JP Jour. Geometry & Topology **4**(1) (2004), 23-34

GEOMETRIC CONSTRUCTION OF A CLASSIFYING SPACE FOR THE FIBRE OF THE DOUBLE SUSPENSION

YASUHIKO KAMIYAMA

Department of Mathematics, *University of the Ryukyus*
Nishihara-Cho, *Okinawa* 903-0213, *Japan*
e-mail: kamiyama@sci.u-ryukyu.ac.jp

Abstract

Let $F(n)$ be the homotopy theoretic fibre of the double suspension $E^2 : S^{2n-1} \to \Omega^2 S^{2n+1}$. It is known that there is a fibration $S^{2n-1} \to \Omega^2 S^{2n+1} \to BF(n)$ such that $\Omega BF(n) = F(n)$. In this paper, we construct $BF(n)$ in terms of $(n+1)$-tuples of polynomials.

1. Introduction

Let $F(n)$ be the homotopy theoretic fibre of the double suspension $E^2 : S^{2n-1} \to \Omega^2 S^{2n+1}$. In [2], Gray constructed a fibration

$$S^{2n-1} \xrightarrow{E^2} \Omega^2 S^{2n+1} \to BF(n) \tag{1.1}$$

such that $\Omega BF(n) = F(n)$. For $n = 1$, we can set $BF(1) = \Omega^2 S^3 \langle 3 \rangle$ (where $S^3\langle 3\rangle$ denotes the 3-connected cover of S^3), since $\Omega^2 S^3 \simeq S^1 \times \Omega^2 S^3 \langle 3 \rangle$. In order to construct $BF(n)$ for all n, let $J^l(n)$ denote the

2000 Mathematics Subject Classification: 55P35.

Key words and phrases: double suspension, rational function.

Received October 21, 2003

l-th stage of the James construction which builds ΩS^{2n+1}, and let $W^l(n)$ be the homotopy theoretic fibre of the inclusion

$$j^l(n) : J^l(n) \hookrightarrow J(n) \simeq \Omega S^{2n+1}. \tag{1.2}$$

For a certain map

$$\psi^l(n) : W^l(n) \to S^{2(l+1)n-1}, \tag{1.3}$$

we denote the fibre of $\psi^l(n)$ by $BF^l(n)$. Let p be a prime. It is shown in [2] that if $1 \le l,\ l' \le p-1$, then we have a homotopy equivalence localized at $p : BF^l(n) \simeq BF^{l'}(n)$. Then, localized at p, we define $BF(n)$ in (1.1) to be $BF^l(n)$ for $1 \le l \le p-1$. Thus for the construction of $BF(n)$, it is important to construct a map $\psi^l(n)$ in (1.3). An alternate but equivalent construction of $\psi^l(n)$ is given in [6].

The purpose of this paper is to construct the map $\psi^l(n)$ in terms of $(n+1)$-tuples of polynomials. More precisely, we define a finite dimensional space $X_k^l(n)$ and a map

$$\varphi_k^l(n) : X_k^l(n) \to S^{2(l+1)n-1} \tag{1.4}$$

integrally so that if we form the direct limit $k \to \infty$, then we obtain $\psi^l(n)$. Thus if we denote the fibre of $\varphi_k^l(n)$ by $B_k^l(n)$, then we have $B_\infty^l(n) \simeq BF^l(n)$. Hence, $B_k^l(n)$ is a certain refinement of $BF^l(n)$.

Let $\mathrm{Rat}_k(n)$ denote the space of based holomorphic maps of degree k from the Riemannian sphere S^2 to the complex projective space $\mathbf{C}P^n$. The basepoint condition we assume is that $f(\infty) = [1, \ldots, 1]$. Such holomorphic maps are given by rational functions:

$$\mathrm{Rat}_k(n) = \{(p_0(z), \ldots, p_n(z)) : \text{each } p_i(z) \text{ is monic, degree-}k \text{ polynomial and such that there are no roots common to all } p_i(z)\}.$$

There is an inclusion

$$i_k(n) : \mathrm{Rat}_k(n) \hookrightarrow \Omega_k^2 \mathbf{C}P^n \simeq \Omega^2 S^{2n+1}. \tag{1.5}$$

Segal [7] proved that $i_k(n)$ is a homotopy equivalence up to dimension $k(2n-1)$ (i.e., $i_k(n)$ induces isomorphisms in homotopy groups in dimensions less than $k(2n-1)$, and an epimorphism in dimension $k(2n-1)$).

Later, the stable homotopy type of $\mathrm{Rat}_k(n)$ was described in [1] in terms of Snaith's stable summands of $\Omega^2 S^{2n+1}$. The Segal theorem is sharp only for $n = 1$ as it follows from results of [1] that $i_k(n)$ is a homotopy equivalence up to dimension $(k+1)(2n-1)-1$.

We generalize the definition of $\mathrm{Rat}_k(n)$ as follows. We set

$$X_k^l(n) = \{(p_0(z), \ldots, p_n(z)) : \text{each } p_i(z) \text{ is a monic, degree-}k \text{ polynomial and such that there are at most } l \text{ roots common to all } p_i(z)\}.$$

In [3] for $l = 1$ and in [4] for general l, the stable homotopy type of $X_k^l(n)$ was described in terms of stable summands of $W^l(n)$, where $W^l(n)$ is defined in (1.2). But a stability theorem as in [7] is not known. Far from it: we do not know an unstable map $X_k^l(n) \to W^l(n)$ which is a generalization of $i_k(n)$ for $l = 0$ (compare (1.5)). Our first result is then:

Theorem A. *For $l \geq 1$, there is a map*

$$\alpha_k^l(n) : X_k^l(n) \to W^l(n)$$

which satisfies the following properties: For a prime p and the homomorphism $\alpha_k^l(n)_* : H_*(X_k^l(n); \mathbf{Z}/p) \to H_*(W^l(n); \mathbf{Z}/p)$,

(i) *$\alpha_k^l(n)_*$ is injective.*

(ii) *$\mathrm{Im}\, \alpha_k^l(n)_*$ is spanned by monomials in $H_*(W^l(n); \mathbf{Z}/p)$ of weight $\leq k$.*

From Theorem A, we obtain:

Theorem B. *For* $l \geq 1$, $\alpha_k^l(n)$ *is a homotopy equivalence up to dimension* $\left(\left[\frac{k}{l+1}\right]+1\right)(2(l+1)n-1)-1$, *where* $\left[\frac{k}{l+1}\right]$ *is as usual the largest integer* $\leq \frac{k}{l+1}$.

Remarks.

1. We construct the map $\alpha_k^l(n)$ in Section 2. The construction essentially uses the fact that

$$X_k^l(n)/X_k^{l-1}(n) \simeq \Sigma^{2ln}(\mathrm{Rat}_{k-l}(n) \vee S^0).$$

Note that $\alpha_k^l(n)$ is defined integrally.

2. The structures of $H_*(X_k^l(n); \mathbf{Z}/p)$ and $H_*(W^l(n); \mathbf{Z}/p)$ were determined in [3] for $l = 1$ and in [4] for general l. We recall them in Section 2.

In Section 2 we construct a map $\varphi_k^l(n): X_k^l(n) \to S^{2(l+1)n-1}$ integrally (compare (2.1)). Let $B_k^l(n)$ be the fibre of $\varphi_k^l(n)$.

Theorem C. *Let* $l \geq 1$.

(i) *We have the following homotopy commutative diagram*:

$$\begin{tikzcd}
B_k^l(n) \arrow[r] \arrow[d, "\beta_k^l(n)"'] & X_k^l(n) \arrow[r, "\varphi_k^l(n)"] \arrow[d, "\alpha_k^l(n)"'] & S^{2(l+1)n-1} \arrow[d, equal] \\
BF^l(n) \arrow[r] & W^l(n) \arrow[r, "\psi^l(n)"] & S^{2(l+1)n-1},
\end{tikzcd}$$

where $\beta_k^l(n)$ *is the restriction of* $\alpha_k^l(n)$ *to the fibres.*

(ii) $\beta_k^l(n)$ *is a homotopy equivalence up to dimension* $\left(\left[\frac{k}{l+1}\right]+1\right)$ $(2(l+1)n-1)-2$.

Since we can use $BF^1(n)$ as $BF(n)$ in (1.1) integrally, we study $B_k^1(n)$ in detail.

Theorem D.

(i) *Let p be a prime and we write $k = ps + i$, where $0 \leq i \leq p - 1$. Then there are homotopy equivalences localized at p:*

$$B_{ps}^1(n) \simeq B_{ps+1}^1(n) \quad \textit{and} \quad B_{ps+2}^1(n) \simeq B_{ps+i}^1(n) \ (2 \leq i \leq p-1),$$

where we consider the latter only for odd primes p.

(ii) *We have*

$$H_*(B_{2s}^1(n); \mathbf{Z}/2) \cong H_*(X_{2s}^1(n); \mathbf{Z}/2) \otimes H_*(\Omega S^{4n-1}; \mathbf{Z}/2).$$

(iii) *For an odd prime p and $s \geq 0$, there is a homotopy equivalence localized at p:*

$$\Sigma^2 \operatorname{Rat}_{ps+1}(n) \simeq \Sigma^2(S^{2n-1} \times B_{ps+2}^1(n)).$$

Remarks.

1. From results of [1], there is a homotopy equivalence localized at p:

$$\operatorname{Rat}_{ps+1}(n) \simeq \operatorname{Rat}_{ps+i}(n) \ (1 \leq i \leq p-1).$$

Similarly, from results of [3], there are homotopy equivalences localized at p:

$$X_{ps}^1(n) \simeq X_{ps+1}^1(n) \quad \text{and} \quad X_{ps+2}^1(n) \simeq X_{ps+i}^1(n) \ (2 \leq i \leq p-1).$$

Theorem D(i) is a consequence of the latter.

2. For an odd prime p, we know $H_*(B_{ps+2}^1(n); \mathbf{Z}/p)$ from Theorem D (iii) (compare Lemma 2.5). In particular, $\beta_{ps+2}^1(n)_* : H_*(B_{ps+2}^1(n); \mathbf{Z}/p) \to H_*(BF^1(n); \mathbf{Z}/p)$ is injective. On the other hand, $H_*(B_{ps}^1(n); \mathbf{Z}/p)$ is somewhat complicated and contains unstable elements (i.e., $\beta_{ps}^1(n)_*$ is not injective).

3. For all k, l and n, there is a map

$$\nu_k^l(n) : \mathrm{Rat}_k(n) \to B_k^l(n)$$

so that when localized at p and $1 \le l \le p-1$, $\nu_\infty^l(n)$ is the map $\Omega^2 S^{2n+1} \to BF(n)$ in (1.1). It is interesting to study how different the fibre of $\nu_k^l(n)$ is from S^{2n-1}. We discuss this briefly in Section 2 (compare Lemma 2.7).

2. Proofs

Note that as sets we have

$$X_k^l(n) = \coprod_{q=0}^{l} \mathbf{C}^q \times \mathrm{Rat}_{k-q}(n),$$

where $\mathbf{C}^q \times \mathrm{Rat}_{k-q}(n)$ corresponds to the subspace of $X_k^l(n)$ consisting of elements $(p_0(z), \ldots, p_n(z))$ such that there are exactly q roots common to all $p_i(z)$. Hence,

$$X_k^l(n) = X_k^{l-1}(n) \coprod \mathbf{C}^l \times \mathrm{Rat}_{k-l}(n).$$

It is known that the normal bundle of $\mathbf{C}^l \times \mathrm{Rat}_{k-l}(n)$ in $X_k^l(n)$ is trivial (compare [3] and [4]). In $X_k^l(n)$, we pinch an open set $X_k^{l-1}(n)$ to a point. Then we have a map

$$\pi_1 : X_k^l(n) \to X_k^l(n)/X_k^{l-1}(n) \cong \Sigma^{2ln}((\mathbf{C}^l \times \mathrm{Rat}_{k-l}(n)) \vee S^0).$$

Let

$$\pi_2 : \Sigma^{2ln}\, \mathrm{Rat}_{k-l}(n) \vee S^{2ln} \to \Sigma^{2ln}\, \mathrm{Rat}_{k-l}(n)$$

be the map pinching S^{2ln} to a point. We set

$$p_k^l(n) = \pi_2 \circ \pi_1 : X_k^l(n) \to \Sigma^{2ln}\, \mathrm{Rat}_{k-l}(n).$$

Recall that there is an inclusion (compare (1.5)):

$$i_{k-l}(n) : \mathrm{Rat}_{k-l}(n) \hookrightarrow \Omega^2 S^{2n+1}.$$

Taking the adjoint of $i_{k-l}(n)$ and the $(2ln-2)$-fold suspensions, we have

$$\Sigma^{2ln-2}(\mathrm{Ad}(i_{k-l}(n))) : \Sigma^{2ln}\,\mathrm{Rat}_{k-l}(n) \to S^{2(l+1)n-1}.$$

Let $\varphi_k^l(n) : X_k^l(n) \to S^{2(l+1)n-1}$ be the composition

$$\varphi_k^l(n) = \Sigma^{2ln-2}(\mathrm{Ad}(i_{k-l}(n))) \circ p_k^l(n). \tag{2.1}$$

Let $f^l(n) : S^{2(l+1)n-1} \to J^l(n)$ be the map which may be used to attach a cell to obtain $J^{l+1}(n)$. In particular, $f^1(n)$ is the Whitehead product $[e_{2n}, e_{2n}]$, where e_{2n} denotes the generator of $\pi_{2n}(S^{2n})$ represented by the identity map. Let $h_k^l(n) : X_k^l(n) \to J^l(n)$ be the composition

$$h_k^l(n) = f^l(n) \circ \varphi_k^l(n).$$

Recall the map $j^l(n)$ in (1.2). Since $j^l(n) \circ f^l(n)$ is null homotopic, so is $j^l(n) \circ h_k^l(n)$. Hence, there is a lifting

$$\widetilde{h}_k^l(n) : X_k^l(n) \to W^l(n). \tag{2.2}$$

Recall that $H_*(\Omega^2 S^{2n+1}; \mathbf{Z}/p)$ is given as follows. There is a (torsion free) generator $\iota_{2n-1} \in H_{2n-1}(\Omega^2 S^{2n+1}; \mathbf{Z}/p) \cong \mathbf{Z}/p$, and the following hold:

(i) For $p = 2$,

$$H_*(\Omega^2 S^{2n+1}; \mathbf{Z}/2) \cong \mathbf{Z}/2\,[\iota_{2n-1}, Q_1(\iota_{2n-1}), \ldots, Q_1 \cdots Q_1(\iota_{2n-1}), \ldots].$$

(ii) For an odd prime p,

$$H_*(\Omega^2 S^{2n+1}; \mathbf{Z}/p) \cong \bigwedge(\iota_{2n-1}, Q_1(\iota_{2n-1}), \ldots, Q_1 \cdots Q_1(\iota_{2n-1}), \ldots)$$
$$\otimes\, \mathbf{Z}/p\,[\beta Q_1(\iota_{2n-1}), \ldots, \beta Q_1 \cdots Q_1(\iota_{2n-1}), \ldots].$$

In (i) and (ii), Q_1 is the first Dyer-Lashof operation (it takes a class of dimension d to a class of dimension $dp + p - 1$) and β is the mod p Bockstein operation.

For each monomial in (i) and (ii), we define a weight function w in the usual manner, that is, (1) $w(\iota_{2n-1}) = 1$; (2) $w(Q_1^d(\iota_{2n-1})) = w(\beta Q_1^d(\iota_{2n-1})) = p^d$, where $Q_1^d = \underbrace{Q_1 \cdots Q_1}_{d \text{ times}}$; (3) $w(x * y) = w(x) + w(y)$, where $*$ is the loop sum Pontryagin product.

The structure of $H_*(W^l(n); \mathbf{Z}/p)$ is given in [4]. For simplicity, we recall only the case $l = 1$ (compare also [3] and [5]).

(i) For $p = 2$,

$$H_*(W^1(n); \mathbf{Z}/2) \cong \mathbf{Z}/2\,[Q_1(\iota_{2n-1}), \ldots, Q_1 \cdots Q_1(\iota_{2n-1}), \ldots].$$

(ii) For an odd prime p, there is a (torsion free) generator $x_{4n-1} \in H_{4n-1}(W^1(n); \mathbf{Z}/p) \cong \mathbf{Z}/p$ so that

$$H_*(W^1(n); \mathbf{Z}/p) \cong \bigwedge(x_{4n-1}, Q_1(\iota_{2n-1}), \ldots, Q_1 \cdots Q_1(\iota_{2n-1}), \ldots) \otimes \mathbf{Z}/p\,[\beta Q_1(\iota_{2n-1}), \ldots, \beta Q_1 \cdots Q_1(\iota_{2n-1}), \ldots]. \quad (2.3)$$

We set $w(x_{4n-1}) = 2$. The structure of $H_*(X_k^l(n); \mathbf{Z}/p)$ is given as follows.

Proposition 2.4 [4]**.** *For a prime p, $H_*(X_k^l(n); \mathbf{Z}/p)$ is isomorphic to the subspace of $H_*(W^l(n); \mathbf{Z}/p)$ spanned by monomials of weight $\leq k$.*

Proof of Theorem A. From the construction of the map $h_k^l(n)$ and Proposition 2.4, we see that we can choose a lifting $\tilde{h}_k^l(n)$ in (2.2) so that $\tilde{h}_k^l(n)_* : H_*(X_k^l(n); \mathbf{Z}/p) \to H_*(W^l(n); \mathbf{Z}/p)$ satisfies the properties of Theorem A (i) and (ii). Setting $\alpha_k^l(n) = \tilde{h}_k^l(n)$, we obtain Theorem A.

Proof of Theorem B. Among elements of $H_*(W^l(n); \mathbf{Z}/p)$ which are not contained in $\operatorname{Im} \alpha_k^l(n)_*$, the element of least degree is given as follows. Note that $X_k^l(n) \cong \mathbf{C}^{k(n+1)}$ $(k \le l)$ and $X_{l+1}^l(n) \cong \mathbf{C}^{l+1} \times (\mathbf{C}^{(l+1)n})^*$. Hence, when $k \ge l+1$, the element of $H_*(X_k^l(n); \mathbf{Z}/p)$ of least positive degree has degree $2(l+1)n-1$. We write the element by $x_{2(l+1)n-1}$ (compare (2.3) for $l = 1$). When $p = 2$, $x_{2(l+1)n-1}^i$ is non-trivial in $H_*(W^l(n); \mathbf{Z}/2)$ for all $i \ge 1$ (compare [4]). Hence, the least degree element of $H_*(W^l(n); \mathbf{Z}/p)$ which is not contained in $\operatorname{Im} \alpha_k^l(n)_*$ occurs when $p = 2$ and is of the form $x_{2(l+1)n-1}^i$. Since $w(x_{2(l+1)n-1}) = l+1$, we have $(l+1)i \ge k+1$, i.e., $i \ge \left[\frac{k}{l+1}\right] + 1$. Hence, $\alpha_k^l(n)_* : H_*(X_k^l(n); \mathbf{Z}/p)$ $\to H_*(W^l(n); \mathbf{Z}/p)$ is an isomorphism for $* \le \left(\left[\frac{k}{l+1}\right]+1\right)(2(l+1)n-1)-1$. From the universal coefficient theorem and the Whitehead theorem, $\alpha_k^l(n)_* : \pi_*(X_k^l(n)) \to \pi_*(W^l(n))$ is an isomorphism for $* < \left(\left[\frac{k}{l+1}\right]+1\right)$ $(2(l+1)n-1)-1$ and an epimorphism for $* = \left(\left[\frac{k}{l+1}\right]+1\right)(2(l+1)n-1)$ -1. This completes the proof of Theorem B.

Proof of Theorem C. The map $\psi^l(n)$ is defined as the following composition (compare [2, p. 304]):

$$W^l(n) \to W^l(n) \cup CW^{l-1}(n) \to \Sigma^l (S^{2n-1})^{(l+1)}.$$

Noting the map $p_k^l(n) : X_k^l(n) \to \Sigma^{2ln} \operatorname{Rat}_{k-l}(n)$, it is easy to show that the restriction of $\psi^l(n)$ to $X_k^l(n)$ is homotopic to $\varphi_k^l(n)$. Hence (i) follows. (ii) is an immediate consequence of Theorem B and the five lemma. This completes the proof of Theorem C.

Proof of Theorem D. From the structure of $H_*(W^1(n); \mathbf{Z}/p)$ and Proposition 2.4, each monomial in $H_*(X_k^1(n); \mathbf{Z}/p)$ has weight 0 or 2 mod p. Hence, localized at p, we have

$$X^1_{ps}(n) \simeq X^1_{ps+1}(n) \ \text{ and } \ X^1_{ps+2}(n) \simeq X^1_{ps+i}(n) \ (2 \le i \le p-1).$$

Now (i) is clear from the fibration of the first row of Theorem C(i).

(ii) is proved by comparing the mod 2 Serre spectral sequence for the fibrations:

$$\begin{tikzcd} \Omega S^{4n-1} \arrow[r] \arrow[d, equal] & B^1_{2s}(n) \arrow[r] \arrow[d, "\beta^1_{2s}(n)"'] & X^1_{2s}(n) \arrow[d, "\alpha^1_{2s}(n)"'] \\ \Omega S^{4n-1} \arrow[r] & BF^1(n) \arrow[r] & W^1(n) \end{tikzcd}$$

For the rest of this paper we prove (iii).

Lemma 2.5. *Let p be an odd prime.*

(i)

$$H_*(BF^1(n); \mathbf{Z}/p) \cong \bigwedge(Q_1(\iota_{2n-1}), \ldots, Q_1 \cdots Q_1(\iota_{2n-1}), \ldots)$$
$$\otimes \mathbf{Z}/p\,[\beta Q_1(\iota_{2n-1}), \ldots, \beta Q_1 \cdots Q_1(\iota_{2n-1}), \ldots].$$

(ii) $\beta^1_{ps+2}(n)_* : H_*(B^1_{ps+2}(n); \mathbf{Z}/p) \to H_*(BF^1(n); \mathbf{Z}/p)$ *is injective so that* $\operatorname{Im}\beta^1_{ps+2}(n)_*$ *is spanned by monomials in* $H_*(BF^1(n); \mathbf{Z}/p)$ *of weight* $\le ps$.

Proof. (i) is clear from the fibration $BF^1(n) \to W^1(n) \to S^{4n-1}$. To prove (ii), let M_k be the subspace of $H_*(BF^1(n); \mathbf{Z}/p)$ spanned by monomials of weight $\le k$. From (2.3) and Proposition 2.4, $H_*(X^1_{ps+2}(n); \mathbf{Z}/p)$ is isomorphic to $M_{ps} \oplus x_{4n-1} \otimes M_{ps}$. (Recall that $w(x_{4n-1}) = 2$.) From the mod p Serre spectral sequence for the fibration $B^1_{ps+2}(n) \to X^1_{ps+2}(n) \to S^{4n-1}$, we have $H_*(B^1_{ps+2}(n); \mathbf{Z}/p) \cong M_{ps}$. Hence (ii) holds. This completes the proof of Lemma 2.5.

Since $\pi_1 \,|\, X^{l-1}_k(n)$ is null homotopic, so is $\varphi^l_k(n) | X^{l-1}_k(n)$. Hence, the inclusion $X^{l-1}_k(n) \hookrightarrow X^l_k(n)$ lifts to a map $X^{l-1}_k(n) \to B^l_k(n)$ (compare the fibration of the first row of Theorem C(i)). Restricting to $\mathrm{Rat}_k(n)$, there is a map

$$\nu_k^l(n) : \mathrm{Rat}_k(n) \to B_k^l(n).$$

Note that when localized at p and $1 \le l \le p-1$, $\nu_\infty^l(n)$ is the map $\Omega^2 S^{2n+1} \to BF(n)$ in (1.1). In particular, we consider the map

$$\nu_k^1(n) : \mathrm{Rat}_k(n) \to B_k^1(n). \tag{2.6}$$

Let $C_k(n)$ be the fibre of (2.6).

Lemma 2.7. *For an odd prime p and* $k = ps + i$ *with* $2 \le i \le p-1$,

$$H_*(C_k(n); \mathbf{Z}/p) \cong H_*(S^{2n-1}; \mathbf{Z}/p).$$

Proof. The lemma follows easily from Lemma 2.5 and the mod p Serre spectral sequence for the fibration (2.6).

Lemma 2.7 implies that localized at p, there is a fibration

$$S^{2n-1} \to \mathrm{Rat}_{ps+2}(n) \to B_{ps+2}^1(n). \tag{2.8}$$

Let $F \to E \xrightarrow{\pi} B$ be a fibration with a retraction $\Sigma^r E \to \Sigma^r F$. Then we have a homotopy equivalence

$$\Sigma^r E \simeq \Sigma^r (F \times B).$$

(Compare the proof of [2, Proposition 7].) We use this for $r = 2$ and apply to (2.8). A retraction $\Sigma^2 \mathrm{Rat}_{ps+2}(n) \to S^{2n+1}$ is constructed as the adjoint of $i_{ps+2}(n)$ in (1.5). Then

$$\Sigma^2 \mathrm{Rat}_{ps+2}(n) \simeq \Sigma^2 (S^{2n-1} \times B_{ps+2}^1(n)).$$

Localized at p, we have $\mathrm{Rat}_{ps+1}(n) \simeq \mathrm{Rat}_{ps+2}(n)$. Hence Theorem D(iii) holds. This completes the proof of Theorem D.

References

[1] F. R. Cohen, R. L. Cohen, B. M. Mann and R. J. Milgram, The topology of rational functions and divisors of surfaces, Acta Math. 166 (1991), 163-221.

[2] B. Gray, On the iterated suspension, Topology 27 (1988), 301-310.

[3] Y. Kamiyama, Geometric approximation of the fibre of the Freudenthal suspension, Bull. Lond. Math. Soc. 30 (1998), 635-642.

[4] Y. Kamiyama, Polynomial model for homotopy fibres associated with the James construction, Math. Z. 237 (2001), 149-180.

[5] R. J. Milgram, Unstable homotopy from the stable point of view, Lect. Notes Math., Vol. 368, Springer, Berlin, Heidelberg, New York, 1974.

[6] J. C. Moore and J. A. Neisendorfer, Equivalence of Toda-Hopf invariants, Israel J. Math. 66 (1989), 300-318.

[7] G. B. Segal, The topology of spaces of rational functions, Acta Math. 143 (1979), 39-72.

■