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A NOTE ON THE ALGORITHM OF DIFFERENTIATION VII FOR EQUIPPED POSETS

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Abstract

As a fourth part of a series of papers concerning morphisms of equipped posets. In this paper, the authors discuss some categorical properties of the short generalized version of the algorithm of differentiation VII for equipped posets introduced by Rodriguez and Zavadskij in [4].

1. Introduction

The algorithm of differentiation VII (D-VII) was introduced in 2003 by Zavadskij to classify equipped posets of tame and finite growth

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representation type [5, 6]. Soon afterwards, Rodriguez and Zavadskij defined a short version of this algorithm [4]. Such a version allows to describe D-VII as a composition of functors whose categorical properties can be investigated more easily. Actually, in [2], Cañadas gave the categorical properties of D-VII by establishing suitable categorical equivalences induced by the short version of such algorithm. To do that, some subsets of equipped posets considered in the original version were deleted. The purpose of this paper is to describe categorical properties of the short generalized version of the algorithm VII without such changes.

Authors refer to the interested reader to [1, 2] and [3] to precise notation and definitions. However, for the sake of clarity, we include here main definitions and notation for categories of equipped posets.

A poset $(\mathcal{P}, \leq)$ is called *equipped* if all the order relations between its points $x \leq y$ are separated into strong (denoted $x \trianglelefteq y$) and weak (denoted $x \preceq y$) in such a way that

$$x \leq y \trianglelefteq z \text{ or } x \trianglelefteq y \leq z \text{ implies } x \trianglelefteq z, \tag{1}$$

i.e., a composition of a strong relation with any other relation is strong.

In general, relations $\trianglelefteq$ and $\preceq$ are not order relations. These relations are antisymmetric but not reflexive. In particular, $\preceq$ is not reflexive (meanwhile $\trianglelefteq$ is transitive) [4].

We let $x \leq y$ denote an arbitrary relation in an equipped poset $(\mathcal{P}, \leq)$. The order $\leq$ on an equipped poset $\mathcal{P}$ gives rise to the relations $\prec$ and $\lhd$ of *strict inequality*: $x \prec y$ (respectively, $x \lhd y$) in $\mathcal{P}$ if and only if $x \preceq y$ (respectively, $x \trianglelefteq y$) and $x \neq y$.

A point $x \in \mathcal{P}$ is called *strong* (*weak*) if $x \trianglelefteq x$ (respectively, $x \preceq x$). These points are denoted $\circ$ (respectively, $\otimes$) in diagrams. We also denote $\mathcal{P}^{\circ} \subseteq \mathcal{P}$ (respectively, $\mathcal{P}^{\otimes} \subseteq \mathcal{P}$) the subset of strong points (respectively, weak points) of $\mathcal{P}$. If $\mathcal{P}^{\otimes} = \varnothing$, then the equipment is *trivial* and the poset $\mathcal{P}$ is ordinary.

Remark 1. Note that, if $x \preceq y$ in an equipped poset $(\mathcal{P}, \leq)$ and there exists $t \in \mathcal{P}$ such that $x \leq t \leq y$, then $x, y \in \mathcal{P}^{\otimes}$, $x \preceq t$ and $t \preceq y$. Otherwise, if $x \trianglelefteq t$ or $t \trianglelefteq y$, then by definition, it is obtained the contradiction $x \trianglelefteq y$.

If $\mathcal{P}$ is an equipped poset and $a \in \mathcal{P}$, then the subsets of $\mathcal{P}$ denoted $a^{\vee}, a_{\wedge}, a^{\triangledown}, a_{\vartriangle}, a^{\blacktriangledown}, a_{\blacktriangle}, a^{\curlyvee}$ and $a_{\curlywedge}$ are defined in such a way that

$$a^{\vee} = \{x \in \mathcal{P} \,|\, a \leq x\}, \quad a_{\wedge} = \{x \in \mathcal{P} \,|\, x \leq a\},$$

$$a^{\triangledown} = \{x \in \mathcal{P} \,|\, a \trianglelefteq x\}, \quad a_{\vartriangle} = \{x \in \mathcal{P} \,|\, x \trianglelefteq a\},$$

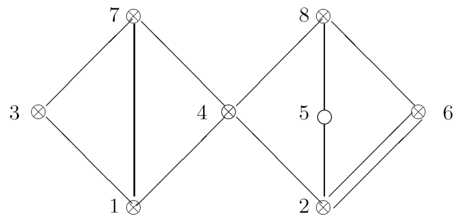
$$a^{\blacktriangledown} = a^{\vee} \backslash a, \quad a_{\blacktriangle} = a_{\wedge} \backslash a,$$

$$a^{\curlyvee} = \{x \in \mathcal{P} \,|\, a \preceq x\}, \quad a_{\curlywedge} = \{x \in \mathcal{P} \,|\, x \preceq a\}.$$

Subset $a^{\vee}$ $(a_{\wedge})$ is called the *ordinary upper* (*lower*) *cone* associated to the point $a \in \mathcal{P}$ and subset $a^{\triangledown}$ $(a_{\vartriangle})$ is called the *strong upper* (*lower*) *cone* associated to the point $a \in \mathcal{P}$, whereas subsets $a^{\blacktriangledown}$ and $a_{\blacktriangle}$ are called *truncated cones* (*upper* and *lower*) associated to the point $a \in \mathcal{P}$.

In general, subsets $a^{\curlyvee}$ and $a_{\curlywedge}$ are not cones. Note that, if $x \in \mathcal{P}^{\circ}$, then $x^{\curlyvee} = x_{\curlywedge} = \varnothing$.

The diagram of an equipped poset $(\mathcal{P}, \leq)$ may be obtained via its Hasse diagram (with strong $(\circ)$ and weak points $(\otimes)$). In this case, a new line is added to the line connecting two points $x, y \in \mathcal{P}$ with $x \lhd y$ if and only if such relation cannot be deduced of any other relations in $\mathcal{P}$. In Figure 1, we show an example of this kind of diagrams.



$1^{\curlyvee} = \{1, 3, 4, 8\}$ $1^{\triangledown} = \{7\}$
$2^{\curlyvee} = \{2, 4, 7\}$ $2^{\triangledown} = \{5, 6, 8\}$
$3^{\curlyvee} = \{3, 7\}$ $3^{\triangledown} = \varnothing$
$4^{\curlyvee} = \{4, 7, 8\}$ $4^{\triangledown} = \varnothing$
$5^{\curlyvee} = \varnothing$ $5^{\triangledown} = \{5, 8\}$
$6^{\curlyvee} = \{6, 8\}$ $6^{\triangledown} = \varnothing$
$7^{\curlyvee} = \{7\}$ $7^{\triangledown} = \varnothing$
$8^{\curlyvee} = \{8\}$ $8^{\triangledown} = \varnothing$

Figure 1

For an equipped poset $(\mathcal{P}, \leq)$ and $A \subset \mathcal{P}$, we define the subsets $A^{\triangledown}$, $A^{\curlyvee}$ and $A^{\vee}$ in such a way that

$$A^{\triangledown} = \bigcup_{a \in A} a^{\triangledown}, \quad A^{\curlyvee} = \bigcup_{a \in A} a^{\curlyvee}, \quad A^{\vee} = \bigcup_{a \in A} a^{\vee}.$$

Subsets $A_{\triangle}$, $A_{\curlywedge}$ and $A_{\wedge}$ are defined in the same way.

If $\mathcal{P}$ is an equipped poset, then a *chain* $C = \{c_i \in \mathcal{P} \,|\, 1 \leq i \leq n,$ $c_{i-1} < c_i$ if $i \geq 2\} \subseteq \mathcal{P}$ is a *weak chain* if and only if $c_{i-1} \prec c_i$ for each $i \geq 2$. If $c_1 \prec c_n$, then we say that C is a *completely weak chain*. Moreover, a subset $X \subset \mathcal{P}$ is *completely weak* if $X = X^{\otimes}$ and weak relations are the only relations between points of X. Often we let $\{c_1 \prec c_2 \prec \cdots \prec c_n\}$ denote a weak chain which consists of points $c_1, c_2, ..., c_n$. An ordinary chain C is denoted in the same way (by using the corresponding symbol <).

For an equipped poset $\mathcal{P}$ and $A, B \subset \mathcal{P}$, we write $A < B$ if $a < b$ for each $a \in A$ and $b \in B$. Notation $A \prec B$ and $A \lhd B$ are assumed in the same way.

Let $F \subset G$ be an arbitrary quadratic field extension with $G = F(\mathbf{u})$ for some fixed element $\mathbf{u} \in G$. Then each element $x \in G$ can be written uniquely in the form $\alpha + \mathbf{u}\beta$ with $\alpha, \beta \in F$ in this case (analogously to the case $(F, G) = (\mathbb{R}, \mathbb{C})$) α is called the *real part* of x and β is the *corresponding imaginary part* of x.

The complexification of a real vector space can be generalized to the case (F, G), where $G = F(\mathbf{u})$ is a quadratic extension of F. In this case, we assume that $\mathbf{u}$ is a root of the minimal polynomial $t^2 + \mu t + \lambda, \quad \lambda \neq 0$ $(\lambda, \mu \in F)$. In particular, if U_0 is an F-space, then the corresponding complexification is the G-vector space U_0^2 also denoted $\tilde{U}_0$. As in the case $(\mathbb{R}, \mathbb{C})$, we write $U_0^2 = U_0 + \mathbf{u}U_0 = \tilde{U}_0$.

To each G-subspace W of $\tilde{U}_0$, it is possible to associate the following F-subspaces of U_0,

$$W^+ = \operatorname{Re} W_F = \operatorname{Im} W_F \quad \text{and} \quad W^- = gen\{\alpha \in U_0 \,|\, (\alpha, 0)^t \in W\} \subset W^+,$$

and for a G-space Z, we have the following property:

$$\tilde{Z}^+ = F(Z) \text{ is called the } F\text{-}hull \text{ of } Z \text{ such that } Z \subset F(Z).$$

The *category of representations of an equipped poset* over a pair of fields (F, G) is defined as a system of the form

$$U = (U_0; U_x \,|\, x \in \mathcal{P}), \tag{2}$$

where U_0 is a finite dimensional F-space and for each $x \in \mathcal{P}$, U_x is a G-subspace of $\tilde{U}_0$ such that

$$x \leq y \Rightarrow U_x \subset U_y,$$

$$x \trianglelefteq y \Rightarrow F(U_x) \subset U_y.$$

For each $x \in \mathcal{P}$, we let $\underline{U_x}$ denote the *radical subspace* of U_x such that $\underline{U_x} = \sum_{z \lhd x} F(U_z) + \sum_{z \prec x} U_z$.

We let $\operatorname{rep}\mathcal{P}$ denote the category whose objects are the representations of an equipped poset $\mathcal{P}$ over a pair of fields (F, G). In this case, a morphism $\varphi : (U_0; U_x \,|\, x \in \mathcal{P}) \to (V_0; V_x \,|\, x \in \mathcal{P})$ between two representations U and V is an F-linear map $\varphi : U_0 \to V_0$ such that

$$\tilde{\varphi}(U_x) \subset V_x \text{ for each } x \in \mathcal{P},$$

where $\tilde{\varphi} : \tilde{U}_0 \to \tilde{V}_0$ is the complexification of φ, i.e., the application G-linear induced by φ and defined in such a way that if $z = x + \mathbf{u}y \in \tilde{U}_0$, then $\tilde{\varphi}(z) = \varphi^2(z) = \varphi(x) + \mathbf{u}\varphi(y)$. The composition between morphisms of $\operatorname{rep} \mathcal{P}$ is defined in a natural way.

The short generalized version of the algorithm of differentiation VII (denoted VII_s) was defined by Rodriguez and Zavadskij in the following way [1, 2, 4]:

A triple of points (a, b, c) of an equipped poset $\mathcal{P}$ is said to be VII_s-*suitable* if the points a, c are weak, b is a strong point incomparable with a, c and

$$\mathcal{P} = a^{\nabla} + b_{\triangle} + \{a \prec X \prec c \prec Y\},$$

where $\{a \prec X \prec c \prec Y\}$ is a completely weak set containing arbitrary subsets X, Y (probably empty). Actually, in [2], it is assumed that $Y = \varnothing$ to obtain categorical properties of the algorithm of differentiation VII. In this paper, such properties are discussed in the case $Y \neq \varnothing$.

The *derived* or (a, b, c)-*derived* equipped poset *with relations* $\mathcal{P}'_{(a,b,c)}$ of the poset $\mathcal{P}$ is a pair

$$\mathcal{P}'_{(a,b,c)} = (\mathcal{P}^{(s)}_{(a,b,c)} \,|\, \Sigma_{(a,b,c)}),$$

where

$$\mathcal{P}^{(s)}_{(a,b,c)} = (\mathcal{P} \backslash c) + \{c^-, c^+\}$$

is an equipped poset such that the pairs $c^- \prec c^+$, $X \prec c^+$ and $c^- \prec Y$ are completely weak, $a \lhd c^+$, $c^- < b$ and the partial order in $\mathcal{P}^{(s)}_{(a,b,c)}$ is induced by these relations and by the initial order in $\mathcal{P}\backslash c$, it is assumed that each of the points c^+, c^- inherits the order relations of the point c with the points of the subset $a^\nabla + b_\triangle$. Further, $\Sigma_{(a,b,c)}$ is a set of two formal relations

$$\Sigma_{(a,b,c)} = \{c^+ \subset \tilde{a} + \widehat{Y};\, b(a+X) \subset c^-\}$$

which means that the category $\operatorname{rep}\mathcal{P}'_{(a,b,c)}$ is a full subcategory of the category whose objects W satisfy the relations

$$W_b \cap W_X \subset W_{c^-}, \quad W_{c^+} \subset F(W_a) + \widehat{W}_Y,$$

where

$$U_X = \sum_{x\in X} U_x, \quad U_X^+ = \sum_{x\in X} U_x^+,$$

$$\widehat{U}_X = \bigcap_{x\in X} U_x, \quad (\widehat{U}_X)^- = \bigcap_{x\in X} U_x^-, \tag{3}$$

$$U_\varnothing = 0, \quad \widehat{U}_\varnothing = U_0. \tag{4}$$

Figure 2 shows a diagram for this differentiation:

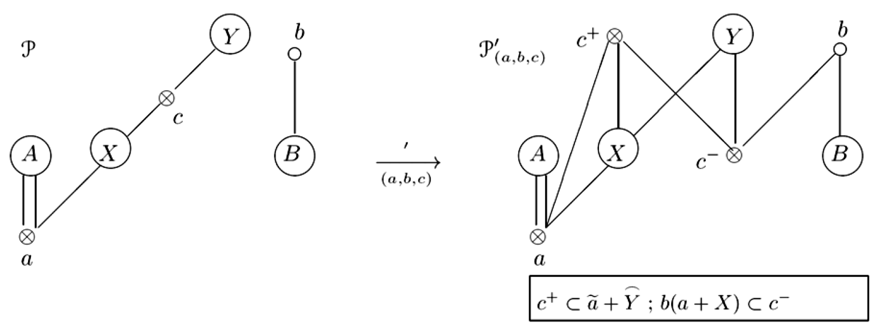


Figure 2

The *functor of differentiation* $D_{(a,b,c)} : \operatorname{rep}\mathcal{P} \to \operatorname{rep}\mathcal{P}'_{(a,b,c)}$ is defined by the following identities for an object $U' = D_{(a,b,c)}(U)$:

$$U'_0 = U_0,$$

$$U'_{c^+} = U_c + F(U_a),$$

$$U'_{c^-} = U_c \cap U_b,$$

$$U'_x = U_x \text{ for the remaining points } x \in \mathcal{P}'_{(a,b,c)},$$

$$\varphi' = \varphi \text{ for a linear map-morphism } \varphi : U_0 \to V_0. \tag{5}$$

If $\mathcal{P}$ is an equipped poset and $A \subset \mathcal{P}$, then $P(A) = P(\min A) = (F;\, P_x \mid x \in \mathcal{P})$, $P_x = G$ if $x \in A^{\vee}$ and $P_x = 0$ otherwise. In particular, $P(\varnothing) = (F;\, 0, \ldots, 0)$.

If $a, b \in \mathcal{P}^{\otimes}$, then $T(a)$ and $T(a, b)$ denote indecomposable objects with matrix representation of the following form:

$$T(a) = \begin{array}{c} a \\ \boxed{\begin{matrix} 1 \\ \mathbf{u} \end{matrix}} \end{array}, \quad a \in \mathcal{P}^{\otimes}, \quad T(a, b) = \begin{array}{cc} a & b \\ \hline \multicolumn{1}{|c|}{1} & \multicolumn{1}{c|}{0} \\ \multicolumn{1}{|c|}{\mathbf{u}} & \multicolumn{1}{c|}{1} \\ \hline \end{array} \text{ with } a \prec b.$$

If we consider the notation used in (2) for objects in $\operatorname{rep}\mathcal{P}$, then the object $T(a)$ may be described in such a way that $T(a) = (T_0;\, T_x \mid x \in \mathcal{P})$, where $T_0 = F^2$ and

$$T_x = \begin{cases} \widetilde{T}_0 = G^2, & \text{if } x \in a^{\triangledown}, \\ G\{(1, \mathbf{u})^t\}, & \text{if } x \in a^{\curlyvee}, \\ 0, & \text{otherwise,} \end{cases}$$

where $(1, \mathbf{u})^t$ is the column of coordinates with respect to an ordered basis of T_0.

On the other hand, representation $T(a, b)$ may be described in such a way that $T(a, b) = (T_0;\, T_x \mid x \in \mathcal{P})$, where $T_0 = F^2$ and

$$T_x = \begin{cases} G\{(1, \mathbf{u})^t\}, & \text{if } a \preceq x \prec b, \\ \tilde{T}_0 = G^2, & \text{if } x \in a^{\triangledown} \cup b^{\vee}, \\ 0, & \text{otherwise.} \end{cases}$$

If $a \in \mathcal{P}^{\otimes}$ and $B \subset \mathcal{P}$ is a subset completely weak such that $a \prec B$, then we let $T(a, B)$ denote the representation of $\mathcal{P}$ which satisfies the following conditions with $T_0 = F^2$:

$$T_x = \begin{cases} G\{(1, \mathbf{u})^t\}, & \text{if } x \in a^{\curlyvee} \setminus B, \\ \tilde{T}_0 = G^2, & \text{if } x \in a^{\triangledown} + B^{\vee}, \\ 0, & \text{otherwise.} \end{cases}$$

In particular, $T(a, \varnothing) = T(a)$.

Note that, if $Y = \varnothing$ in an equipped poset $\mathcal{P}$ with a triple of points (a, b, c) VII_s-suitable, then $P'(a) = P(a)$, and $T'(a, c) = T'(a) = T(a)$.

2. Preliminaries Results

In [4] was proved the following theorem:

Theorem 2. *Let* (a, b, c) *be a* VII_s *triple of points of an equipped poset* $\mathcal{P}$ *and* $\mathcal{P}'_{(a,b,c)} = (\mathcal{P}^s_{(a,b,c)}, \Sigma_{(a,b,c)})$ *the corresponding derived equipped poset with relations. Then the short generalized differentiation functors* $D_{(a,b,c)}$ *induce bijections between indecomposables,*

$$\operatorname{Ind}\mathcal{P}\setminus[T(a, c), T(a, Y)] \rightleftarrows \operatorname{Ind}\mathcal{P}'_{(a,b,c)}\setminus[T(a, Y)].$$

The following results were proved in [2]:

Lemma 3. *Let* $X_1 \subset \cdots \subset X_n$, $Y_1 \subset \cdots \subset Y_n$, $X_i \subset \tilde{U}_0$, $Y_i \subset \tilde{V}_0$ *be two chains of G-subspaces. Furthermore, let* $X \subset U_0$, $Y \subset V_0$ *and* φ *be two F-subspaces and an F-linear map, respectively, such that* $\varphi \in [X, Y]$

and $\tilde{\varphi}(X_i) \subset Y_i$ *for* $1 \leq i \leq n$, *then* $\varphi \in \sum_{i=1}^{n+1}[(\tilde{X} + X_{i-1})^{-}, Y \cap Y_i^{-}]$, *where* $X_0 = 0$ *and* $Y_{n+1}^{-} = V_0$.

Lemma 4. *Let* U *and* V *be two representations of an equipped poset* $\mathcal{P} = a^{\triangledown} + b_{\triangle} + \{a \prec X \prec c\}$, *where* $a, c \in \mathcal{P}^{\otimes}$, $b \in \mathcal{P}^{\circ}$ *is a strong point incomparable with* a *and* c, $\{a \prec X \prec c\}$ *is a completely weak set containing an arbitrary set* X (*eventually empty*). *Then for an F-linear map* $\varphi : U_0 \to V_0$, *we have the following equivalences*:

(a) $\varphi \in {}_U\langle T(a)\rangle_V \Leftrightarrow \varphi \in [(U_b + U_c)^{-}, V_a^{+}], \tilde{\varphi}(U_c) \subset V_a$,

(b) $\varphi \in {}_U\langle T(a, c)\rangle_V \Leftrightarrow \varphi \in [(U_b + U_{a+X})^{-}, V_a^{+} \cap V_c^{-}]$,

$\tilde{\varphi}(U_c) \subset \tilde{V}_a^{+} \cap \tilde{V}_c^{-}, \tilde{\varphi}(U_{a+X}) \subset V_a \cap \tilde{V}_c^{-}$,

(c) $\varphi \in {}_U\langle P(a)\rangle_V \Leftrightarrow \varphi \in [U_b^{+}, V_a^{-}]$,

where if $X \subset U_0$, $Y \subset V_0$ *are the corresponding subspaces of the finite dimensional k-vector spaces* U_0 *and* V_0, *then* $[X, Y]$ *is a subspace of* $\mathrm{Hom}_k(U_0, V_0)$ *such that*

$$\varphi \in [X, Y] \text{ if and only if } X \subset \mathrm{Ker}\, \varphi \text{ and } \mathrm{Im}\, \varphi \subset Y.$$

For a category $\mathcal{A}$, we let $\langle U_i \mid i \in I\rangle_{\mathcal{A}}$ denote the ideal consisting of all morphisms passed through finite direct sums of the objects U_i. That is, if $\varphi : U \to V \in \langle U_i \mid i \in I\rangle_{\mathcal{A}}$, then there exist morphisms $f, g \in \mathcal{A}$ such that $\varphi = U \xrightarrow{f} \bigoplus_i U_i^{m_i} \xrightarrow{g} V$ with $m_i = 0$ for almost all i.

Corollary 5. *Let* U *and* V *be the representations of an equipped poset* $\mathcal{P} = a^{\triangledown} + b_{\triangle} + \{a \prec c_1 \prec \cdots \prec c_n\}$, *where* $\{a \prec c_1 \prec \cdots \prec c_n\}$ *is a completely weak chain incomparable with the strong point b. Then for an*

F-linear map $\varphi : U_0 \to V_0$, *we have the following equivalences if* $1 \le i \le n$ $(U_{c_0} = U_a)$:

(a) $\varphi \in {}_U\langle T(a, c_i)\rangle_V \Leftrightarrow \varphi \in [(U_b + U_{c_{i-1}})^-, V_a^+ \cap V_{c_i}^-]$,

$$\tilde{\varphi}(U_{c_n}) \subset \tilde{V}_{c_i}^-, \tilde{\varphi}(U_{c_i^\curlyvee}) \subset \tilde{V}_a^+ \cap \tilde{V}_{c_i}^-, \tilde{\varphi}(U_{c_{i-1}}) \subset V_a \cap \tilde{V}_{c_i}^-,$$

(b) $\varphi \in {}_U\langle T(a)\rangle_V \Leftrightarrow \varphi \in [(U_b + U_{c_n})^-, V_a^+], \tilde{\varphi}(U_{c_n}) \subset V_a$.

Lemma 4 allows us to conclude the following result.

Lemma 6. $\varphi \in {}_{U'}\langle T(a)\rangle_{V'}$ *if and only if* $\tilde{\varphi}(U_{a+X}) \subset V_a$, *and* $\varphi \in [(U_b + U_{a+X})^-, V_a^+]$.

If we fix two objects $U, V \in \mathcal{R}$ being $\mathcal{R} = \operatorname{rep} \mathcal{P}$, then assuming the following notation:

$R = \mathcal{R}(U, V)$, $R' = \mathcal{R}'(U', V')$, $I = {}_U\langle T(a), T(a, c)\rangle_V$ is the ideal which consists of morphisms passing through objects $T(a)$ and $T(a, c)$ in $\mathcal{R}$. Further, $I' = {}_{U'}\langle T(a)\rangle_{V'}$ is the ideal which consists of morphisms passing through the object $T(a)$ in $\mathcal{R}'$.

Since $\varphi = \varphi'$, $R \subset R'$ and for two arbitrary objects $U, V \in \mathcal{R}$ the following inclusions hold:

$$I \subset I' \subset R', \quad I \subset R \subset R'.$$

Proposition 7. $R \cap I' = I$ *and* $R + I' = R'$.

Lemma 8. *For each object* $W \in \operatorname{rep} \mathcal{P}'_{(a, b, c)}$, *there exists an object* $U \in \operatorname{rep} \mathcal{P}$ *such that* $U' \simeq W \oplus T^m(a)$ *for some* $m \ge 0$.

Proof. Suppose that $W \in \operatorname{rep} \mathcal{P}'_{(a, b, c)}$, we present subspace W_{c^+} in the form $W_{c^+} = \underline{W_{c^+}} \oplus K_{c^+}$ with $K_{c^+} = L_b \oplus R_{c^+} \oplus K_{b_{c^+}}$, $\tilde{K}_{c^+}^- = \tilde{K}_{c^+}^- \cap$

$W_b \oplus L_b$, L_b a complement, $R_{c^+} \cap W_b = 0$, $R_{c^+} \cap \widetilde{W}^-_{c^+} = 0$ and $K_{b_{c^+}} \subset W_b \cap W_{c^+}$.

We write subspace $K_{b_{c^+}}$ in the form $K_{b_{c^+}} = K_{b_{c^+}} \cap F(W_a \cap W_b) \oplus Z_*$. Therefore, if we fix a basis of the complement $Z_* = G\{e_{b_j} \mid 1 \leq j \leq t\}$, then it is possible to define a new F-subspace E_0 such that $E_0 = F\{e_{q_j^a}, e_{r_j^a}\}$. If we now build a subspace R_{a_*} such that $R_{a_*} = N_{a_*} = G\{e_{q_j^a} + \mathbf{u}e_{r_j^a}\}$, then the representation $U \in \operatorname{rep}\mathcal{P}$ given by the following formulas satisfies the required condition:

$$U_0 = E_0 \oplus W_0,$$

$$U_x = W_x + R_{a_*}^{(c_\curlywedge \setminus c) \cap \{x\}},$$

$$U_c = U_{c_\curlywedge \setminus c} + W_{c^-} + L_b + R_{c^+} + G\{e_{q_j^a} + e_{b_j}\},$$

$$U_x = W_x + F(R_{a_*}^{a^\nabla \cap \{x\}}) \text{ for the remaining points } x \in \mathcal{P}. \tag{6}$$

We are done. □

The representation *reduced derived* W of a representation $U \in \operatorname{rep}\mathcal{P}$ denoted $W = U^\downarrow$ (unique up to isomorphisms) is the maximal direct summand of U' without direct summands of the form $T(a)$, namely,

$$U' = U^\downarrow \oplus T^m(a).$$

In [4], it is proved that $(W^\uparrow)^\downarrow \simeq W$ and $(U^\downarrow)^\uparrow \simeq U$ if U and W are reduced objects, where $W^\uparrow = U$ is the object obtained in Lemma 8. Therefore, Theorem 2, Proposition 7 and Lemma 8 prove the following main result concerning differentiation VII_s for equipped posets with a triple of points VII_s-suitable and $Y = \varnothing$.

Theorem 9. *Let* $\mathcal{P}$ *be an equipped poset with a triple of points* (a, b, c) VII_s *-suitable. Then the functor of differentiation*

$$D_{(a,b,c)} : \operatorname{rep}\mathcal{P} \to \operatorname{rep}\mathcal{P}'_{(a,b,c)},$$

defined by formulae (5) *induces an equivalence between the quotient categories*

$$\mathcal{R}/\mathcal{I} \xrightarrow{\sim} \mathcal{R}'/\mathcal{I}',$$

where $\mathcal{R} = \operatorname{rep}\mathcal{P}$, $\mathcal{R}' = \operatorname{rep}\mathcal{P}'_{(a,b,c)}$ *and*

$$\mathcal{I} = \langle T(a), T(a, c)\rangle_{\mathcal{R}}, \quad \mathcal{I}' = \langle T(a)\rangle_{\mathcal{R}'}.$$

In particular, the functor induces mutually inverse bijections between classes of indecomposables

$$\operatorname{Ind}\mathcal{P}\backslash[T(a), T(a, c)] \rightleftarrows \operatorname{Ind}\mathcal{P}'_{(a,b,c)}\backslash[T(a)]$$

realized by the operations $\uparrow$ *and* $\downarrow$.

Corollary 10. *If* $\Gamma(\mathcal{R})$ *and* $\Gamma(\mathcal{R}')$ *are the Gabriel quivers of the categories* $\mathcal{R}$ *and* $\mathcal{R}'$, *then*

$$\Gamma(\mathcal{R})\backslash[T(a), T(a, c)] \simeq \Gamma(\mathcal{R}')\backslash[T(a)].$$

3. Differentiation VII_s with $Y \neq \varnothing$

In this section, we discuss categorical properties of the algorithm of differentiation $D_{(a,b,c)}$ for a poset $\mathcal{P}$ such that $\mathcal{P} = a^{\nabla} + b_{\triangle} + \{a \prec X \prec c \prec Y\}$ with $Y \neq \varnothing$. To do that, we consider the full subcategory $\mathcal{L} \subset \operatorname{rep}\mathcal{P}$ whose objects $U \in \mathcal{L}$ satisfy the condition $U_a^+ \subset \bigcap_{y \in Y} U_y^-$ (clearly, objects $U' \in \mathcal{L}' \subset \operatorname{rep}\mathcal{P}'_{(a,b,c)}$ satisfy this condition). In this case, $T(a, Y), T(a, c) \in \mathcal{L}$, $T(a) \notin \mathcal{L}$. Furthermore,

$$T'(a) = T(a), \quad P'(a) = P(a), \quad T'(a, c) = T'(a, Y) = T(a, Y).$$

For a poset $\mathcal{P} = a^{\triangledown} + b_{\vartriangle} + \{a \prec X \prec c \prec Y\}$, $Y \neq \varnothing$, Lemma 4 has the following interpretation:

Lemma 11. *Let U and V be the representations of an equipped poset* $\mathcal{P}$ *with a suitable triple of points* VII_s*-suitable* (a, b, c). *Then if* $X' = a + X$, $Y' = a + X + c + Y$, *then the following equivalences hold for an F-linear map* $\varphi : U_0 \to V_0$:

(a) $\varphi \in {}_U\langle T(a, Y)\rangle_V \Leftrightarrow \varphi \in [(U_b + U_c)^-, V_a^+]$, $\tilde{\varphi}(U_{Y'}) \subset \tilde{V}_a^+$,

furthermore $\tilde{\varphi}(U_{X'+c}) \subset V_a$,

(b) $\varphi \in {}_U\langle T(a, c)\rangle_V \Leftrightarrow \varphi \in [(U_b + U_{X'})^-, V_a^+ \cap V_c^-]$,

$\tilde{\varphi}(U_{Y'}) \subset \tilde{V}_c^-$, $\tilde{\varphi}(U_{c^{\curlyvee}}) \subset \tilde{V}_a^+ \cap \tilde{V}_c^-$, $\tilde{\varphi}(U_{X'}) \subset V_a \cap \tilde{V}_c^-$,

(c) $\varphi \in {}_{U'}\langle T(a, Y)\rangle_{V'} \Leftrightarrow \varphi \in [(U_b + U_{X'})^-, V_a^+]$,

$\tilde{\varphi}(U_{Y'\setminus c}) \subset \tilde{V}_a^+$, $\tilde{\varphi}(U_{X'}) \subset V_a$.

Fixing two objects $U, V \in \mathcal{L}$ with $\mathcal{L} \subset \operatorname{rep}\mathcal{P}$ and defining L, L' as R, R' in the case $Y = \varnothing$, we define $J = {}_U\langle T(a, Y), T(a, c)\rangle_V$ as the ideal consisting of morphisms passing through objects $T(a, c)$ and $T(a, Y)$ in $\mathcal{L}$. We let $J' = {}_{U'}\langle T(a, Y)\rangle_{V'}$ denote the ideal consisting of morphisms passing through the indecomposable $T(a, Y)$ in $\mathcal{L}'$.

Since $\varphi = \varphi'$, $L \subset L'$ and for arbitrary objects $U, V \in \mathcal{L}$, the following inclusions hold:

$$J \subset J' \subset L', \quad J \subset L \subset L'.$$

Corollary 12. $L \cap J' = J$.

Proof. Suppose that $\varphi \in L \cap J'$. Then $\tilde{\varphi}(U_x) \subset V_x$, for each $x \in \mathcal{P}$, in particular, $\tilde{\varphi}(U_c) \subset V_c$. Lemma 11 allows to conclude $\varphi \in [(U_b + U_{X'})^-,$

$V_a^+]$. Therefore, $\varphi \in [(U_b + U_c)^-, V_a^+] + [(U_b + U_{X'})^-, V_a^+ \cap V_c^-]$. Since φ can be written as a sum of the form $\varphi = \varphi_1 + \varphi_2$, where $\varphi_1, \varphi_2 \in J$, Lemmas 3 and 11 allow us to conclude $\varphi \in J$. □

Arguments used in the proof of Proposition 7 prove the following result for an equipped poset $\mathcal{P}$ which can be written in the form $\mathcal{P} = a^\triangledown + b_\vartriangle + \{a \prec X \prec c \prec Y\}$, $Y \neq \varnothing$:

Proposition 13. $L + J' = L'$.

Lemma 14. *For each object $W \in \mathcal{L}'$, there exists an object $U \in \mathcal{L}$ such that $U' \simeq W \oplus T^m(a, Y)$ for some $m \geq 0$.*

Proof. We describe space W_{c^+} as in Lemma 8, then the conclusion can be obtained by using the same arguments described in such lemma. Actually, space U_0 and subspace U_x, for each point $x \in \mathcal{P} \backslash a^\triangledown + Y$, have the form described in formulas (6). In this case, for each point $x \in a^\triangledown + Y$, subspace U_x is presented in the form:

$$U_x = W_x + F(R_{a_*}^{(a^\triangledown + Y) \cap \{x\}}). \tag{7}$$

□

Corollary 12, Proposition 13 and Lemma 14 prove the following theorem.

Theorem 15. *Let $\mathcal{P}$ be an equipped poset with a triple of points (a, b, c), VII_s-suitable (a, b, c). Then the functor of differentiation*

$$D_{(a,b,c)} : \operatorname{rep} \mathcal{P} \to \operatorname{rep} \mathcal{P}'_{(a,b,c)}$$

defined by formulae (5) *induces an equivalence between the quotient categories*

$$\mathcal{L}/\mathcal{J} \xrightarrow{\sim} \mathcal{L}'/\mathcal{J}',$$

where

$$\mathcal{J} = \langle T(a, c), T(a, Y)\rangle_{\mathcal{L}}, \quad \mathcal{J}' = \langle T(a, Y)\rangle_{\mathcal{L}'}. \qquad \square$$

Corollary 16. *If* $\Gamma(\mathcal{L})$ *and* $\Gamma(\mathcal{L}')$ *are the Gabriel quivers of the categories* $\mathcal{L}$ *and* $\mathcal{L}'$, *then*

$$\Gamma(\mathcal{L})\backslash[T(a, c), T(a, Y)] \simeq \Gamma(\mathcal{L}')\backslash[T(a, Y)]. \qquad \square$$

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