9, 13)$, $S_{\mu\nu}(v_2) = (11, 13)$, $S_{\mu\nu}(v_3) = (9, 15)$, $S_{\mu\nu}(v_4) = (9, 13)$.

Definition 3.4. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the minimum μ-status of G is denoted by $m[S_\mu(G)]$ and is defined as $m[S_\mu(G)] = \min(S_\mu(v_i)), \forall v_i \in V$.

Definition 3.5. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the minimum ν-status of G is denoted by $m[S_\nu(G)]$ and is defined as $m[S_\nu(G)] = \min(S_\nu(v_i)), \forall v_i \in V$.

Definition 3.6. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the minimum $\mu\nu$-status of G is denoted by $m[S_{\mu\nu}(G)]$ and is defined as $m[S_{\mu\nu}(G)] = (m[S_\mu(G)], m[S_\nu(G)])$.

Definition 3.7. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the maximum μ-status of G is denoted by $M[S_\mu(G)]$ and is defined as $M[S_\mu(G)] = \max(S_\mu(v_i), \forall v_i \in V)$.

Definition 3.8. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the maximum ν-status of G is denoted by $M[S_\nu(G)]$ and is defined as $M[S_\nu(G)] = \max(S_\nu(v_i), \forall v_i \in V)$.

Definition 3.9. Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. Then the maximum $\mu\nu$-status of G is denoted by $M[S_{\mu\nu}(G)]$ and is defined as $M[S_{\mu\nu}(G)] = (M[S_\mu(G)], M[S_\nu(G)])$.

Example 3.2. In the following Figure 3.2, the IFG is $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_1)\}$.

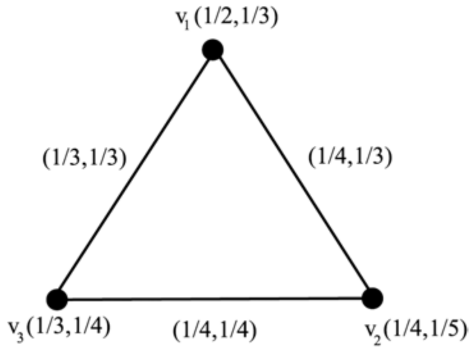


Figure 3.2. Intuitionistic fuzzy graph.

By routine calculations, we have

$$\delta_\mu(v_1, v_2) = 4,\ \delta_\mu(v_1, v_3) = 3,\ \delta_\mu(v_2, v_3) = 4,\ \delta_\nu(v_1, v_2) = 3,$$

$$\delta_\nu(v_1, v_3) = 3,\ \delta_\mu(v_2, v_3) = 4,\ S_\mu(v_1) = 7,\ S_\mu(v_2) = 8,$$

$$S_\mu(v_3) = 7,\ S_\nu(v_1) = 6,\ S_\mu(v_2) = 7,\ S_\mu(v_3) = 7.$$

Therefore, $S_{\mu\nu}(v_1) = (7, 6)$, $S_{\mu\nu}(v_2) = (8, 7)$, $S_{\mu\nu}(v_3) = (7, 7)$. $m[S_{\mu\nu}(G)] = (7, 6)$, $M[S_{\mu\nu}(G)] = (8, 7)$.

Definition 3.10. The total μ-status of an IFG G is denoted by $t[S_\mu(G)]$ and is defined as $t[S_\mu(G)] = \sum_{\forall v_i \in V} S_\mu(v_i)$.

Definition 3.11. The total ν-status of an IFG G is denoted by $t[S_\nu(G)]$ and is defined as $t[S_\nu(G)] = \sum_{\forall v_i \in V} S_\nu(v_i)$.

Definition 3.12. An IFG $G : \langle \mu, \nu, V, E \rangle$ is said to be *self median* if all the vertices have the same status. In other words, G is self median if and only if $m[S_{\mu\nu}(G)] = M[S_{\mu\nu}(G)]$.

Example 3.3. In the following Figure 3.3, the IFG is $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$.

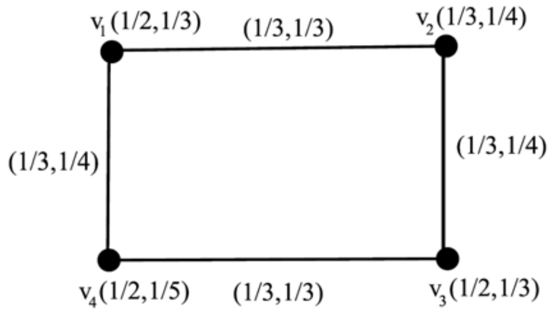


Figure 3.3. Self median IFG.

By routine calculations, we have

$$\delta_\mu(v_1, v_2) = 3,\ \delta_\mu(v_1, v_3) = 6,\ \delta_\mu(v_1, v_4) = 3,\ \delta_\mu(v_2, v_3) = 3,$$

$$\delta_\mu(v_2, v_4) = 6,\ \delta_\mu(v_3, v_4) = 3,\ \delta_\nu(v_1, v_2) = 3,\ \delta_\nu(v_1, v_3) = 7,$$

$$\delta_\nu(v_1, v_4) = 4,\ \delta_\nu(v_2, v_3) = 4,\ \delta_\nu(v_2, v_4) = 7,\ \delta_\nu(v_3, v_4) = 3,$$

$$S_\mu(v_1) = 12,\ S_\mu(v_2) = 12,\ S_\mu(v_3) = 12,\ S_\mu(v_4) = 12,\ S_\nu(v_1) = 14,$$

$$S_\nu(v_2) = 14,\ S_\nu(v_3) = 14,\ S_\nu(v_4) = 14.$$

Therefore, $S_{\mu\nu}(v_1) = (12, 14)$, $S_{\mu\nu}(v_2) = (12, 14)$, $S_{\mu\nu}(v_3) = (12, 14)$, $S_{\mu\nu}(v_4) = (12, 14)$ and $t[S_{\mu\nu}(G)] = (48, 56)$. Here, $S_{\mu\nu}(v_i) = (12, 14)$, $\forall v_i \in V$. Hence G is self median intuitionistic fuzzy graph.

Definition 3.13. Let $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ and $G_2 : \langle \mu, \nu, V_2, E_2 \rangle$ be two IFGs. Then the union of G_1 and G_2 is an IFG, denoted by $G_1 \cup G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \rangle$ and is defined as

$$(\mu_1 \cup \mu_1')(v_i) = \begin{cases} \mu_1(v_i), & v_i \in V_1 - V_2, \\ \mu_1'(v_i), & v_i \in V_2 - V_1, \\ \max(\mu_1(v_i), \mu_1'(v_i)), & v_i \in V_1 \cap V_2, \end{cases}$$

$$(\nu_1 \cup \nu_1')(v_i) = \begin{cases} \nu_1(v_i), & v_i \in V_1 - V_2, \\ \nu_1'(v_i), & v_i \in V_2 - V_1, \\ \min(\nu_1(v_i), \nu_1'(v_i)), & v_i \in V_1 \cap V_2, \end{cases}$$

$$(\mu_2 \cup \mu_2')(v_i, v_j)$$

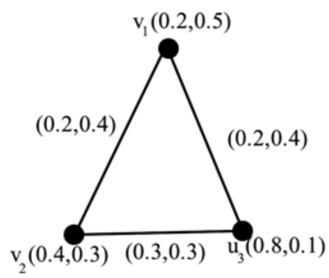
$$= \begin{cases} \mu_{2ij}, & e_{ij} \in E_1 - E_2, \\ \mu_{2ij}', & e_{ij} \in E_2 - E_1, \\ \max(\mu_2(v_i, v_j), \mu_2'(v_i, v_j)), & (v_i, v_j) \in E_1 \cap E_2, \\ \min((\mu_1 \cup \mu_1')(v_i), \max(\mu_1(v_j), \mu_1'(v_j))), & v_i \in V_1 - V_2, v_j \in V_1 \cap V_2 \text{ and} \\ & e_{ij} \in E_1 - E_2 \text{ or } e_{ij} \in E_2 - E_1, \end{cases}$$

$$(\nu_2 \cup \nu_2')(v_i, v_j)$$

$$= \begin{cases} \nu_{2ij}, & e_{ij} \in E_1 - E_2, \\ \nu_{2ij}', & e_{ij} \in E_2 - E_1, \\ \min((\nu_1 \cup \nu_1')(v_i), (\nu_1 \cup \nu_1')(v_j)), & (v_i, v_j) \in E_1 \cap E_2, \\ \max((\nu_1 \cup \nu_1')(v_i), \min(\nu_1(v_j), \nu_1'(v_j))), & v_i \in V_1 - V_2, v_j \in V_1 \cap V_2 \text{ and} \\ & e_{ij} \in E_1 - E_2 \text{ or } e_{ij} \in E_2 - E_1, \end{cases}$$

where (μ_1, ν_1) and (μ_1', ν_1') refer the vertex membership and non-membership of G_1 and G_2, respectively; (μ_2, ν_2) and (μ_2', ν_2') refer the edge membership and non-membership of G_1 and G_2, respectively.

Example 3.4. In the following Figure 3.5, the IFGs are $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ and $G_2 : \langle \mu, \nu, V_2, E_2 \rangle$ such that $V_1 = \{v_1, v_2, u_3\}$, $E_1 = \{(v_1, v_2), (v_1, u_3), (u_3, v_2)\}$, $V_2 = \{v_1, v_2, v_3, v_4\}$, $E_2 = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$ and $V_1 \cup V_2 = \{v_1, v_2, v_3, v_4, u_3\}$, $E_1 \cup E_2 = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, u_3), (u_3, v_2)\}$.



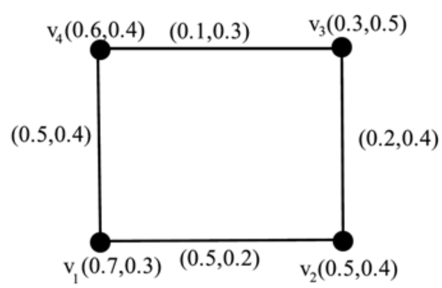


Figure 3.4. G_1 and G_2.

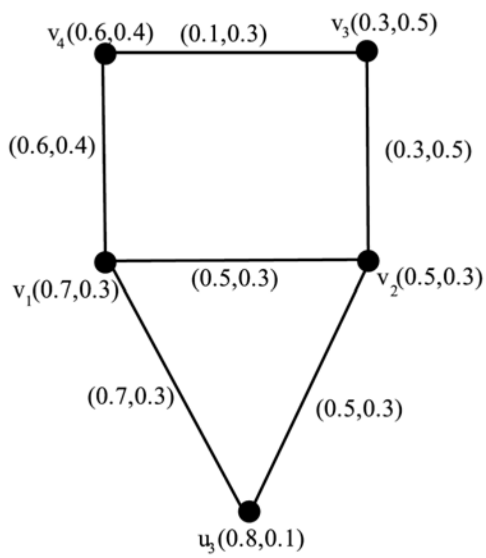


Figure 3.5. $G_1 \bigcup G_2$.

Definition 3.14. Let $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ and $G_2 : \langle \mu, \nu, V_2, E_2 \rangle$ be two IFGs. Then the intersection of G_1 and G_2 is an IFG, denoted by $G_1 \bigcap G_2 = \langle V_1 \bigcap V_2, E_1 \bigcap E_2 \rangle$ and is defined as

$$(\mu_1 \cap \mu_1')(v_i) = \begin{cases} \mu_1(v_i), & v_i \in V_1 - V_2, \\ \mu_1'(v_i), & v_i \in V_2 - V_1, \\ \min(\mu_1(v_i), \mu_1'(v_i)), & v_i \in V_1 \bigcap V_2, \end{cases}$$

$$(\nu_1 \cap \nu_1')(v_i) = \begin{cases} \nu_1(v_i), & v_i \in V_1 - V_2, \\ \nu_1'(v_i), & v_i \in V_2 - V_1, \\ \max(\nu_1(v_i),\ \nu_1'(v_i)), & v_i \in V_1 \cap V_2, \end{cases}$$

$$(\mu_2 \cap \mu_2')(v_i,\ v_j)$$

$$= \begin{cases} \mu_{2ij}, & e_{ij} \in E_1 - E_2, \\ \mu_{2ij}', & e_{ij} \in E_2 - E_1, \\ \min((\nu_1 \cap \nu_1')(v_i),\ \min(\nu_1(v_j),\ \nu_1'(v_j))), & v_i \in V_1 - V_2,\ v_j \in V_1 \cap V_2 \text{ and} \\ & e_{ij} \in E_1 - E_2 \text{ or } e_{ij} \in E_2 - E_1, \\ \min(\mu_2(v_i,\ v_j),\ \mu_2'(v_i,\ v_j)), & (v_i,\ v_j) \in E_1 \cap E_2, \end{cases}$$

$$(\nu_2 \cap \nu_2')(v_i,\ v_j)$$

$$= \begin{cases} \nu_{2ij}, & e_{ij} \in E_1 - E_2, \\ \nu_{2ij}', & e_{ij} \in E_2 - E_1, \\ \max((\nu_1 \cap \nu_1')(v_i),\ \max(\nu_1(v_j),\ \nu_1'(v_j))), & v_i \in V_1 - V_2,\ v_j \in V_1 \cap V_2 \text{ and} \\ & e_{ij} \in E_1 - E_2 \text{ or } e_{ij} \in E_2 - E_1, \\ \max(\nu_2(v_i,\ v_j),\ \nu_2'(v_i,\ v_j)), & (v_i,\ v_j) \in E_1 \cap E_2, \end{cases}$$

where $(\mu_1,\ \nu_1)$ and $(\mu_1',\ \nu_1')$ refer the vertex membership and non-membership of G_1 and G_2, respectively; $(\mu_2,\ \nu_2)$ and $(\mu_2',\ \nu_2')$ refer the edge membership and non-membership of G_1 and G_2, respectively.

Example 3.5. In the following Figure 3.6, the IFGs are $G_1 : \langle \mu,\ \nu,\ V_1,\ E_1 \rangle$ and $G_2 : \langle \mu,\ \nu,\ V_2,\ E_2 \rangle$ such that $V_1 = \{v_1,\ v_2,\ u_3\}$, $E_1 = \{(v_1,\ v_2),\ (v_1,\ u_3),\ (u_3,\ v_2)\}$, $V_2 = \{v_1,\ v_2,\ v_3,\ v_4\}$, $E_2 = \{(v_1,\ v_2),\ (v_2,\ v_3),\ (v_3,\ v_4),\ (v_4,\ v_1)\}$ and $V_1 \cap V_2 = \{v_1,\ v_2,\ v_3,\ v_4,\ u_3\}$, $E_1 \cap E_2 = \{(v_1,\ v_2),\ (v_2,\ v_3),\ (v_3,\ v_4),\ (v_4,\ v_1),\ (v_1,\ u_3),\ (u_3,\ v_2)\}$.

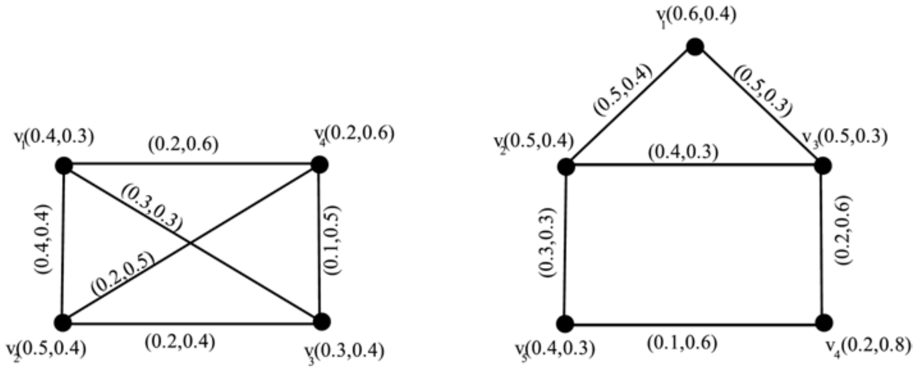


Figure 3.6. G_1 and G_2.

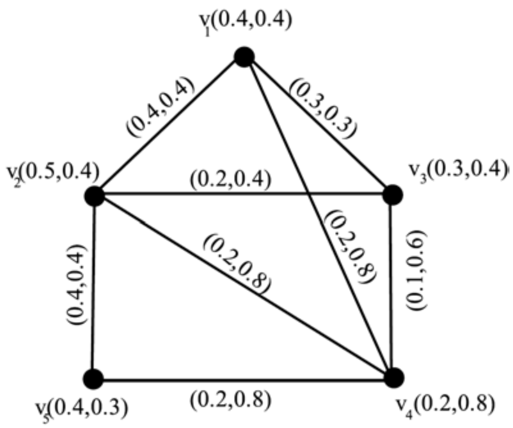


Figure 3.7. $G_1 \cap G_2$.

Theorem 3.1. *Let $G : (\mu, \nu, V, E)$ be an IFG where crisp graph G is an odd cycle. Then G is self median IFG if and only if (μ_2, ν_2) is a constant function.*

Proof. If (μ_2, ν_2) is a constant function, say $\mu_2 = c_1$ and $\nu_2 = c_2$ for all $v_i, v_j \in E$, then $S_\mu(v_i) = l_1$, $S_\nu(v_i) = l_2$, $\forall v_i \in V$. Hence G is self median IFG.

Conversely, suppose that G is self median IFG.

Let $e_1, e_2, \ldots, e_{2n+1}$ be the edges of G in that order. Let us assume that (μ_2, ν_2) is not a constant function. Then $S_\mu(v_i) \neq S_\mu(v_j)$ and $S_\nu(v_i) \neq S_\nu(v_j)$, for some i, j, where $i \neq j$, $i, j = 1, 2, \ldots, n$. Hence G is not self median, which is a contradiction. Therefore, (μ_2, ν_2) is a constant. □

Theorem 3.2. *Let $G : \langle \mu, \nu, V, E \rangle$ be an IFG where crisp graph G is an even cycle. Then G is self median IFG if and only if either (μ_2, ν_2) is a constant function or alternate edges have same membership values and non-membership values.*

Proof. If either (μ_2, ν_2) is a constant function or alternate edges have same membership values and non-membership values, then G is a self median IFG. Conversely, suppose G is a self median IFG. Let $e_1, e_2, \ldots, e_{2n}$ be the edges of even cycle G^* in that order. Since G is a self median IFG and also crisp graph G is an even cycle, $S_\mu(v_i)$ is same for all $i = 1, 2, \ldots, n$. That is $S_\mu(v_1) = S_\mu(v_2) = \cdots = S_\mu(v_n)$. Let us assume that (μ_2, ν_2) is not a constant function and alternate edges have different membership values and non-membership values. Then $S_\mu(v_i) \neq S_\mu(v_j)$ and $S_\nu(v_i) \neq S_\nu(v_j)$, for some i, j, where $i \neq j$, $i, j = 1, 2, \ldots, n$. Hence G is not a self median, which is a contradiction. Therefore, (μ_2, ν_2) is a constant function and alternate edges have same membership values and non-membership values. □

Proposition 3.3. *Every complete IFG is not a self median IFG.*

Theorem 3.4. *In a complete IFG $G : \langle \mu, \nu, V, E \rangle$, $M[S_\mu(G)] = (n-1)r_\mu(G)$, $m[S_\nu(G)] = (n-1)r_\nu(G)$.*

Proof. Let $G : \langle \mu, \nu, V, E \rangle$ be a complete IFG and n be the number of vertices in G, say $v_1, v_2, \ldots, v_n$. Let $\mu_1(v_1)$ be the least. Then $\mu_2(v_1, v_i) = \mu_1(v_1)$, $\forall i = 2, 3, \ldots, n$, which implies $\delta_\mu(v_1, v_i) = \frac{1}{\mu_1(v_1)}$ and $\delta_\mu(v_i, v_j)$

$= \frac{1}{\mu_{2ij}}$, where $i \neq j$, $i = 2, 3, \ldots, n$, $j = 1, 2, \ldots, n$. But

$$\frac{1}{\mu_1(v_1)} > \frac{1}{\mu_{2ij}}, \text{ since } \mu_1(v_1) \text{ is least.} \tag{2}$$

Therefore, $e_\mu(v_i) = \max(\delta_\mu(v_i, v_j)) = \frac{1}{\mu_1(v_1)}$, where $i \neq j$, $\forall i$, $j = 1, 2, \ldots, n$, and $r_\mu(G) = \min(e_\mu(v_i)) = \frac{1}{\mu_1(v_1)}$. The μ-status is given by $S_\mu(v_1) = \sum_{v_1 \neq v_i} \delta_\mu(v_1, v_i) = (n-1) \cdot \frac{1}{\mu_1(v_1)}$, $\forall i = 2\ 3, \ldots, n$, since G is complete and $S_\mu(v_i) = \sum_{v_i \neq v_j} \delta_\mu(v_i, v_j)$, where $i = 2, 3, \ldots, n$ and $j = 1, 2, \ldots, n$. But by equation (2), $S_\mu(v_1) > S_\mu(v_i)$, $\forall i = 2, 3, \ldots, n$. Therefore,

$$\begin{aligned} M[S_\mu(G)] &= S_\mu(v_1) \\ &= \sum_{i=2}^{n} \delta_\mu(v_1, v_i) \\ &= (n-1) \cdot \frac{1}{\mu_1(v_1)} \\ M[S_\mu(G)] &= (n-1) r_\mu(G). \end{aligned}$$

Let $\nu_1(v_1)$ be the greatest. Then $\nu_2(v_1, v_i) = \nu_1(v_1)$, $\forall i = 2, 3, \ldots, n$ which implies $\delta_\nu(v_1, v_i) = \frac{1}{\nu_1(v_1)}$ and $\delta_\nu(v_i, v_j) = \frac{1}{\nu_{2ij}}$, where $i \neq j$, $i = 2, 3, \ldots, n$, $j = 1, 2, \ldots, n$. But

$$\frac{1}{\nu_1(v_1)} < \frac{1}{\nu_{2ij}}, \text{ since } \nu_1(v_1) \text{ is greatest.} \tag{3}$$

Therefore, $e_\nu(v_i) = \min(\delta_\nu(v_i, v_j)) = \frac{1}{\nu_1(v_1)}$, where $i \neq j$, $\forall i$, $j = 1, 2,$

..., n, and $r_\nu(G) = \min(e_\mu(v_i)) = \dfrac{1}{\nu_1(v_1)}$. The ν-status is given by $S_\nu(v_1) = \sum_{v_1 \neq v_i} \delta_\nu(v_1, v_i) = (n-1) \cdot \dfrac{1}{\nu_1(v_1)}$, $\forall i = 2, 3, ..., n$, since G is complete and $S_\nu(v_i) = \sum_{v_i \neq v_j} \delta_\nu(v_i, v_j)$, where $i = 2, 3, ..., n$ and $j = 1, 2, ..., n$. But by equation (3), $S_\nu(v_1) < S_\nu(v_i)$, $\forall i = 2, 3, ..., n$. Therefore,

$$m[S_\mu(G)] = S_\nu(v_1)$$

$$= \sum_{i=2}^{n} \delta_\nu(v_1, v_i)$$

$$= (n-1) \cdot \frac{1}{\nu_1(v_1)}$$

$$m[S_\mu(G)] = (n-1) r_\nu(G). \qquad \square$$

Theorem 3.5. *Every constant IFG is a self centered IFG.*

Proof. Let $G : \langle \mu, \nu, V, E \rangle$ be a constant IFG and let P_1, P_2 be the path covers of G.

Case (i). Let the crisp graph G be of odd cycle. By Theorem 2.2, (μ_2, ν_2) is a constant function. If (μ_2, ν_2) is a constant function, then $\delta_\mu(v_i, v_j) = d_\mu(G)$, $\forall (v_i, v_j) \in P_1$ and $\delta_\nu(v_i, v_j) = r_\mu(G)$, $\forall (v_i, v_j) \in P_2$ which implies that $e_\mu(v_i) = k_1$ and $e_\nu(v_i) = k_2$, $\forall v_i \in V$. Therefore, $r_\mu(G) = e_\mu(v_i)$, $r_\nu(G) = e_\nu(v_i)$. Hence G is self centered IFG.

Case (ii). Let the crisp graph $G : (\mu, \nu, V, E)$ be of even cycle. By Theorem 2.3, either (μ_2, ν_2) is a constant function or alternate edges have same membership and non-membership values. Suppose that (μ_2, ν_2) is a constant function, then $e_\mu(v_i) = k_1$, $e_\nu(v_i) = k_2$, $\forall v_i \in V$. Therefore, $r_\mu(G) = e_\mu(v_i)$, $r_\nu(G) = e_\nu(v_i)$. Hence G is self centered IFG.

Now suppose that the alternate edges have the same membership and non-membership values. Then $\delta_\mu(v_i, v_j) = d_\mu(G)$, $\forall (v_i, v_j) \in P_1$ and $\delta_\nu(v_i, v_j) = r_\mu(G)$, $\forall (v_i, v_j) \in P_2$ which implies that $e_\mu(v_i) = k_1$ and $e_\nu(v_i) = k_2$, $\forall v_i \in V$. Therefore, $r_\mu(G) = e_\mu(v_i)$, $r_\nu(G) = e_\nu(v_i)$. Hence G is self centered IFG. □

Theorem 3.6. *Every self median IFG is a self centered IFG.*

Proof. Proof follows from Theorems 2.2, 2.3, 3.1, 3.2 and then from Theorem 3.5. □

Note 3.7. Every self centered IFG is not necessarily self median IFG.

Theorem 3.8. *Every constant IFG is a self median IFG.*

Proof. Proof follows from Theorems 2.2, 2.3, 3.1 and 3.2. □

4. Operations on Complete Intuitionistic Fuzzy Graphs

In this section, we study about the operations on complete IFG. We can conceptualize about the operation of two IFGs G_1 and G_2 to be the complete IFG or not by considering G_1 and G_2 are complete IFG and also we discussed for self centered IFG and self median IFG under the same consideration.

Theorem 4.1. *Let* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *be complete IFG. Then* G_1 *and* G_2 *are complete IFG if and only if* $G_1 \circ G_2$ *is a complete IFG.*

Proof. Given that $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ and $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ are complete IFG, where (μ_1, ν_1) and (μ_2, ν_2) are membership and non-membership values of a vertex and edge in G_1, (μ_1', ν_1') and (μ_2', ν_2') are membership and non-membership values of a vertex and an edge in G_2. Let $G = G_1 \circ G_2 = (V, E)$ be the composition of two graphs G_1 and G_2, where $V = V_1 \times V_2$ and

$$\begin{aligned} E = &\{(u_1, u_2)(u_1, v_2) : u_1 \in V_1, \forall (u_2, v_2) \in E_2\} \\ &\cup \{(u_1, v_2)(v_1, v_2) : v_2 \in V_2, \forall (u_1, v_1) \in E_1\} \\ &\cup \{(u_1, u_2)(v_1, v_2) : (u_1, v_1) \in E_1, \forall u_2 \neq v_2\}. \end{aligned}$$

Let $u_1, v_1 \in V_1$ and $u_2, v_2 \in V_2$. Then by the definition of composition of IFGs G_1 and G_2, we have

$$(\mu_1 \circ \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2)),$$

$$(\mu_1 \circ \mu_1')(u_1, v_2) = \min(\mu_1(u_1), \mu_1'(v_2)),$$

$$(\mu_1 \circ \mu_1')(v_1, v_2) = \min(\mu_1(v_1), \mu_1'(v_2)),$$

$$(\mu_1 \circ \mu_1')(v_1, u_2) = \min(\mu_1(v_1), \mu_1'(u_2)),$$

$$\begin{aligned} (\mu_2 \circ \mu_2')(u_1, u_2)(u_1, v_2) &= \min(\mu_1(u_1), \mu_2'(u_2, v_2)) \\ &= \min(\mu_1(u_1), \min(\mu_1'(u_2), \mu_1'(v_2))), \\ &\quad \text{since } G_2 \text{ is complete} \\ &= \min(\min(\mu_1(u_1), \mu_1'(u_2)), \min(\mu_1(u_1), \mu_1'(v_2))) \\ &= \min((\mu_1 \circ \mu_1')(u_1, u_2), (\mu_1 \circ \mu_1')(u_1, v_2)), \\ &\quad \forall u_1 \in V_1 \text{ and } (u_2, v_2) \in E_2, \end{aligned} \tag{4}$$

$$\begin{aligned} (\mu_2 \circ \mu_2')(u_1, u_2)(v_1, u_2) &= \min(\mu_1'(u_2), \mu_2(u_1, v_1)) \\ &= \min(\mu_1'(u_2), \min(\mu_1(u_1), \mu_1(v_1))), \\ &\quad \text{since } G_1 \text{ is complete} \end{aligned}$$

$$= \min(\min(\mu_1(u_1), \mu_1'(u_2)), \min(\mu_1(v_1), \mu_1'(u_2)))$$

$$= \min((\mu_1 \circ \mu_1')(u_1, u_2), (\mu_1 \circ \mu_1')(v_1, u_2)),$$

$$\forall u_2 \in V_2 \text{ and } (u_1, v_1) \in E_1, \tag{5}$$

$$(\mu_2 \circ \mu_2')(u_1, u_2)(v_1, v_2) = \min(\mu_1'(u_2), \mu_1'(v_2), \mu_2(u_1, v_1))$$

$$= \min(\mu_1'(u_2), \mu_1'(v_2), \min(\mu_1(u_1), \mu_1(v_1))),$$

$$\text{since } G_1 \text{ is complete}$$

$$= \min(\mu_1'(u_2), \mu_1'(v_2), \mu_1(u_1), \mu_1(v_1))$$

$$= \min(\min(\mu_1(u_1), \mu_1'(u_2)), \min(\mu_1(v_1), \mu_1'(v_2)))$$

$$= \min((\mu_1 \circ \mu_1')(u_1, u_2), (\mu_1 \circ \mu_1')(v_1, v_2)),$$

$$\forall (u_1, u_2)(v_1, v_2) \in E - E'', \tag{6}$$

where

$$E'' = \{(u_1, u_2)(u_1, v_2) : u_1 \in V_1, \forall (u_2, v_2) \in E_2\}$$

$$\bigcup \{(u_1, v_2)(v_1, v_2) : v_2 \in V_2, \forall (u_1, v_1) \in E_1\},$$

$$(\nu_1 \circ \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2)),$$

$$(\nu_1 \circ \nu_1')(u_1, v_2) = \max(\nu_1(u_1), \nu_1'(v_2)),$$

$$(\nu_1 \circ \nu_1')(v_1, v_2) = \max(\nu_1(v_1), \nu_1'(v_2)),$$

$$(\nu_1 \circ \nu_1')(v_1, u_2) = \max(\nu_1(v_1), \nu_1'(u_2)),$$

$$(\nu_2 \circ \nu_2')(u_1, u_2)(u_1, v_2) = \max(\nu_1(u_1), \nu_2'(u_2, v_2))$$

$$= \max(\nu_1(u_1), \max(\nu_1'(u_2), \nu_1'(v_2))),$$

$$\text{since } G_2 \text{ is complete}$$

$$= \max(\max(\nu_1(u_1), \nu_1'(u_2)), \max(\nu_1(u_1), \nu_1'(v_2)))$$

$$= \max((\nu_1 \circ \nu_1')(u_1, u_2), (\nu_1 \circ \nu_1')(u_1, v_2)),$$

$$\forall u_1 \in V_1 \text{ and } (u_2, v_2) \in E_2, \tag{7}$$

$$(\nu_2 \circ \nu_2')(u_1, u_2)(v_1, u_2) = \max(\nu_1'(u_2), \nu_2(u_1, v_1))$$

$$= \max(\nu_1'(u_2), \max(\nu_1(u_1), \nu_1(v_1))),$$

since G_1 is complete

$$= \max(\max(\nu_1(u_1), \nu_1'(u_2)), \max(\nu_1(v_1), \nu_1'(u_2)))$$

$$= \max((\nu_1 \circ \nu_1')(u_1, u_2), (\nu_1 \circ \nu_1')(v_1, u_2)),$$

$$\forall u_2 \in V_2 \text{ and } (u_1, v_1) \in E_1, \tag{8}$$

$$(\nu_2 \circ \nu_2')(u_1, u_2)(v_1, v_2) = \max(\nu_1'(u_2), \nu_1'(v_2), \nu_2(u_1, v_1))$$

$$= \max(\nu_1'(u_2), \nu_1'(v_2), \max(\nu_1(u_1), \nu_1(v_1))),$$

since G_1 is complete

$$= \max(\nu_1'(u_2), \nu_1'(v_2), \nu_1(u_1), \nu_1(v_1))$$

$$= \max(\max(\nu_1(u_1), \nu_1'(u_2)), \max(\nu_1(v_1), \nu_1'(v_2)))$$

$$= \max((\nu_1 \circ \nu_1')(u_1, u_2), (\nu_1 \circ \nu_1')(v_1, v_2)),$$

$$\forall (u_1, u_2)(v_1, v_2) \in E - E'', \tag{9}$$

where

$$E'' = \{(u_1, u_2)(u_1, v_2) : u_1 \in V_1, \forall (u_2, v_2) \in E_2\}$$

$$\bigcup \{(u_1, v_2)(v_1, v_2) : v_2 \in V_2, \forall (u_1, v_1) \in E_1\}.$$

From equations (4)-(8) and (9), $G_1 \circ G_2$ is a complete IFG.

Conversely, if $G_1 \circ G_2$ is a complete IFG, then we need to claim that G_1 and G_2 are complete IFG. Suppose that G_1 and G_2 are not complete IFG. Then by the definition of composition of graphs, there is no edge existing between some vertices, which implies that $G_1 \circ G_2$ is not a complete IFG. It is a contradiction to the fact that $G_1 \circ G_2$ is a complete IFG. Hence G_1 and G_2 are complete IFG. □

Theorem 4.2. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 \circ G_2$ *is a self centered IFG.*

Proof. Proof follows from Theorem 4.1 and then from Theorem 2.1. □

Theorem 4.3. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 \circ G_2$ *is not a self median IFG.*

Proof. Proof follows from Theorem 4.1 and then from Proposition 3.3. □

Theorem 4.4. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 \times G_2$ *is a* ν*-self centered IFG.*

Proof. Given that $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ and $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ are complete IFG, where (μ_1, ν_1) and (μ_2, ν_2) are membership and non-membership values of a vertex and edge in G_1, (μ_1', ν_1') and (μ_2', ν_2') are membership and non-membership values of a vertex and edge in G_2. Let $G = G_1 \times G_2 = (V_1 \times V_2, E)$ be the Cartesian product of two graphs G_1 and G_2, where

$$E = \{(u_1, u_2)(u_1, v_2) : u_1 \in V_1, \forall (u_2, v_2) \in E_2\}$$

$$\cup \{(u_1, v_2)(v_1, v_2) : v_2 \in V_2, \forall (u_1, v_1) \in E_1\}$$

$$\cup \{(u_1, u_2)(v_1, v_2) : (u_1, v_1) \in E_1, \forall u_2 \neq v_2\}.$$

Let $u_1, v_1 \in V_1$ and $u_2, v_2 \in V_2$. Then by the definition of Cartesian products of IFGs $G_1 \times G_2$, we have

$$(\nu_1 \times \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2)),$$

$$(\nu_1 \times \nu_1')(u_1, v_2) = \max(\nu_1(u_1), \nu_1'(v_2)),$$

$$(\nu_1 \times \nu_1')(v_1, v_2) = \max(\nu_1(v_1), \nu_1'(v_2)),$$

$$(\nu_1 \times \nu_1')(v_1, u_2) = \max(\nu_1(v_1), \nu_1'(u_2)),$$

$$\begin{aligned}(\nu_2 \times \nu_2')(u_1, u_2)(u_1, v_2) &= \max(\nu_1(u_1), \nu_2'(u_2, v_2)) \\ &= \max(\nu_1(u_1), \max(\nu_1'(u_2), \nu_1'(v_2))) \\ &= \max(\max(\nu_1(u_1), \nu_1'(u_2)), \max(\nu_1'(u_1), \nu_1'(v_2))) \\ &= \max((\nu_1 \times \nu_1')(u_1, u_2), (\nu_1 \times \nu_1')(u_1, v_2)), \\ &\quad \forall u_1 \in V_1 \text{ and } (u_2, v_2) \in E_2, \end{aligned} \tag{10}$$

$$\begin{aligned}(\nu_2 \times \nu_2')(u_1, u_2)(v_1, u_2) &= \max(\nu_1'(u_2), \nu_2(u_1, v_1)) \\ &= \max(\nu_1'(u_2), \max(\nu_1(u_1), \nu_1(v_1))) \\ &= \max(\max(\nu_1(u_1), \nu_1'(u_2)), \max(\nu_1(v_1), \nu_1'(u_2))) \\ &= \max((\nu_1 \times \nu_1')(u_1, u_2), (\nu_1 \times \nu_1')(v_1, u_2)), \\ &\quad \forall u_2 \in V_2 \text{ and } (u_1, v_1) \in E_1. \end{aligned} \tag{11}$$

Hence $G_1 \times G_2$ is a ν-strong IFG. Let $u_1 \in V_1$ and $\nu_1(u_1) = l$ be the greatest value among all other vertices in V. Then

$$\nu_1(u_1, x) = l, \quad \forall x \in V_2 \tag{12}$$

and

$$\nu_2(u_1, x)(u_k, v_j) = l, \quad \forall (u_k, v_j) \in V, \text{ since } G_1 \times G_2 \text{ is a } \nu\text{-strong IFG.} \tag{13}$$

Then there exists a path cover P such that every vertex of $G : \langle \mu, \nu, V, E \rangle$ is incident to some path of P. Therefore, from equations (12) and (13),

$$\nu_2(u_1, x)(u_k, v_j) = l, \quad \forall (u_1, x)(u_k, v_j) \in P. \tag{14}$$

Hence $\frac{1}{\nu_2(u_1, u_2)(u_k, v_j)} = \frac{1}{l}$, which is the least. That is,

$$\delta_\nu(u_1, x)(u_k, v_j) = r_\nu(G), \quad \forall (u_1, x)(u_k, v_j) \in P. \tag{15}$$

Hence by Theorem 2.4, $G : \langle \mu, \nu, V, E \rangle$ is a ν-self centered IFG. □

Corollary 4.5. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 \times G_2$ *is not necessarily a self centered IFG.*

Example 4.1. The following example proves that $G_1 \times G_2$ is not a self centered IFG. Consider an IFG, $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ and $G_2 : \langle \mu, \nu, V_2, E_2 \rangle$, such that $V_1 = \{v_1, v_2, v_3\}$, $E_1 = \{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}$, and $V_2 = \{u_1, u_2\}$, $E_2 = \{(u_1, u_2)\}$.

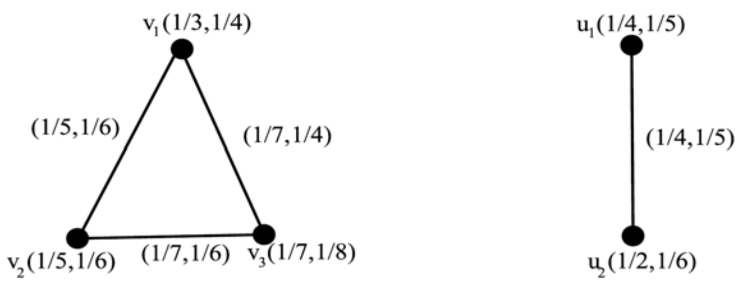


Figure 4.1. G_1 and G_2.

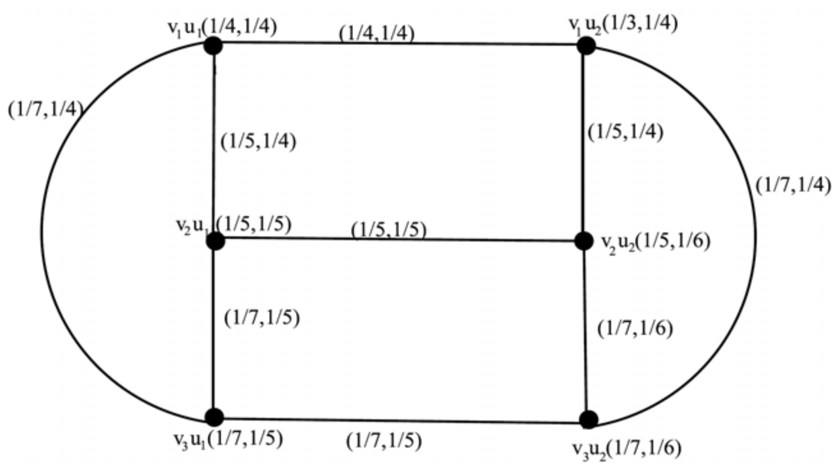


Figure 4.2. $G_1 \times G_2$.

$\delta(v_1u_1, v_1u_2) = (4, 4)$, $\delta(v_1u_1, v_2u_1) = (5, 4)$, $\delta(v_1u_1, v_3u_1) = (7, 4)$,

$\delta(v_1u_1, v_2u_2) = (9, 8)$, $\delta(v_1u_1, v_3u_2) = (11, 8)$, $\delta(v_1u_2, v_2u_2) = (5, 4)$,

$\delta(v_1u_2, v_2u_1) = (9, 8)$, $\delta(v_1u_2, v_3u_1) = (11, 8)$, $\delta(v_1u_2, v_3u_2) = (7, 4)$,

$\delta(v_2u_1, v_2u_2) = (5, 5)$, $\delta(v_2u_1, v_3u_1) = (7, 5)$, $\delta(v_2u_1, v_3u_2) = (12, 10)$,

$\delta(v_2u_2, v_3u_1) = (12, 10)$, $\delta(v_2u_2, v_3u_2) = (7, 6)$, $\delta(v_3u_1, v_3u_2) = (7, 5)$,

$e(v_1u_1) = (11, 4)$, $e(v_1u_2) = (11, 4)$, $e(v_2u_1) = (12, 4)$, $e(v_2u_2) = (12, 4)$,

$e(v_3u_1) = (12, 4)$, $e(v_3u_2) = (12, 4)$, $r(G) = (11, 4)$, $d(G) = (12, 4)$.

Hence $G_1 \times G_2$ is not self centered IFG.

Corollary 4.6. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 \times G_2$ *is not necessarily a self median IFG.*

Example 4.2. From Figures 4.1 and 4.2, $S_{\mu\nu}(v_1u_1) = (36, 28)$, $S_{\mu\nu}(v_1u_2) = (36, 28)$, $S_{\mu\nu}(v_2u_1) = (38, 32)$, $S_{\mu\nu}(v_2u_2) = (38, 33)$, $S_{\mu\nu}(v_3u_1) = (44, 32)$, $S_{\mu\nu}(v_3u_2) = (44, 33)$. Here $S_{\mu\nu}(v_1u_1) = S_{\mu\nu}(v_1u_2)$, $S_{\mu\nu}(v_1u_1) \neq S_{\mu\nu}(v_2u_1) \neq S_{\mu\nu}(v_2u_2) \neq S_{\mu\nu}(v_3u_1) \neq S_{\mu\nu}(v_3u_2)$. Hence $G_1 \times G_2$ is not a self median IFG.

Theorem 4.7. *Let* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *be IFG. Then* G_1 *and* G_2 *are complete IFG if and only if* $G_1 + G_2$ *is a complete IFG.*

Proof. If G_1 and G_2 are complete, then we claim that $G_1 + G_2$ is a complete IFG. If G_1 has n-vertices and G_2 has m-vertices, then $G_1 + G_2$ has $m + n$-vertices and each vertex is having $(m + n) - 1$ edges. By Definition 2.2,

$$\mu_2(v_i, v_j) = \min(\mu_{1i}, \mu_{1j}),$$

$$\nu_2(v_i, v_j) = \max(\nu_{1i}, \nu_{1j}), \quad \forall v_i, v_j \in V.$$

Hence $G_1 + G_2$ is a complete IFG.

Conversely, if $G_1 + G_2$ is a complete IFG, then we claim that G_1 and G_2 are complete IFG. Suppose that G_1 and G_2 are not complete IFG, then some vertices in $G_1 + G_2$ have less than $(m+n)-1$ edges with

$$\mu_2(v_i, v_j) = \min(\mu_{1i}, \mu_{1j}),$$

$$\nu_2(v_i, v_j) = \max(\nu_{1i}, \nu_{1j}), \quad \forall (v_i, v_j) \in E_1 \cup E_2 \cup E'.$$

which implies that $G_1 + G_2$ is not a complete IFG, which is a contradiction to the fact that $G_1 + G_2$ is a complete IFG. Hence G_1 and G_2 are complete IFG. □

Theorem 4.8. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 + G_2$ *is a self centered IFG.*

Proof. Proof follows from Theorem 4.7 and then from Theorem 2.1. □

Theorem 4.9. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 + G_2$ *is not a self median IFG.*

Proof. Proof follows from Theorem 4.7 and then from Proposition 3.3. □

Theorem 4.10. *Let* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *be IFG. Then* G_1 *and* G_2 *are complete IFG if and only if* $G_1 \cup G_2$ *is a complete IFG, where* $G_1 \cap G_2 = \varnothing$.

Proof. Let $G = G_1 \cup G_2 = (V, E)$ be the union of two graphs G_1 and G_2, where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$.

Let $(v_i, v_j) \in E_1 - E_2$ and $v_i, v_j \in V_1 - V_2$. Then

$$(\mu_2 \cup \mu_2')(v_i, v_j) = \mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$$

$$= \min((\mu_1 \cup \mu_1')(v_i), (\mu_1 \cup \mu_1')(v_j))$$

and

$$(\nu_2 \cup \nu'_2)(v_i, v_j) = \nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$$

$$= \max((\nu_1 \cup \nu'_1)(v_i), (\nu_1 \cup \nu'_1)(v_j)).$$

Similarly, if $(v_i, v_j) \in E_2 - E_1,$ then

$$(\mu_2 \cup \mu'_2)(v_i, v_j) = \min((\mu_1 \cup \mu'_1)(v_i), (\mu_1 \cup \mu'_1)(v_j)),$$

$$(\nu_2 \cup \nu'_2)(v_i, v_j) = \max((\nu_1 \cup \nu'_1)(v_i), (\nu_1 \cup \nu'_1)(v_j)).$$

Therefore, $G_1 \cup G_2$ is a complete IFG. Conversely, if $G_1 \cup G_2$ is a complete IFG, where $G_1 \cap G_2 = \emptyset,$ then $G_1 \cup G_2$ is a disconnected IFG. That is, it has components G_1 and $G_2,$ which are complete. Hence $G_1 \cup G_2$ is a complete IFG. □

Theorem 4.11. *Let* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *be IFG. Then* G_1 *and* G_2 *are complete IFG if and only if* $G_1 \cup G_2$ *is a self centered IFG, where* $G_1 \cap G_2 = \emptyset.$

Proof. Proof follows from Theorem 4.10 and Theorem 2.1. □

Theorem 4.12. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are complete IFG, then* $G_1 \cup G_2$ *is not a self median IFG, where* $G_1 \cap G_2 = \emptyset.$

Proof. Proof follows from Definition 3.13 and Proposition 3.3. □

Theorem 4.13. *If* $G_1 : \langle \mu, \nu, V_1, E_1 \rangle$ *and* $G_2 : \langle \mu', \nu', V_2, E_2 \rangle$ *are constant IFG, then* $G_1 \cup G_2$ *is a self median IFG, where* $G_1 \cap G_2 = \emptyset.$

Proof. If G_1 and G_2 are constant IFG, where $G_1 \cap G_2 = \emptyset,$ then $G_1 \cup G_2$ is a disconnected IFG. That is, it has components G_1 and $G_2,$ which are constant and therefore $G_1 \cup G_2$ is also constant IFG. Hence by Theorem 3.8, $G_1 \cup G_2$ is a self median IFG. □

Theorem 4.14. *If* $G : \langle \mu, \nu, V, E \rangle$ *is self centered IFG, then* $\langle C(G) \rangle \cong G.$

Proof. If G is self centered IFG, then we have $e_{\mu}(v_i) = e_{\mu}(v_j)$ and $e_{\nu}(v_i) = e_{\nu}(v_j)$. That is, $r_{\mu}(G) = e_{\mu}(v_i)$ and $r_{\nu}(G) = e_{\nu}(v_i)$, $i = 1, 2, ..., n$. $C(G)$ is the set of central vertices. Here $C(G) = \{v_1, ..., v_n\}$. Hence, $\langle C(G) \rangle \cong G$. □

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