



## **A STUDY AND MODELING OF NON-CUMULATIVE ATTENUATION IN COILED OPTICAL FIBER WITH SMALL RADII OF CURVATURE**

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### **Abstract**

Recently, the usage of optical fiber coils has increased significantly, especially in the design of physics and chemical sensors. Therefore, it is important to test the theoretical current models developed to predict the power loss throughout an optical fiber coiled bent. In this paper, a pioneer and popular model, the Marcuse model of power loss and a model derived from this, the Schermer and Cole model, were studied and evaluated for optical fiber coils of small radii. Power attenuation in a bent fiber data was collected using an optical time domain reflectometer (OTDR), and they were compared to the theoretical

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predictions of the Marcuse model and Schermer and Cole model. It was observed that the models predicted correctly the attenuation behavior for usual curvature radii, however, these failed to predict accurately the attenuation behavior for small curvature radii, underestimating considerably the actual power loss. The main factor that Marcuse's model (coefficient of power loss  $2\alpha$ ) fails to predict the behavior of the attenuation when the fiber is bent at small radii of curvature is because we consider the parameter  $\beta$  (mode propagation of the wave guide) as a constant. Schermer and Cole [16] made clarifying remarks about the propagation constant  $\beta$ , but even so its model does not predict correctly the behavior of the attenuation when the fiber is bent in small radii of curvature. When the fiber is bent at small radii of curvature, it is possible that new mechanisms of light leaking are present, due to the extreme distortion of the mode configuration. In this paper, we propose a new model function of the optical path, where the parameter  $\beta$  is not constant, but it depends on the number of loops and the radii of curvature. Theoretical behavior of this model correctly predicts the behavior of the experimental data of attenuation. The terms of this model are based in the form  $(a/b)$ , thus correspond exponential behavior of basis  $(a/b)$ , that have the modes LP.

## 1. Introduction

Nowadays, optical fibers are being used in a wide variety of applications, i.e., telecommunications, aerospace industry, civil engineering and sensors, measuring micro cracks in large concrete structures [1], and thermal effects [2]. Optical fiber sensors have been evolving consistently and now are capable of measuring different physical and chemical parameters, such as temperature, strength and pressure on a structure. They can be used as well to locate hydrocarbon leaks in pipelines [3], and to control unmanned sailing and radio-controlled vehicles [4]. They have been implemented in the use of optical gyroscopes [5, 6] and monitoring temperature distribution in transformers [7].

The coiled optical fiber with continuous curvatures is a geometric design

that has been implemented, recently. It is employed for various applications: gyroscopes, automated ships, radio-controlled vehicles and hydrocarbon leaks detection are a few examples. Then it is quite important to have an analytic model that can describe or at least predict accurately the light power behavior inside a bent fiber, especially its attenuation.

Currently, there are a few models that try to predict the said behavior with a certain degree of accuracy. Some models have been developed that calculate the error in the reflection of the coiled optical fiber [8]. Also, analytical models have been proposed that predict the attenuation behavior in single bent and multimode optical fibers [9-16]. The Marcuse model for power loss [15], apparently the most popular, was developed a few decades ago. This model is very good, and it is widely used by scientists. However, it was developed before the need of coiled optical fiber with continuous loops, and it does not take into consideration the light power behavior after the first loop. Also, this model predicts quite well the power loss for most curvatures, except for those below a critical radius.

In this work, the Marcuse model was tested as well as a new model developed by Schermer and Cole [16], that was developed by improving some simplifying assumptions made by Marcuse. They were compared to experimental results of power attenuation in coiled optical fiber. It turns out that both the models could be used as a base to develop a new model for coiled wave guides, that could characterize the power loss behavior and non-cumulative attenuation in continuous loops.

A new model had to be developed by focusing on the propagation constant  $\beta$ , and making it a function of both the curvature radius and the number of loops. The new developed model works quite well, as found when compared to the experimental results.

## 2. Experimental

A step graded Thorlabs multimode fiber was used (the fiber characteristics are listed in Tables 1 and 2). The fiber was coiled around steel cylinders of 18 different radii (from 1.5 to 10mm), and the power loss was

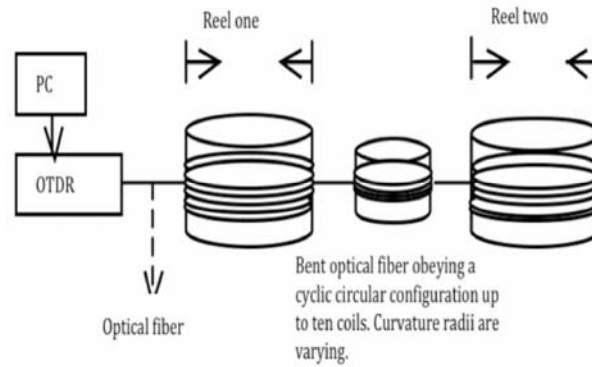
measured for different number of loops around each cylinder as well (from 1 to 10 loops), as shown in Figure 1. The power attenuation for each radius of curvature and the number of loops was measured three times to make sure the launching conditions were steady. The equipment used to measure the power loss was an optical time domain reflectometer (OTDR), model FM8513 from Tektronics.

**Table 1.** Parameters: optical fiber parameters

Parameters	Optical fiber
Core diameter	$125 \pm 0.7\mu\text{m}$
Cladding diameter	$245 \pm 5\mu\text{m}$
Numerical aperture	$0.275 \pm 0.015$

**Table 2.** OTDR parameters

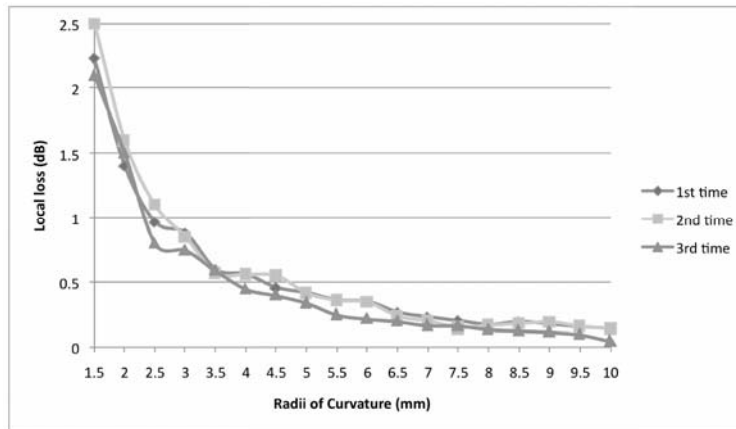
Parameters	OTDR
Wavelength	850nm to 1300nm
Pulse width	10 $\mu\text{s}$ /1000m
Output	26.5dB
Operating range of wavelength	850nm $\pm$ 20nm



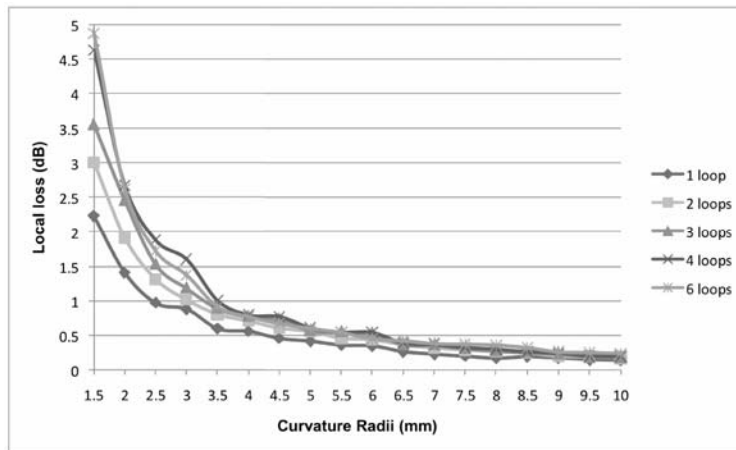
**Figure 1.** Diagram showing an optical fiber wrapped around several cylinders with different diameters.

### 3. Experimental Results

Figure 2 shows the local loss results for a coiled optical fiber with only one loop but different radii of curvature. It can be easily observed that the results are very similar; thus the launching conditions were the same through the whole experimentation and was not a significant factor causing variance in the results obtained from the OTDR.

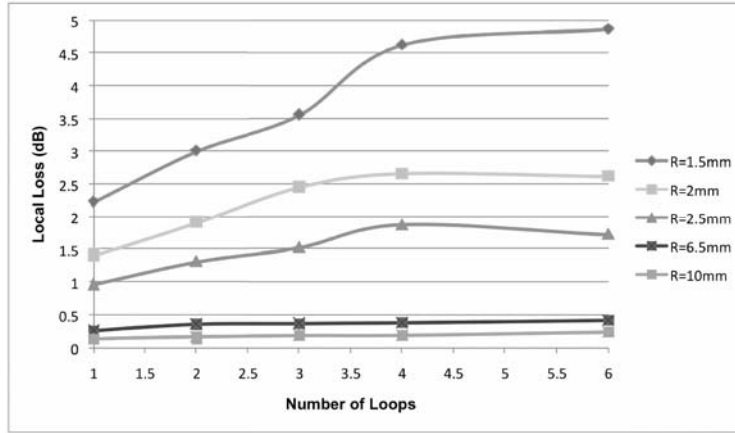


**Figure 2.** The attenuation behavior of power loss in a coiled optical fiber with one loop and different radii of curvature.



**Figure 3.** The attenuation behavior of power loss in a coiled optical fiber with several loops.

Figure 3 shows the local loss results. As expected, the smaller the curvature radii are, the greater the losses are produced. However, it is evident that for any radius smaller than 4.5mm, the power loss increases significantly, especially with a radius of 1.5mm. The power loss for any radius greater than 5mm seems to be pretty much the same, because the difference between them is almost undetectable. So it is observed that for a radius of 1.5mm, the power loss is large, while it is regular for radii among 2 to 3mm and small for radius larger than 3.5mm.



**Figure 4.** The attenuation behavior of power loss in a coiled optical fiber with different radii of curvature.

The effect the number of loops around each cylinder has on the power loss is illustrated in Figure 4. In this figure, the power loss behavior of five radii is graphed as a function of the number of loops. Previous studies indicate that most of the power loss is lost in the first quarter of the loop [21]; nevertheless, the results found in the experimentation are quite interesting and worthy to analyze:

- For smaller radii, i.e., below 2.5mm, the loops have an average increment of 30% in respect to the previous loop, instead of the 100% that the Marcuse model predicts, until it becomes steady after the fourth loop.

- For larger radii, i.e., larger than 6.5mm, there is a leap in power loss in the first and second loops (around 30%) and then it becomes almost steady just after the second loop.

#### 4. Current Attenuation Models

A number of optic fiber bend loss models were investigated [9-16]. However, we have selected only two models to work with: the Marcuse model [15] and the Schermer and Cole model (S&C) [16]. Those models were selected because they are the result of fundamental analysis and they are easy to handle. Here is the formula of the Marcuse model [15]:

$$2\alpha = \frac{\sqrt{\pi}\kappa^2 \exp\left[-\frac{2}{3}\left(\frac{\gamma^3}{\beta^2}\right)R\right]}{e_v \gamma^{3/2} V^2 \sqrt{R} K_{v-1}(\gamma a) K_{v+1}(\gamma a)}, \quad e_v = \begin{cases} 2, & v = 0, \\ 1, & v \neq 0, \end{cases} \quad (1)$$

where  $\kappa$  is the core field decay rate,  $R$  is the curvature radius,  $V$  is a normalized frequency parameter,  $\gamma$  is the cladding field decay rate, and  $\beta$  is the mode propagation constant.

Here is the main expression of the Schermer and Cole model (S&C) [16]:

$$2\alpha = \frac{\sqrt{\pi}\kappa^2 \exp\left[-\frac{2}{3}\left(\frac{\gamma^3}{\beta^2}\right)(R + a) - 2\gamma a\right]}{2\sqrt{R + a} \gamma^{3/2} V^2 K_{m-1}(\gamma a) K_{m+1}(\gamma a)}, \quad (2)$$

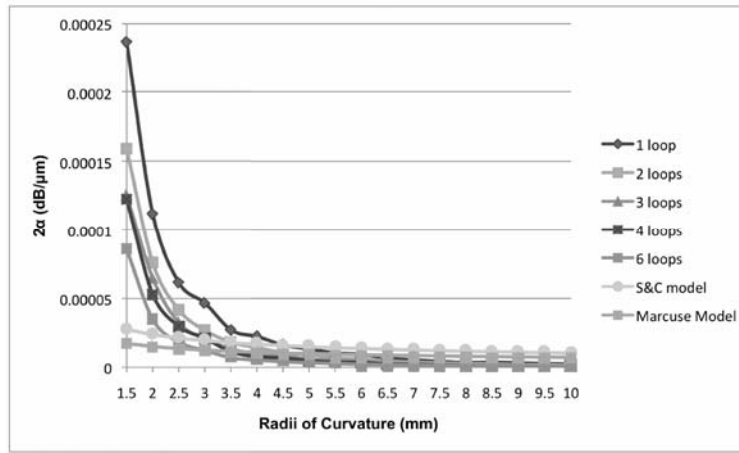
where  $a$  is the fiber radius and the rest of the parameters are similar to those in the Marcuse model.

#### 5. Comparison between the Previous Models and the Experimental Data

In order to obtain the values of the power loss parameter  $2\alpha$ , the values of the experimental losses were divided by the fiber length coiled in each cylinder. Then the experimental data was compared with the predicted values of the Marcuse model. It was noted that unexpectedly, for the radii smaller

than 6mm, the power loss parameter is not a constant but a function of the number of loops (or more properly, a function of the optical path inside the fiber). On the other hand, the parameter  $2\alpha$  calculated for a single loop is larger than those calculated for additional loops.

Figure 5 shows this comparison. It can be clearly seen that the Marcuse model underestimates the power loss for any radius smaller than 4.5mm. Although this model works very well in a simple curved wave guide, the need of developing a new one that can be applied to coiled optical fiber with continuous loops was considered, due to the new power attenuation mechanisms described above.



**Figure 5.** Comparison between the Marcuse model predictions of power loss, the Schermer and Cole model, and the experimental data in a coiled optical fiber with several loops.  $2\alpha$  is the power loss coefficient.

Also, it is important to remark the following points:

- (a) For the previous radii, the power loss parameter  $2\alpha$  is not a constant but a function of the number of loops (or more properly a function of the coiled arc length).
- (b) The parameter  $2\alpha$  calculated for a single loop is larger than any other calculated for additional loops.



(c) Although those models work very well in a simple curved wave guide, the need to develop a new one that can be applied to coiled optical fiber with continuous loops is evident.

## 6. Bending Loss Model Improvement for Coiled Optical Fiber

Now it is well known that the Marcuse model is based on the Maxwell equations of the electric field in a cylindrical coordinate system, and it consists of an analytic formula that predicts the power loss coefficient  $2\alpha$  (see equation (1)).

The term  $\beta$  is highlighted. Marcuse defined it as the *propagation constant of the fiber*. There are several approximations to obtain such a constant.

The value used to evaluate the Marcuse model can be:

$$\beta = n_2k + d, \quad (3)$$

where  $n$  is the cladding refraction index and  $k = 2\pi/\lambda$ , the free space propagation constant, and  $d$  is a fitting value. To evaluate the Marcuse model, the restriction  $kcl \leq \beta \leq kco$  was as well considered. The new model presented by Schermer and Cole [16] was as well revised for the purposes of this document. This model implements several corrections. They corrected some simplifying assumptions made by Marcuse in order to improve the accuracy of the model. Here the propagation constant  $\beta$  is assumed to change with position.

This new model improved the accuracy of the Marcuse model for non-critical radii of curvature. However, this new model does not predict the significant increment in power loss in small radii of curvature as it can be seen in Figure 5. The importance of having a new model that does so was already stated, so the models were modified in order to make them predict accurately the high attenuation in small curvatures and the regular power loss already predicted by the models.

When the experimental data was obtained, and the attenuation was observed, it was concluded that the power loss does not behave linearly with respect to the curvature radius. Due to this reason, it was intended to modify the Marcuse model, so it could predict the power loss as a function of the curvature radius in the coiled fiber, taking into consideration, the clarifying remarks made by Schermer and Cole [16] about the propagation constant  $\beta$ .

With non-linear regression techniques, the term  $\beta$  has been modified in order that it becomes a function of the curvature radius and the number of loops. The present new formula for the propagation constant is:

$$\beta = n_2 k + \exp \left[ \frac{\theta_1}{r} + \frac{\theta_2}{2n\pi r} + \theta_3 \right], \quad (4)$$

where  $r$  is the radius,  $n$  is the number of loops and, in the particular case of our experimentation, the fitting terms  $\theta$  have the following values:

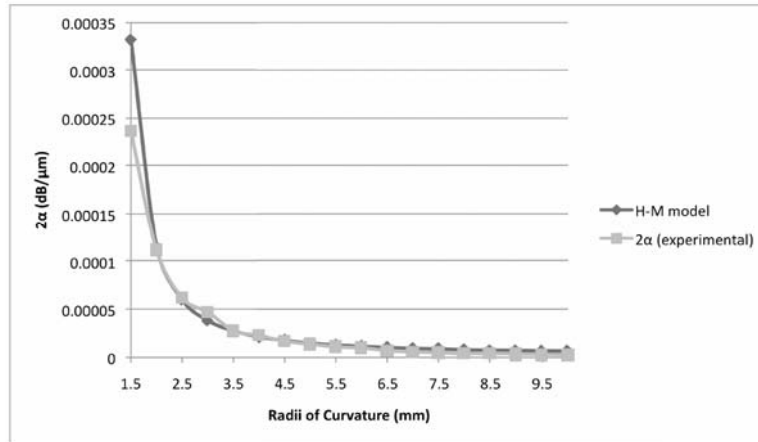
$$\theta_1 = 9472.4, \quad \theta_2 = 73010 \quad \text{and} \quad \theta_3 = -34.765.$$

The factor  $\beta$  was changed in the Marcuse model in order to obtain a better prediction for the attenuation behavior. It is quite important to mention that this new formula is only the first approach to the real behavior of the attenuation in a coiled optical fiber, both in big and small radii of curvature.

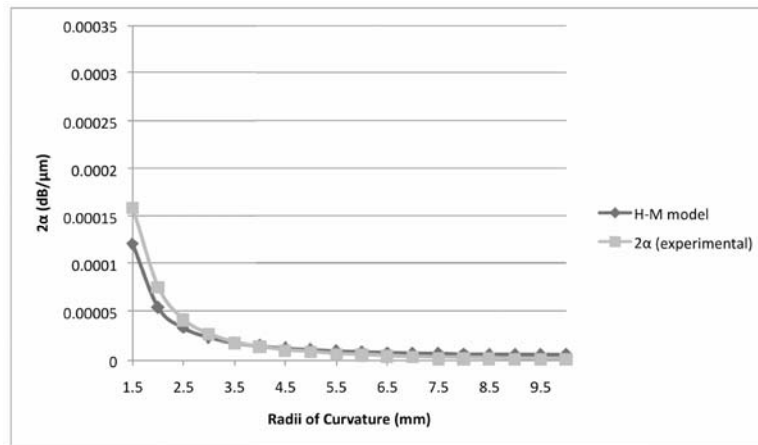
The results obtained with this model are shown in Figures 6-10:

(H-M is the model developed by the authors).

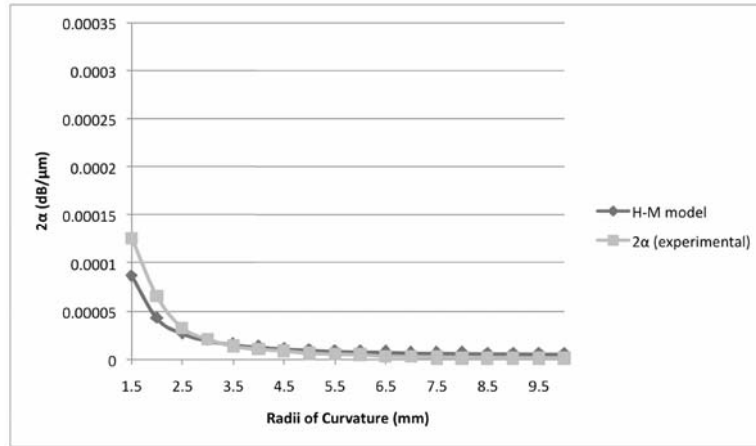
It can be observed that the new model predicts accurately the power loss in a coiled optical fiber with a different number of loops and different radii of curvature as well. It is recognized that there is still room for improvement in the model with a radius of curvature of 1.5mm, however, the improvement is quite easy to recognize from the previous models.



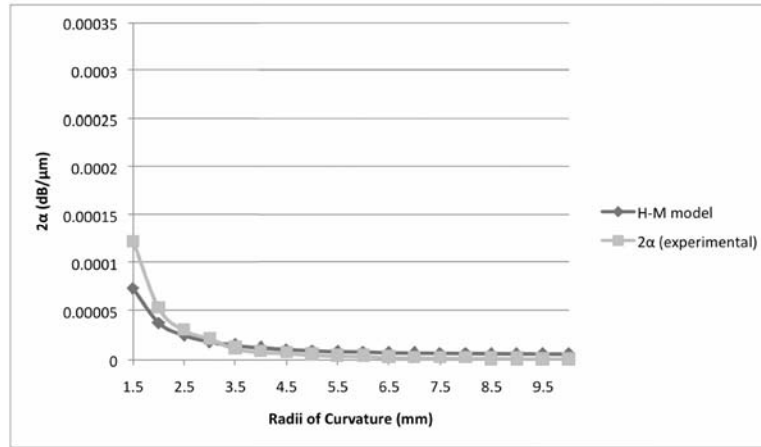
**Figure 6.** Comparison between the H-M model predictions of power loss and the experimental data in a coiled optical fiber with one loop.



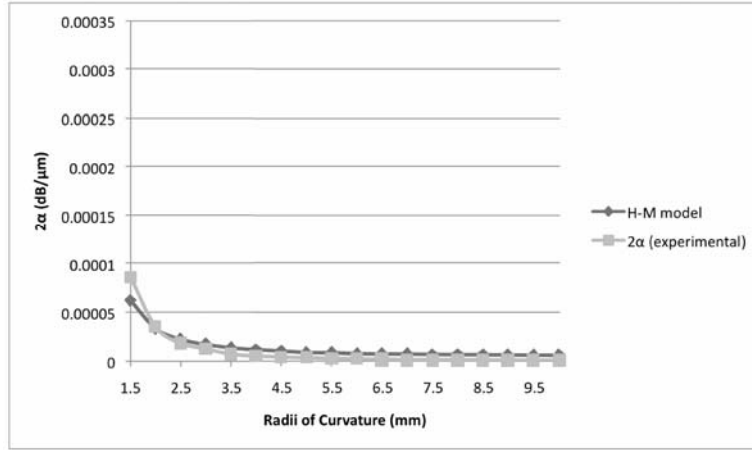
**Figure 7.** Comparison between the H-M model predictions of power loss and the experimental data in a coiled optical fiber with two loops.



**Figure 8.** Comparison between the H-M model predictions of power loss and the experimental data in a coiled optical fiber with three loops.



**Figure 9.** Comparison between the H-M model predictions of power loss and the experimental data in a coiled optical fiber with four loops.



**Figure 10.** Comparison between the H-M model predictions of power loss and the experimental data in a coiled optical fiber with six loops.

## 7. Discussion

The results presented here seem to indicate that there are new mechanisms affecting light leakage in a coiled fiber with a number of loops. Those mechanisms produce, at the first loop, a power loss parameter value of  $2\alpha$  [15] much larger than theoretically predicted, when the radii are smaller than a critical value (in the present case, smaller than 6mm). However, the rest of the light power still confined in the fiber seems to leak more slowly on subsequent loops, producing the decrease of the previous parameter.

It is important to mention that, from conclusions drawn from the graphs, it seems that the parameter  $2\alpha$  stops being constant. In other words, the attenuation stops being cumulative to become a function of the number of loops.

It has been proved that the power lost in a curved optical wave guide is lost due to mode coupling [17]. It seems that inside a bent wave guide, the even modes couple to their immediate odd neighbors and vice-versa [18]. The loss is proportional to the bending angle and number of turns, since mode coupling from propagating modes to irradiative modes induces the bending loss.

It is important to mention that the bending tends to distort the fiber modes, and causes them to shift away from the center of curvature. Also, the orientations of the LP modes shift upon bending, and rotational symmetry is destroyed with several bending radii. Some modes start to interact and merge with each other. Such profound changes in the fiber mode distributions generate significant changes in the modal propagation constants compared to the straight or quasi-straight fibers [19].

In any wave guide, different modes will have different degrees of confinement within the fiber core. Thus,  $\beta$  is one of the quantities that is necessary to determine for each mode of a guide. At a given frequency, different modes will have different values of  $\beta$ . This point is highlighted because it is fundamental for the present study.

## 8. Conclusions

In this work, the Marcuse model and the Schermer and Cole model were tested as a tool to predict the attenuation behavior in coiled optical fiber with different radii of curvature. It was found that those models only predict accurately the power loss for non-critical radii. Indeed, for smaller radii values to  $6000\mu\text{m}$ , the models underestimate the  $2\alpha$  values. In fact, it is possible that new mechanisms of light leaking are present, due to the extreme distortion of the modes configuration into the fiber at small radii. Those mechanisms cannot be described by a model that considers the power loss parameter  $2\alpha$ , and more specifically, the mode propagation constant of the wave guide ( $\beta$ ) as constants.

Because of these possible mechanisms, a new model was developed as a function of optical path, considering the exponential behavior of basis  $(a/b)$ , that has the modes LP. This new model correctly predicts the behavior of the experimental data of attenuation, when the fiber is bent in radii of curvatures large and small.

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