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A MATHEMATICAL MODEL ABOUT HUB LOCATION PROBLEM BASED MINIMUM SPANNING TREE ALGORITHM

Yingci Zhang, Xiao Wang and Yan Peng

School of Computer Science
Sichuan University of Science and Engineering
Xueyuan Road No. 180
Zigong, P. R. China

Abstract

This paper introduces a mathematical model of binary levels tree hub location problem to determine hub locations from potential locations. Usually, the cost of connection link between two places is very high. The network we designed must contain some hub nodes and a route between all nodes. Switching, transshipment and distribution of people or commodity are acted by hub nodes. With the graph theory, we design the network of non-loop graph with minimum number of arcs when traversing all nodes. Minimum spanning tree (MST) algorithm was applied to minimize the connection cost and allocate non-hub nodes to the located hubs. In this paper, we also describe the proposed model and corresponding mathematical model. Then we use MATLAB tools to analyse data instances and conclude the effectiveness of the model we proposed by minimum spanning tree (MST) algorithm.

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I. Introduction

The focus of studies about hub location problem (HLP) in recent years is concentrated on the origin and destination nodes exist in the network. The basic demand of these nodes is transmission needed be satisfied. In the networks, some special nodes are regarded as hub nodes which act as switching, transshipment and distribution facilities [1]. So, transferring people or commodity from its origin to its destination point must be passed within hub facilities to take advantage of economies of scale [1].

Usually, people study hub location problem from two basic models, single allocation and multiple allocations HLP. People advanced single allocation uncapacitated hub location problem (SAUHLP) [2] firstly in 1987 then a linear integer formulation in 1994. An exchange heuristic, tabu search [3, 4] and GRASP methods were used in large size problem to find a better result. And some solution approaches are proposed for SAUHLP such as a new mathematical formulation along with tight LP [5] relaxation.

Meanwhile, Skorin-Kapov et al. [5] studied a capacitated single allocation HLP by introducing a formulation with two networking strategy. If not cost efficient, the connection between origin and destination nodes need not go through a hub network. Soon afterwards a new formulation with two heuristic algorithms was expressed and several researches about multiphase characteristics of single allocation hub location problem with capacity constraint or applied branch were done. From 2007, a new bi-criteria approach [6] was used to minimize the run time and total cost in hub nodes instead of capacity constraint. People then studied different capacity sizes for each hub in the case of solution methods of single allocation HLP.

Another basic model is multiple allocations HLP that was studied in this field in past two decades as a linear integer programming model in multiple uncapacitated HLP. Then a branch and bound algorithm [7] was used in large scale problems and a bender decomposition algorithm to solve the large size problem has been designed. To visit hubs singly or in pair, people then considered a model for uncapacitated HLP.

Based on capacitated multiple allocations HLP linear model, people illustrated new mixed integer linear formulation [8] along with a heuristic approach then preprocessing procedures and tightening constraints for mixed integer linear programming models [9]. Later a model based on tight integer linear programming was used to solve medium size problem.

There is classic literatures that hub network should designed to interconnected completely to get economies of scale in most distribution networks. In fact, the streaming in hub networks connection links is much less than the streaming in the connection link between non-hub nodes to hub node. That means it may cause some unrealistic results if we design so. For this reason, a non-fully interconnected design of hub location network [10, 11] was raised which had been proved no condition on locating hub arcs in the network with hub arc location model. In 2009, hub location problem along with hub arc location was studied with a time definite model which had been illustrated model efficiently by an example of truck transportation [12]. Several results show that some hub nodes need not connected together with hub location model in the incomplete hub network.

From a graph theory perspective, the studies on tree hub location problem [13] in digital data service networks proposed some practicable solutions. For example, in a two-level hierarchal network, the backbone network is a star form composed by tree and access network. An advanced case is a three-level hierarchal network including three level hub nodes that the central hubs in top layer were connected with the second layer completely, and the hub nodes in the lowest layer were connected back to the top layer in a star form. Besides more examples such as a single assignment tree hub location problem was introduced by Contreras et al. [14].

We mentioned the researches about HLP in decades above. Based on these, mathematical theories or solutions were applied in the tree hub location problem such as minimum spanning tree (MST) algorithm. The MST algorithm means in a network with several nodes, there exists a loop-free graph with minimum number of arcs when traversing all nodes. The typical applications of MST are telecommunication system and AIR transportation network. For instance, the cost of connection link between two nodes is highly expensive and a route must be placed between all nodes in the AIR transportation network.

In this paper, we study the mathematical model of binary levels tree HLP to determine hub locations from potential locations. Because of the possibility of connections link between non-hub nodes, we apply minimum spanning tree (MST) algorithm to minimize the connection cost and allocate non-hub nodes to the located hubs. In this paper, we also describe the proposed model and corresponding mathematical model. Then we use MATLAB tools to analyse data instances and conclude the effectiveness of the model we proposed by minimum spanning tree (MST) algorithm.

II. The Binary Levels Tree Hub Location Problem

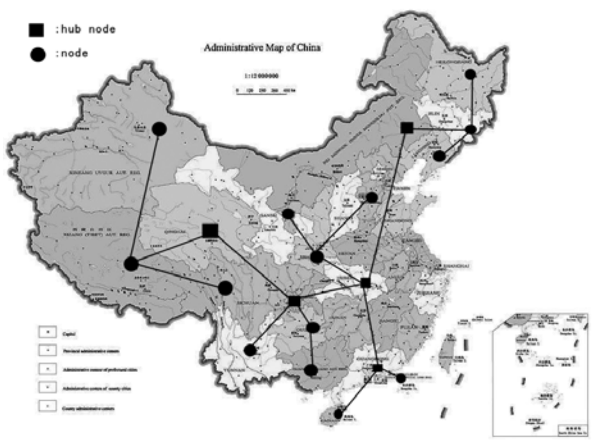

Figure 1. A configuration of the binary levels tree HLP.

We define an undirected subgraph of $G = (V, E)$ in the binary levels tree hub location problem. V is a vertex set including N nodes of all origin, destination and hub nodes. E is an edge vertex set including all connection links in the network. $D1_{hh'}$, $D2_{nm}$, $D3_{nh}$ are hub to hub, node to node and node to hub connection distances in the network, respectively. Regarding all nodes corresponding to each hub as a unit block, we design the network as transmission must be passed through the hub network between blocks. In the case of air transportation system, we provide a configuration of the binary levels tree hub location problem, as shown in Figure 1.

III. The Mathematical Model

In this section, we apply minimum spanning tree (MST) algorithm to study the binary levels tree hub location network and analyse the corresponding mathematical model. There are some notations in the mathematical model.

We define:

N_{hn} (the number of hub nodes);

N_n (the number of nodes);

$D1_{hh'}$ (the distance between hub h and hub h');

$D2_{nm}$ (the distance between node n and node m);

$D3_{nh}$ (the distance between node n and hub node h);

R_h (the hub node's radius);

C_h (the establishment cost of hub node h).

Also, there are some Boolean values as decision variables, as shown in Table 1.

Table 1. Boolean value definition of decision variables

A_h	0	o.w
	1	hub h is active
U_h	0	o.w
	1	hub h is a root node
$C_{hh'}$	0	o.w
	1	exist a directed connection between hub h and h'
C_{nm}	0	o.w
	1	exist a directed connection between node n and m
C_{nh}	0	o.w
	1	exist a directed connection between node n and hub h

Besides the Boolean values above, we define several numeral values as decision variables: F_m (amount of flow reached to node m), $F_{hh'}$ (amount of flow from hub h to hub h'), W_{hn} (amount of weight entered to hub h from node n), W_n (amount of weight entered to node n).

In the mathematical model, we make the objective function as:

$$\min d = d1 + d2 + d3 + d4. \tag{1}$$

In (1):

$$d1 = \sum_{h} \sum_{h'} C_{hh'} \cdot D1_{hh'}.$$

This function minimizes total distance between hub and hub.

$$d2 = \sum_{n} \sum_{m} C_{nm} \cdot D2_{nm}.$$

This function minimizes total distance between node and node.

$$d3 = \sum_{h} \sum_{n} C_{nh} \cdot D3_{nh}.$$

This function minimizes total distance between hub and node.

$$d4 = \sum_{h} R_h \cdot C_h.$$

This function computes establishment cost of hub nodes.

Constraints are described as follows in this proposed model:

$$\sum_{h \in N} R_h \geq 3.$$

This constrain means that the network should have at least three hub nodes because the presupposition is at least three nodes in a tree.

$$A_h \leq U_h, \quad h \in N_n.$$

This constrain means that a hub node can be defined as the root if active.

$$A_h - (A_{h'} - 1) * M \geq C_{hh'},$$

$$A_{h'} \geq C_{hh'},\ h,\ h' \in N_{hn}.$$

These two constrains mean allowing two active hub nodes adjacent.

$$\sum_{h \in N_{hn}} U_h = 1, \quad \sum_{h \in N_{hn}} C_{hh'} = (1 - U_{h'}) * A_{h'}, \quad h' \in N_{hn}.$$

These two constrains limit that only one hub node can be regarded as the root node in the network.

$$\sum_{h \in N_{hn}} C_{hh'} = (1 - U_{h'}) * A_{h'}, \quad h' \in N_{hn}, \tag{2}$$

$$C_{hn} \leq F_{hh'},\ h,\ h' \in N_{hn};\ C_{hn} * M \geq F_{hh'},\ h,\ h' \in N_{hn}. \tag{3}$$

(2), (3) mean the amount of flow between two hub nodes.

$$\sum_{h \in N_{hn}} F_{hh'} - \sum_{h' \in N_{hn}} F_{h'h''} \geq ((-U'_h * M) + 1) * A_h, \quad h' \in N_{hn}, \tag{4}$$

$$\sum_{h \in N_{hn}} F_{hh'} - \sum_{h' \in N_{hn}} F_{h'h''} \leq ((-U'_h * M) + 1) * A_h, \quad h' \in N_{hn}. \tag{5}$$

(4), (5) guarantee the network has no loop.

$$\sum_{n \in N_n} C_{nh} \leq A_h * M, \quad h \in N_{hn}, \tag{6}$$

$$\sum_{n \in N_n} C_{nh} \geq A_h, \quad h \in N_{hn}. \tag{7}$$

As we have mentioned before, there must be at least one connection between active hub node and nodes. This condition is expressed by (6) and (7).

$$\sum_{h \in N_{hn}} \sum_{n \in N_n} C_{nh} \leq m.$$

This constraint means the number of nodes can imply the number of arcs between hub nodes and nodes.

$$\sum_{n \in N_n} C_{nm} + \sum_{h \in N_{hn}} C_{nh} = 1, \quad m \in N_n.$$

This constraint means the connection between each hub nodes and nodes should be only one.

$$\sum_{n \in N_n} C_{nm} + \sum_{h \in N_{hn}} C_{nh} \leq F_m, \quad m \in N_n.$$

This constraint means the amount of flow reached to node is at least one.

$$F'_m - \sum_{m \in N_n} C_{nm} * F_m = 1, \quad n \in N_n.$$

In this constraint, F'_m is the amount of flow reached to node *n*. It ensures there is no closed loop between nodes in the network.

$$\sum_{n \in N} F'_m * C_{nh} \leq 2 * A_h * M, \quad h \in N_{hn}, \tag{8}$$

$$\sum_{n \in N_n} F'_m * C_{nh} \geq 2 * A_h, \quad h \in N_{hn}. \tag{9}$$

(8) and (9) mean that at least two nodes are included to an active hub node.

$$F'_m \geq 1 - (W_n * M), \quad n \in N_n, \tag{10}$$

$$F'_m \leq 1 + (W_n * M), \quad n \in N_n, \tag{11}$$

$$\underset{m \in N_n}{\mathrm{MAX}}\, C_{nm} * (D2_{nm} + W'_n) = W_n, \quad n \in N_n, \tag{12}$$

$$C_{nh} * (D3_{nh} + W_n) = W_{hn}, \quad \forall n \in N_n, \ h \in N_{hn}. \tag{13}$$

(10), (11), (12) mean the weight of each node. W_n is the weight entered to node *m* in (12). (13) means the weight of active hub node from node.

$$W_{hn} \leq R_h, \quad \forall n \in N_n, \ h \in N_{hn}. \tag{14}$$

(14) presents capacity restriction of hub node.

$$A_h \in \{0, 1\}, \quad h \in N_{hn}, \tag{15}$$

$$U_h \in \{0, 1\}, \quad h \in N_{hn}, \tag{16}$$

$$C_{hh} \in \{0, 1\}, \quad \forall h \in N_{hn}, \ \ h' \in N_{hn}, \tag{17}$$

$$C_{nm} \in \{0, 1\}, \quad \forall n \in N_n, \ \ m \in N_n, \tag{18}$$

$$C_{nh} \in \{0, 1\}, \quad \forall h \in N_{hn}, \ \ n \in N_n. \tag{19}$$

These five constraints are the Boolean values defined as decision variables.

$$F_m \geq 0, \quad m \in N_n, \tag{20}$$

$$F_{hh'} \geq 0, \quad \forall h \in N_{hn}, \ \ h' \in N_{hn}, \tag{21}$$

$$W_{hn} \geq 0, \quad \forall h \in N_{hn}, \ \ n \in N_n, \tag{22}$$

$$W_n \geq 0, \quad n \in N_n. \tag{23}$$

These four constraints are several numeral values as decision variables.

IV. Analysis for Data Instances

In this section, we use MATLAB tools to analyse data instances with 6 potential hub locations and 12 non-hub locations. The data in the following tables demonstrate the effectiveness of the model we proposed by minimum spanning tree (MST) algorithm.

Table 2. Establishing cost and radius of each hub node

Hub	Establishing cost	Radius
1	16	28
2	12	10
3	16	24
4	11	15
5	6	9
6	10	14

Table 3. Distances between hub nodes

Distance	1	2	3	4	5	6
1	0	14	12	13	16	11
2	14	0	13	11	12	9

3	12	13	0	16	18	12
4	13	11	16	0	11	19
5	16	12	18	11	0	7
6	11	9	12	19	7	0

Table 4. Distances between nodes and hub nodes

Distance	1	2	3	4	5	6
1	8	12	13	10	4	13
2	7	19	14	13	6	13
3	9	8	10	21	18	13
4	8	9	11	7	7	16
5	15	8	16	12	21	12
6	20	6	10	10	24	13
7	12	20	21	6	7	13
8	11	6	9	17	12	11
9	17	16	8	10	10	6
10	9	11	11	6	20	19
11	14	8	11	5	7	9
12	15	11	4	6	15	7

Table 5. Distances between nodes

Distance	1	2	3	4	5	6	7	8	9	10	11	12
1	0	8	11	10	∞	∞	7	6	11	9	7	12
2	8	0	18	∞	13	∞	∞	19	16	10	11	9
3	11	18	0	7	10	19	∞	6	4	8	16	11
4	10	∞	7	0	∞	20	∞	9	16	10	12	16
5	∞	13	10	∞	0	10	11	6	21	3	14	9
6	∞	∞	19	20	10	0	26	10	6	11	∞	16
7	7	∞	∞	∞	11	26	0	7	9	6	16	12
8	6	19	6	9	6	10	7	0	12	7	∞	16
9	11	16	4	16	21	6	9	12	0	3	7	11
10	9	10	8	10	3	11	6	7	3	0	∞	5
11	7	11	16	12	14	∞	16	∞	7	∞	0	6
12	12	9	11	16	9	16	12	16	11	5	6	0

Table 6. Active hub nodes

Hub	1	2	3	4	5	6
			T	T	T	T

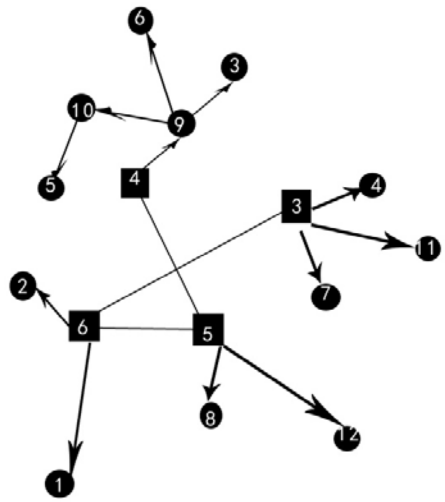

Figure 2. A configuration of numerical example results.

Binary levels tree hub location network is drawn in Figure 2.

V. Conclusion

In this paper, we studied the mathematical model of binary levels tree HLP to determine hub locations from potential locations. By hub facility, we modeled the problem of AIR transportation from origin to destination nodes using the concept of hub and non-hub locations. And based on graph theory, we defined the network case of the binary levels tree hub location problem as an undirected subgraph with two sets. One is a set of nodes that are hub and non-hub facilities and another is a set of arcs that are connections links. Because of the possibility of connections link between non-hub nodes, we apply minimum spanning tree (MST) algorithm to minimize the connection cost and allocate non-hub nodes to the located hubs. In this paper, we also described the proposed model and corresponding mathematical model. Then we used MATLAB tools to analyse data instances and concluded the effectiveness of the model we proposed by minimum spanning tree (MST) algorithm. The tree structure between non-hub nodes was created and every optimal path was analysed. By using MATLAB tool, the result shows that the mathematical model of binary levels tree HLP is effective indeed.

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