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CONVERGENCE OF A GENERAL ITERATIVE SCHEME FOR A FINITE FAMILY OF I_i-ASYMPTOTICALLY QUASI-NONEXPANSIVE MAPPINGS IN CONVEX METRIC SPACES AND APPLICATIONS

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Abstract

In this paper, we obtain the strong convergence of a kind of Ishikawa type iterative scheme with errors to a common fixed point of a finite family of I_i-asymptotically quasi-nonexpansive mappings in convex metric spaces. Our results improve and extend the corresponding results of Temir [8], Thianwan [9], and Khan and Ahmed [5].

1. Introduction and Preliminaries

Let (X, d) be a metric space and C be a nonempty closed convex subset of X. Let T be a self-mapping of C. Then T is said to be *asymptotically*

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nonexpansive if there exists $v_n \in [0, \infty)$ with $\lim_{n\to\infty} v_n = 0,$ such that

$$d(T^n x, T^n y) \le (1 + v_n) d(x, y), \quad \forall x, y \in C.$$

Let $F(T) = \{x \in X : Tx = x\};$ if $F(T) \neq \emptyset,$ then T is called *asymptotically quasi-nonexpansive* if there exists $v_n \in [0, \infty)$ with $\lim_{n\to\infty} v_n = 0,$ such that

$$d(T^n x, p) \le (1 + v_n) d(x, p), \quad \forall x \in C, \ p \in F(T).$$

Furthermore, it is *I*-asymptotically quasi-nonexpansive if there exists sequence $\{v_n\} \subset [0, \infty)$ with $\lim_{n\to\infty} v_n = 0$ such that

$$d(T^n x, p) \le (1 + v_n) d(I^n x, p), \quad \forall x \in C, \ p \in F(T),$$

where $I : C \to C$ is asymptotically nonexpansive mappings with $\{u_n\} \subset [0, \infty).$

Remark 1.1 (See [11])**.** From above definitions, it is easy to see that if $F(T)$ is nonempty, then an asymptotically nonexpansive must be asymptotically quasi-nonexpansive. But the converse does not hold.

In 1970, Takahashi [7] introduced initially a notion of convex metric space and studied the fixed point theorems for nonexpansive mappings. The convex metric space is a more general space and each normed linear space is a special example of a convex metric space. But there are many examples of convex metric spaces which are not embedded in any normed linear space (see [7]). Later on, some authors discussed the existence of fixed points and the convergence of the iterative processes for nonexpansive mappings in convex metric spaces (see, for example, [11]).

There are a number of results on fixed points of asymptotically nonexpansive and asymptotically quasi-nonexpansive mappings in Banach spaces and metric spaces. For example, the strong and weak convergences of the sequence of certain iterates to a fixed point of asymptotically quasi-nonexpansive mappings were studied by Schu [6]. Subsequently, Tian [10] gave some sufficient and necessary conditions for an Ishikawa iteration process of asymptotically quasi-nonexpansive mappings to converge to a fixed point in convex metric spaces. Recently, Wang and Liu [11] gave some results for an Ishikawa type iteration process with errors to approximate a fixed point of two uniformly quasi-Lipschitzian mappings in generalized convex metric spaces. Temir [8] gave some sufficiency and necessary conditions for implicit iteration process to approximate a fixed point of a finite family of *I*-asymptotically nonexpansive mappings in uniformly convex Banach spaces. Moreover, Thianwan [9] investigated the convergence of fixed point of projection type Ishikawa iteration for two asymptotically nonexpansive mappings in uniformly convex Banach spaces.

Inspired and motivated by the above mentioned works, in this paper, we study the Ishikawa type iteration process with errors to approximate common fixed point for a finite family of I_i-asymptotically nonexpansive mappings, and obtain the strong convergence theorems for such mappings in convex metric spaces.

We restate the following definitions and lemmas:

Definition 1.2 [10]**.** Let (X, d) be a metric space, $K = [0, 1]$, and $\{a_n\}, \{b_n\}, \{c_n\}$ be real sequences in $[0, 1]$ with $a_n + b_n + c_n = 1, (n \geq 1)$. A mapping $w : X^3 \times K^3 \to X$ is said to be a *convex structure* on *X*, if for any $(x, y, z, a_n, b_n, c_n) \in X^3 \times K^3$ and $u \in X$, the following inequality holds:

$$d(w(x, y, z, a_n, b_n, c_n), u) \leq a_n d(x, u) + b_n d(y, u) + c_n d(z, u).$$

If (X, d) is a metric space with convex structure *w*, then (X, d) is called a *convex metric space*. Moreover, a nonempty subset *C* of *X* is said to be *convex* if $w(x, y, z, a_n, b_n, c_n) \in C$, for all $(x, y, z, a_n, b_n, c_n) \in C^3 \times K^3$.

Definition 1.3. Let (X, d) be a metric space with a convex structure $w : X^3 \times K^3 \to X$. Let $T_i : X \to X, i \in \{1, ..., N\}, T_i$ be I_i-asymptotically quasi-nonexpansive mappings, I_i be asymptotically nonexpansive mappings.

Define an iterative scheme $\{x_n\}$ as follows:

$$x_{n+1} = w(y_n, I_{i(n)}^{k(n)} y_n, l_n, a_n, b_n, c_n),$$

$$y_n = w(x_n, T_{i(n)}^{k(n)} x_n, r_n, a'_n, b'_n, c'_n), \quad n \geq 1, \tag{1.1}$$

where $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}, \{c'_n\}$ are real sequences in $(0, 1)$ with $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$, $n = (k(n) - 1)N + i(n)$, $i(n) \in \{1, ..., N\}$. $\{l_n\}$, $\{r_n\}$ are two sequences in X for $n = 0, 1, 2, \ldots$. Then $\{x_n\}$ is called the *Ishikawa type iteration process* with errors for a finite family of I_i-asymptotically nonexpansive mappings T_i.

Lemma 1.4 [1]. *Let* $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ *and* $\{\mu_n\}$ *be four nonnegative real sequences satisfying* $\alpha_{n+1} \leq (1 + \gamma_n)(1 + \mu_n)\alpha_n + \beta_n$, *for all* $n \geq 1$. *If* $\sum_{n=1}^{\infty} \mu_n < \infty$, $\sum_{n=1}^{\infty} \gamma_n < \infty$ *and* $\sum_{n=1}^{\infty} \beta_n < \infty$, *then* $\lim_{n\to\infty} \alpha_n$ *exists.*

2. Main Results

Lemma 2.1. *Let X be a convex metric space with convex structure w, C be a nonempty closed convex subset of X,* $\{T_i : i \in \{1, 2, ..., N\}\} : C \to C$ *be* N I_i*-asymptotically quasi-nonexpansive mappings with sequences* $\{v_{in}\} \subset [0, \infty)$ *such that* $\sum_{n=1}^{\infty} v_{in} < \infty$, $i = 1, 2, ..., N$, *and let* $\{I_i : i \in \{1, ..., N\}\} : C \to C$ *be* N *asymptotically nonexpansive mappings with* $\{u_{in}\} \subset [0, \infty)$ *such that* $\sum_{n=1}^{\infty} u_{in} < \infty$, $i = 1, 2, ..., N$, *and* $F = \bigcap_{i=1}^{N} F(T_i) \cap F(I_i)$. *The Ishikawa type iteration sequence* $\{x_n\}$ *is generated by* (1.1), *in which* $\{l_n\}$ *and* $\{r_n\}$ *are two bounded sequences. If* $F \neq \emptyset$ *and* $\sum_{n=1}^{\infty} (c_n + c'_n) < \infty$, *then*

(1) *F is closed*;

(2) $\lim_{n\to\infty} d(x_n, p)$ *exists for any* $p \in F$.

Proof. (1) Let $\{\xi_n\} \subset F$ be such that $\xi_n \to x$ as $n \to \infty$. In addition, for $\forall i \in \{1, 2, ..., N\}$,

$$d(T_i x, x) \le d(T_i x, \xi_n) + d(\xi_n, x) \le (1 + u_{i1} v_{i1}) d(\xi_n, x).$$

This implies that $T_i x = x$. By the same way, we can get that $x \in F(I_i)$. So, x is a common fixed point of T_i and I_i. Thus, F is a closed set.

(2) Let $u_n = \max\{u_{1n}, u_{2n}, ..., u_{Nn}\}$, $v_n = \max\{v_{1n}, v_{2n}, ..., v_{Nn}\}$. So, we have that $\{u_n\}, \{v_n\} \subset [0, \infty)$ satisfy $\lim_{n\to\infty} u_n = 0, \lim_{n\to\infty} v_n = 0$. For any $p \in F$, we have

$$\begin{aligned} d(x_{n+1}, p) &= d(w(y_n, I_{i(n)}^{k(n)} y_n, l_n, a_n, b_n, c_n), p) \\ &\le a_n d(y_n, p) + b_n d(I_{i(n)}^{k(n)} y_n, p) + c_n d(l_n, p) \\ &\le a_n + b_n(1 + u_n) d(y_n, p) + c_n d(l_n, p) \\ &\le (1 + b_n u_n) d(y_n, p) + c_n d(l_n, p), \end{aligned} \tag{2.1}$$

$$\begin{aligned} d(y_n, p) &= d(w(x_n, T_{i(n)}^{k(n)} x_n, r_n, a'_n, b'_n, c'_n), p) \\ &\le a'_n d(x_n, p) + b'_n d(T_{i(n)}^{k(n)} x_n, p) + c'_n d(r_n, p) \\ &\le (a'_n + b'_n) d(x_n, p) + b'_n(u_n + v_n + u_n v_n) d(x_n, p) + c'_n d(r_n, p) \\ &\le (1 + b'_n(u_n + v_n + u_n v_n)) d(x_n, p) + c'_n d(r_n, p). \end{aligned} \tag{2.2}$$

For all $p \in F$, we have from (2.1) and (2.2) that

$$\begin{aligned} d(x_{n+1}, p) &\le (1 + b_n u_n) d(y_n, p) + c_n d(l_n, p) \\ &\le (1 + \alpha_n)(1 + \beta_n) d(x_n, p) + (1 + u_n)\gamma_n M, \end{aligned} \tag{2.3}$$

where $\alpha_n = b_n u_n$, $\beta_n = b'_n(u_n + v_n + u_n v_n)$, $\gamma_n = c_n + c'_n$, and $M = \sup_{p\in F, n\ge 0}(d(l_n, p) + d(r_n, p))$.

Notice that $\lim_{n\to\infty} u_n = 0$, $\lim_{n\to\infty} v_n = 0$, $\sum_{n=1}^{\infty}(c_n + c'_n) < \infty$, we have that $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \gamma_n = 0$.

By Lemma 1.4, $\lim_{n\to\infty} d(x_n, p)$ exists for each $p \in F$. The proof is completed.

Theorem 2.2. *Let X be a convex metric space with convex structure w, C be a nonempty closed convex subset of X,* $\{T_i : i \in \{1, 2, ..., N\}\} : C \to C$ *be* N I_i*-asymptotically quasi-nonexpansive mappings with sequences* $\{v_{in}\} \subset [0, \infty)$ *such that* $\sum_{n=1}^{\infty} v_{in} < \infty$, $i = 1, 2, ..., N$, *and* $\{I_i : i \in \{1, ..., N\}\} : C \to C$ *be N asymptotically nonexpansive mappings with* $\{u_{in}\} \subset [0, \infty)$ *such that* $\sum_{n=1}^{\infty} u_{in} < \infty$, $i = 1, 2, ..., N$, *and* $F = \bigcap_{i=1}^{N} F(T_i) \bigcap F(I_i)$. *The Ishikawa type iteration sequence* $\{x_n\}$ *is generated by* (1.1), *in which* $\{l_n\}$ *and* $\{r_n\}$ *are two bounded sequences. If* $F \neq \emptyset$ *and* $\sum_{n=1}^{\infty}(c_n + c'_n) < \infty$, *then* $\{x_n\}$ *converges strongly to a common fixed point in F if and only if* $\liminf_{n\to\infty} d(x_n, F) = 0$, *where* $d(x, F) = \inf\{d(x, p) : p \in F\}$.

Proof. The necessity of the conditions is obvious. Thus, we will only prove the sufficiency.

By Lemma 2.1, $\lim_{n\to\infty} d(x_n, p)$ exists for each $p \in F$. Hence, (2.3) and Lemma 1.4 guarantee that $\lim_{n\to\infty} d(x_n, F)$ exists and by the hypothesis $\liminf_{n\to\infty} d(x_n, F) = 0$, we have $\lim_{n\to\infty} d(x_n, F) = 0$.

For any given $\varepsilon > 0$, since $\lim_{n\to\infty} d(x_n, F) = 0$, there exists natural number N_1 such that when $n \geq N_1$, $d(x_n, F) < \frac{\varepsilon}{3}$. Thus, there exists $x^* \in F$ such that for above ε there exists positive integer $N_2 \geq N_1$ such that as $n \geq N_2$, $d(x_n, x^*) < \frac{\varepsilon}{2}$.

Now for arbitrary $n, m \geq N_2$, we have $d(x_n, x_m) \leq d(x_n, x^*) + d(x_m, x^*) < \varepsilon.$ This implies that $\{x_n\}$ is a Cauchy sequence in C, therefore it converges to a point, say $p \in C$. And $\lim_{n\to\infty} d(x_n, F) = 0$ gives that $d(p, F) = 0$. By Lemma 2.1, we know that F is closed. Thus $p \in F$. The proof is completed.

Let $I_i = I$ and $T_i = T$ for $i = 1, 2, \ldots,$ where $T : C \to C$ is an I-asymptotically quasi-nonexpansive mapping and I is an asymptotically nonexpansive mapping. Then Theorem 2.2 reduces to the following corollary.

Corollary 2.3. *Let C, X be same as in Theorem 2.2. Then $T : C \to C$ is I-asymptotically quasi-nonexpansive mapping with sequences $\{v_n\} \subset [0, \infty)$ such that $\sum_{n=1}^{\infty} v_n < \infty$ and $I : C \to C$ is asymptotically nonexpansive mapping with $\{u_n\} \subset [0, \infty)$ such that $\sum_{n=1}^{\infty} u_n < \infty$ and $F = \bigcap_{i=1}^{N} F(T) \cap F(I)$. Let $\{x_n\}$ be the Ishikawa type iteration process defined as follows*:

$$x_{n+1} = w(y_n, I^n y_n, l_n, a_n, b_n, c_n),$$

$$y_n = w(x_n, T^n x_n, r_n, a'_n, b'_n, c'_n), \quad n \geq 1, \tag{2.4}$$

where $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}, \{c'_n\}$ are real sequences in $(0, 1)$ with $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$. If $F \neq \emptyset$ and $\sum_{n=1}^{\infty}(c_n + c'_n) < \infty$, $\{l_n\}$ and $\{r_n\}$ are two bounded sequences. Then $\{x_n\}$ converges strongly to a common fixed point in F if and only if $\liminf_{n\to\infty} d(x_n, F) = 0$, where $d(x, F) = \inf\{d(x, p) : p \in F\}$.

Remark 2.4. Since each convex subset of a Banach space is a convex metric space, therefore, Theorem 2.3, Theorem 2.4 of Thiawan [9] are special cases of Theorem 2.2. And Theorem 3.3 of Temir [8] and Theorem 2.1 of Khan and Ahmed [5] are improved and extended by the above theorem.

3. Applications

In this section, we apply Theorem 2.2 to obtain some convergence theorems for scheme (1.1).

Theorem 3.1. *Let C, F,* $\{T_i\}, \{x_n\}$ *be same as in Theorem* 2.2. *Suppose that there exists a map* T_j *which satisfies the following conditions*: (1) $\liminf_{n\to\infty} d(x_n, T_j x_n) = 0$; (2) *there exists a function* $\varphi : [0, \infty) \to [0, \infty)$ *which is right continuous at* 0, $\varphi(0) = 0$ *and* $\varphi(d(x_n, T_j x_n)) \geq d(x_n, F)$ *for all n*. *Then the sequence* $\{x_n\}$ *converges to a common fixed point in F*.

Proof. Conditions (1) and (2) yield that

$$\liminf_{n\to\infty} d(x_n, F) \leq \liminf_{n\to\infty} \varphi(d(x_n, T_j x_n))$$

$$= \varphi(\liminf_{n\to\infty} d(x_n, T_j x_n)) = \varphi(0) = 0,$$

i.e., $\liminf_{n\to\infty} d(x_n, F) = 0$. By Theorem 2.2, $\{x_n\}$ converges to a common fixed point in *F*.

Remark 3.2. Theorem 3.2 in [4], Theorem 3.2 of Khan and Ahmed [5] are special cases of Theorem 3.1 by suitably choosing the spaces, the iterative scheme and the mappings.

Beg and Azam [1], Deng and Ding [2] discussed fixed points of nonexpansive type mappings on star-shaped subsets of convex metric spaces. Since then, some authors [9] studied fixed points problems on star-shaped convex metric spaces.

Definition 3.3. Let (X, d) be a metric space, $K = [0, 1]$, p be a fixed element of *X*. $\{a_n\}, \{b_n\}, \{c_n\}$ real sequences in $[0, 1]$ with $a_n + b_n + c_n = 1$, $(n \geq 1)$. A *p*-star-shaped structure in *X* is a mapping $w : X^3 \times K^3 \to X$ is said to be a *convex structure* on *X*, if for any $(x, y, z, a_n, b_n, c_n) \in X^3 \times K^3$

and $u \in X$, the following inequality holds:

$$d(w(x, y, z, a_n, b_n, c_n), p) \leq a_n d(x, p) + b_n d(y, p) + c_n d(z, p).$$

A metric space together with a *p*-star-shaped structure is called a *p-star-shaped metric space*. Clearly, a *p*-star-shaped metric space is a convex metric space, but the converse is not true.

Theorem 3.4. *Let X be a p-star-shaped metric space, C, F,* $\{T_i\}$, $\{x_n\}$ *be same as in Theorem* 2.2 *and* $\sum_{n=1}^{\infty}(b_n + b'_n) < \infty$. *Then* $\{x_n\}$ *converges strongly to a common fixed point in F if and only if* $\liminf_{n\to\infty} d(x_n, F) = 0$, *where* $d(x, F) = \inf\{d(x, p) : p \in F\}$.

Proof. For a fixed element $p \in X$ and any $q \in F$. Let $u_n = \max\{u_{1n}, u_{2n}, ..., u_{Nn}\}$, $v_n = \max\{v_{1n}, v_{2n}, ..., v_{Nn}\}$. So, we have that $\{u_n\}, \{v_n\} \subset [0, \infty)$ satisfy $\lim_{n\to\infty} u_n = 0$, $\lim_{n\to\infty} v_n = 0$,

$$\begin{aligned}
d(x_{n+1}, p) &= d(w(y_n, I_{i(n)}^{k(n)} y_n, l_n, a_n, b_n, c_n), p) \\
&\leq a_n d(y_n, p) + b_n d(I_{i(n)}^{k(n)} y_n, q) + c_n d(l_n, p) + b_n d(q, p) \\
&\leq a_n + b_n(1 + u_n) d(y_n, p) + c_n d(l_n, p) + b_n(2 + u_n) d(q, p) \\
&\leq (1 + b_n u_n) d(y_n, p) + c_n d(l_n, p) + b_n(2 + u_n) d(q, p), \qquad (3.1)
\end{aligned}$$

$$\begin{aligned}
d(y_n, p) &= d(w(x_n, T_{i(n)}^{k(n)} x_n, r_n, a'_n, b'_n, c'_n), p) \\
&\leq a'_n d(x_n, p) + b'_n d(T_{i(n)}^{k(n)} x_n, q) + c'_n d(r_n, p) + b'_n d(q, p) \\
&\leq a'_n d(x_n, p) + b'_n (1 + u_n)(1 + v_n) d(x_n, q) + c'_n d(r_n, p) \\
&\quad + b'_n (2 + u_n + v_n + u_n v_n) d(q, p)
\end{aligned}$$

$$\leq (1 + b'_n(u_n + v_n + u_n v_n))d(x_n, p) + c'_n d(r_n, p)$$

$$+ b'_n(2 + u_n + v_n + u_n v_n)d(q, p). \tag{3.2}$$

For all $q \in F$, we have from (3.1) and (3.2) that

$$d(x_{n+1}, p) \leq (1 + b_n u_n)d(y_n, p) + c_n d(l_n, p) + b_n(2 + u_n)d(q, p)$$

$$\leq (1 + \alpha_n)(1 + \beta_n)d(x_n, p) + (1 + u_n)(c_n + c'_n)M + (b_n + b'_n)\gamma_n, \tag{3.3}$$

where $\alpha_n = b_n u_n, \beta_n = b'_n(u_n + v_n + u_n v_n), \gamma_n = (1 + u_n)(2 + u_n + v_n + u_n v_n)$ and $M = \sup_{p \in F, n \geq 0}(d(l_n, p) + d(r_n, p))$.

Notice that $\lim_{n\to\infty} u_n = 0$, $\lim_{n\to\infty} v_n = 0$, $\sum_{n=1}^{\infty}(c_n + c'_n) < \infty$, $\sum_{n=1}^{\infty}(b_n + b'_n) < \infty$, for all $n \in \mathcal{N}$, we have that $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = 0$.

By Lemma 1.4, $\lim_{n\to\infty} d(x_n, p)$ exists for the fixed element $p \in X$. Hence, $\lim_{n\to\infty} d(x_n, q)$ exists for each $q \in F$.

The necessity of the conditions is obvious. Thus, we will only prove the sufficiency.

Since $\lim_{n\to\infty} d(x_n, q)$ exists for each $q \in F$. Hence, (3.3) and Lemma 1.4 imply that $\lim_{n\to\infty} d(x_n, F)$ exists, and by the hypothesis $\liminf_{n\to\infty} d(x_n, F) = 0$, we have $\lim_{n\to\infty} d(x_n, F) = 0$.

As the proof in Theorem 2.2, we may show that $\{x_n\}$ is a Cauchy sequence in C, therefore it converges to a point, say $p' \in C$. And $\lim_{n\to\infty} d(x_n, F) = 0$ gives that $d(p', F) = 0$. By Lemma 2.1, we know that F is closed. Thus $p' \in F$. The proof is completed.

Remark 3.5. Theorem 1 in [2] is special cases of Theorem 3.4 by suitably choosing the iterative scheme and the mappings.

References

[1] Ismat Beg and Akbar Azam, Fixed points on star-shaped subsets of convex metric spaces, Indian J. Pure Appl. Math. 18(7) (1987), 594-596.

[2] Lei Deng and Xie Ping Ding, Fixed points of nonexpansive mappings on star-shaped subsets of a convex metric space, Appl. Math. Mech. 13(2) (1992), 121-127.

[3] K. Goebel and W. A. Kirk, A fixed point theorem for asymptotically nonexpansive mappings, Proc. Amer. Math. Soc. 35 (1972), 171-174.

[4] A. R. Khan, A. A. Domlo, H. Fukhar-ud-din, Common fixed points Noor iteration for a finite family of asymptotically quasi-nonexpansive mappings in Banach spaces, J. Math. Anal. Appl. 341 (2008), 1-11.

[5] A. R. Khan and M. A. Ahmed, Convergence of a general iterative scheme for a finite family of asymptotically quasi-nonexpansive mappings in convex metric spaces and applications, Comput. Math. Appl. 59 (2010), 2990-2995.

[6] J. Schu, Iterative construction of fixed points of asymptotically nonexpansive mappings, J. Math. Anal. Appl. 158 (1991), 407-413.

[7] W. Takahashi, A convexity in metric space and nonexpansive mappings, Kodai. Math. Sem. Rep. 22 (1970), 142-149.

[8] S. Temir, On the convergence theorems of implicit iteration process for a finite family of I-asymptotically nonexpansive mappings, J. Comput. Appl. Math. 225(2) (2009), 398-405.

[9] S. Thianwan, Common fixed points of new iterations for two asymptotically nonexpansive nonself-mappings in a Banach space, J. Comput. Appl. Math. 224 (2009), 688-695.

[10] Y. X. Tian, Convergence of an Ishikawa type iterative scheme for asymptotically quasi-nonexpansive mappings, Comput. Math. Appl. 49 (2005), 1905-1912.

[11] C. Wang and L.-W. Liu, Convergence theorems for fixed points of uniformly quasi-Lipschitzian mappings in convex metric spaces, Nonlinear Anal. 70(5) (2009), 2067-2071.