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VARIABLE SEPARATION SOLUTIONS AND THE ANNIHILATION SOLITONS IN A $(2+1)$-DIMENSIONAL NONLINEAR PARTIAL DIFFERENTIAL EQUATION

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Abstract

With the mapping approach and a linear variable separation method, a series of variable separation solutions (including solitory wave solutions, periodic wave solutions, and rational function solutions) for $(2+1)$-dimensional Burgers equation is derived. Based on the derived solitary wave solution, some specific solitons and the annihilation phenomena of solitons are also obtained.

1. Introduction

The $(2+1)$-dimensional Burgers equation takes the form

$$\begin{aligned} &u_t - uu_y - bvu_x - b^2 u_{xx} - bu_{yy} = 0, \\ &u_x - v_y = 0, \end{aligned} \tag{1}$$

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where b is an arbitrary constant. Some soliton excitations of this equation have been discussed by Liu et al. [1]. In this paper, we will study the variable separation solutions and the annihilation solitons of equation (1) with the Riccati mapping approach [2-9].

2. Variable Separation Solutions of Burgers Equation

By the balancing procedure for (1), ansatz becomes

$$u = f + g\phi + \frac{h}{\phi}, \qquad v = F + G\phi + \frac{H}{\phi}, \tag{2}$$

where $f,\ g,\ h,\ F,\ G,\ H,$ and q are functions of $(x,\ y,\ t)$ to be determined. Substituting (2) and Riccati equation $(\phi' = \sigma + \phi^2)$ into (1) and collecting coefficients of polynomials of ϕ, then setting each coefficient to zero, yields

$$2b^2gq_x^2 + g^2qy + 2bgq_y^2 + bGgq_x = 0, \tag{3}$$

$$\begin{aligned} &bGg_x + fgq_y + b^2gq_{xx} - gq_t + 2b^2g_xq_x \\ &\qquad + gg_y + bgq_{yy} + 2bg_yq_y + bFgq_x = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} &bGgq_x\sigma + bg_{yy} + g^2q_y\sigma + bFg_x + bGf_x + bHgq_x + fq_y \\ &\qquad + 2b^2gq_x^2\sigma + b^2g_{xx} + 2bgq_y^2\sigma - bGhq_x - g_t = 0, \end{aligned} \tag{5}$$

$$\begin{aligned} &bFgq_x\sigma + hg_y - b^2hq_{xx} + b^2f_{xx} + hq_t + bGh_x \\ &\qquad + b^2gq_{xx}\sigma + gh_y + bFf_x - 2b^2h_xq_x + bHg_x - gq_t\sigma \\ &\qquad + 2b^2g_xq_x\sigma - bhq_{yy} - bFhq_x - 2bh_yq_y - fhq_y + fgq_y\sigma \\ &\qquad + bgq_{yy}\sigma + 2bg_yq_y\sigma - f_t = 0, \end{aligned} \tag{6}$$

$$\begin{aligned} &bh_{yy} + bHgq_x\sigma + bFh_x + b^2h_{xx} - bHhq_x - bGhq_x\sigma \\ &\qquad + bHf_x - h_t - h^2q_y + 2bhq_y^2\sigma + 2b^2hq_x^2\sigma + fh_y = 0, \end{aligned} \tag{7}$$

$$-b^2 h q_{xx}\sigma - bFhq_x\sigma - fhq_y\sigma - bhq_{yy}\sigma + bHh_x$$

$$+ hh_y - 2bh_y q_y \sigma + hq_t\sigma - 2b^2 h_x q_x \sigma = 0, \tag{8}$$

$$2bhq_y^2\sigma^2 - bHhq_x\sigma - h^2 q_y\sigma + 2b^2 hq_x^2\sigma^2 = 0, \tag{9}$$

$$-Gq_y + gq_x = 0, \tag{10}$$

$$g_x - G_y = 0, \tag{11}$$

$$f_x + gq_x\sigma - Gq_y\sigma - hq_x + Hq_y = 0, \tag{12}$$

$$h_x - H_y = 0, \tag{13}$$

$$-hq_x + Hq_y = 0. \tag{14}$$

Based on (3)-(14), we have

$$f = 0, \quad g = -2bq_y, \quad h = 2bq_y\sigma,$$

$$F = -\frac{-q_t + b^2 q_{xx} + bq_{yy}}{bq_x}, \quad G = -2bq_x, \quad H = 2bq_x\sigma, \tag{15}$$

with the function q in a linear variable separation form

$$q = \chi(x, t) + \varphi(y), \tag{16}$$

where $\chi \equiv \chi(x, t)$, $\varphi \equiv \varphi(y)$ are two arbitrary variable separation functions of (x, t) and of y, respectively. Based on the solutions of Riccati equation [2], one can obtain the exact solutions of (1).

Case 1. For $\sigma = -1$, we can derive the following solitary wave solutions of (1):

$$u_1 = 2b\frac{\tanh(\chi + \varphi)^2 + 1}{\tanh(\chi + \varphi)}, \tag{17}$$

$$v_1 = \frac{\chi_t - b^2\chi_{xx} + 2b^2\chi_x^2 \tanh(\chi + \varphi)}{b\chi_x} + \frac{2b\chi_x}{\tanh(\chi + \varphi)}, \tag{18}$$

$$u_2 = 2b\frac{\coth(\chi+\varphi)^2+1}{\coth(\chi+\varphi)}, \tag{19}$$

$$v_2 = \frac{\chi_t - b^2\chi_{xx} + 2b^2\chi_x^2\coth(\chi+\varphi)}{b\chi_x} + \frac{2b\chi_x}{\coth(\chi+\varphi)}. \tag{20}$$

Case 2. For $\sigma = 1$, we can obtain the following periodic wave solutions of (1):

$$u_3 = -2b\frac{\tan(\chi+\varphi)^2-1}{\tan(\chi+\varphi)}, \tag{21}$$

$$v_3 = \frac{\chi_t - b^2\chi_{xx} + 2b^2\chi_x^2\tan(\chi+\varphi)}{b\chi_x} + \frac{2b\chi_x}{\tan(\chi+\varphi)}, \tag{22}$$

$$u_4 = 2b\frac{\cot(\chi+\varphi)^2-1}{\cot(\chi+\varphi)}, \tag{23}$$

$$v_4 = \frac{\chi_t - b^2\chi_{xx} + 2b^2\chi_x^2\cot(\chi+\varphi)}{b\chi_x} - \frac{2b\chi_x}{\cot(\chi+\varphi)}. \tag{24}$$

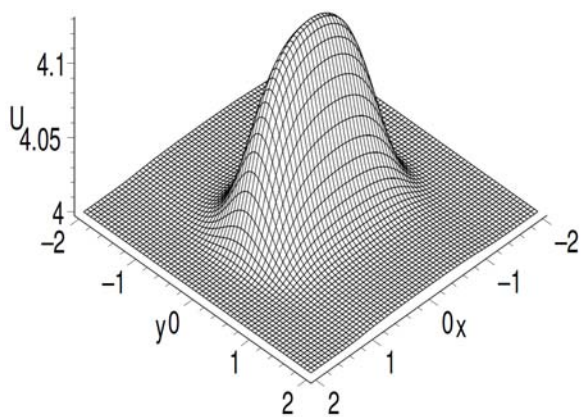

Figure 1. A plot of a soliton structure for the physical quantity U given by the solution (27) with the choice (28).

Case 3. For $\sigma = 0$, we can derive the following variable separation solution of (1):

$$u_5 = \frac{2b}{\chi + \varphi}, \tag{25}$$

$$v_5 = \frac{\chi_t - b^2\chi_{xx}}{b\chi_x} + \frac{2b\chi_x}{\chi + \varphi}. \tag{26}$$

3. Localized Structures of Burgers Equation

In this section, we mainly discuss some new localized coherent excitations in the $(2+1)$-dimensional Burgers equation. For simplification, we only discuss the field u_1 of (17), namely

$$U = u_1 = 2b\frac{\tanh(\chi + \varphi)^2 + 1}{\tanh(\chi + \varphi)}. \tag{27}$$

According to (27), if we choose $\chi(x, t)$ and $\varphi(y)$ as

$$\chi = 1 + \tanh^2(x + ct), \qquad \varphi = 1 + \tanh^2(y), \tag{28}$$

we can obtain a novel soliton structure presented in Figure 1 with fixed parameters $c = 1$, $b = 1$ and $t = -2$.

Furthermore, if we choose $\chi(x, t)$ and $\varphi(y)$ as

$$\chi = 1 + \tanh^6(x + ct), \qquad \varphi = 1 + \tanh^6(y), \tag{29}$$

we can obtain another annihilation soliton structure for the physical quantity U of (27) presented in Figure 2 with fixed parameters $c = 1$, $b = 1$ at different times $t = -10, -5, -1, 0, 1$. From Figure 2, we find that the amplitude and shape of the soliton becomes smaller and smaller with time, and finally it reduces to zero.

4. Summary and Discussion

In summary, via the mapping approach and a variable separation approach, we have found a new family of variable separation solutions to the $(2+1)$-dimensional Burgers equation. Based on the derived solitary wave solutions, we have studied the annihilation phenomena of solitons, which are different from the ones presented in the previous work.

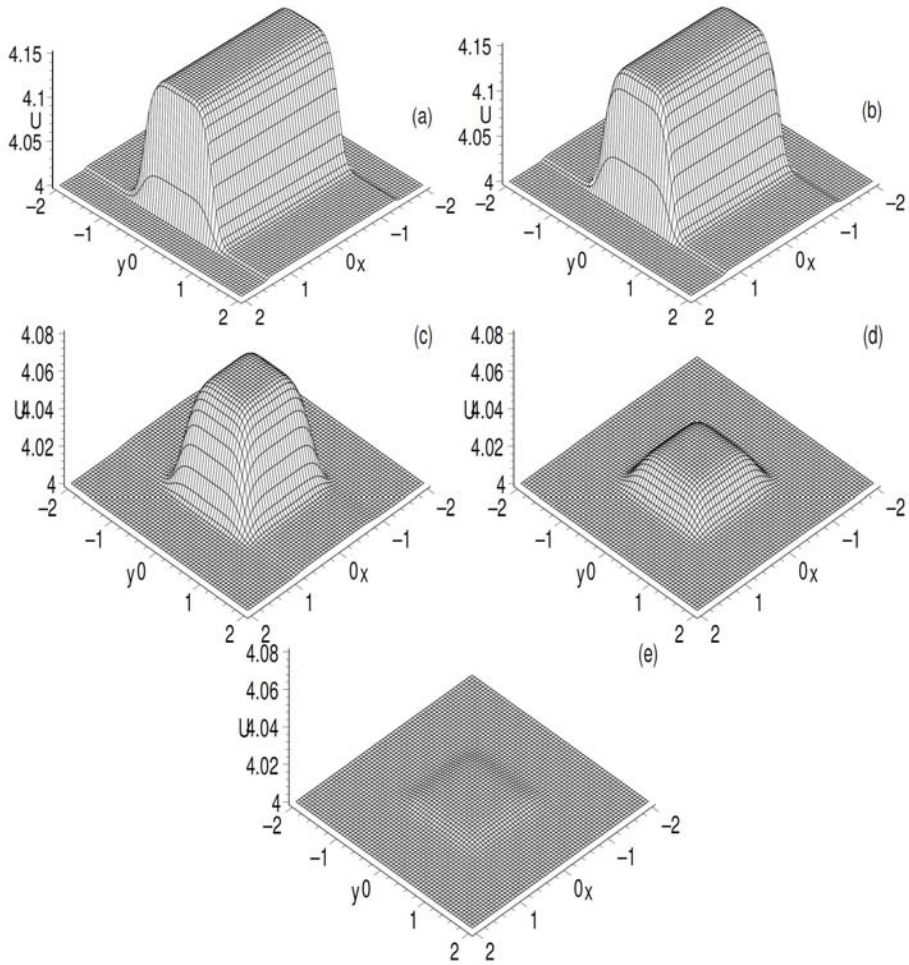

Figure 2. A plot of the annihilation soliton for the solution U by (27) under the condition (29) at different times (a) $t = -10$, (b) $t = -5$, (c) $t = -1$, (d) $t = 0$, (e) $t = 1$, respectively.

References

[1] C.-Liu, S.-H. Ma and J.-P. Fang, Far East J. Dyn. Sys. 15(2) (2011), 107.

[2] J. P. Fang and C. L. Zheng, Acta. Phys. Sin. 4 (2005), 670 (in Chinese).

[3] Z. Yang, S. H. Ma and J. P. Fang, Chin. Phys. B 20 (2011), 040301.

[4] S. H. Ma, J. P. Fang, Q. B. Ren and Z. Yang, Chin. Phys. B 21 (2012), 050511.

[5] S. H. Ma, J. Y. Qiang and J. P. Fang, Acta. Phys. Sin. 56 (2007), 0620 (in Chinese).

[6] S. H. Ma, J. B. Li and J. P. Fang, Commun. Theor. Phys. 48 (2007), 1063.

[7] S. H. Ma, J. P. Fang and C. L. Zheng, Chin. Phys. B 17 (2008), 2767.

[8] S. H. Ma, J. P. Fang, B. H. Hong and C. L. Zheng, Chaos Solitons Fractals 40 (2009), 1352.

[9] S. H. Ma, J. P. Fang and Q. B. Ren, Acta. Phys. Sin. 59 (2010), 4420 (in Chinese).