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SOME RANDOM FIXED POINT THEOREMS AND RANDOM OPERATOR EQUATIONS

Ning Chen and Jiqian Chen

School of Science
Southwest University of Science and Technology
Mianyang, 621010, P. R. China
e-mail: chenning783@163.com
cy783@yahoo.com.cn

Abstract

We obtain some results related to common random solution for a class of random operator equations generalizing several results provided in [4, 5, 7, 18] for Banach spaces. Also, we extend the Altman inequality into the determinant form.

1. Introduction

We obtain the random solution of some random operator equations by using random fixed point theorems. The random functional analysis requires the random fixed point theory. Random operators lie at the heart probabilistic functional analysis and their theory is needed for the study of several classes random operator equations.

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In random contract operator, the random topological degree theory, etc. fully work. The importance of random fixed point theorem is seen amongst others in the existence of solution for the equation and also in multiple solutions problem of the corresponding system. In this paper, we consider the random solution of random operator equations and by using random fixed point theorem and the Altman's type inequality generalize some results obtained in [4, 5, 7, 18].

We shall consider random semi-closed 1-set-contractive operators in a separable real Banach space.

Lemma 1.1 (Lemma 1 in [5]). *Assume that E is a separable real Banach space, X is a non-empty closed convex set in E, and D is a bounded open set in X. Let* $A : \Omega \times \overline{D} \to X$ *be random semi-closed* 1-*set contract operator, and* $e_0 \in D$ *be such that*

$$A(\omega, x) - \mu e_0 \neq \alpha(x - e_0),$$

where $\alpha > \mu \geq 1$, $(\omega, x) \in \Omega \times \partial\Omega$. *Then the operator equation* $A(\omega, x) = \mu x$ *has a random solution in D.*

Theorem A (See [5, Theorem 2]). *Assume that E is a separable real Banach space, X is a non-empty convex and closed set in E, and D is a bounded open set in X. Let* $A : \Omega \times \overline{D} \to X$ *be a random semi-closed* 1-*set contract operator,* $\theta \in D$, *and for a real number* $\mu \geq 1$,

$$\| A(\omega, x) - \mu x \|^{m+\beta+\gamma} \geq \| A(\omega, x) \|^{m+\gamma} \| A(\omega, x) + \mu x \|^{\beta} - \| \mu x \|^{m+\beta+\gamma},$$

$$\forall (\omega\ x) \in \Omega \times \partial D, \quad m > 1, \quad \beta \geq 0, \quad \gamma \geq 0.$$

Then the equation $A(\omega, x) = \mu x$ *has a random solution in D.*

2. Common Solution of an Operator Equation

Theorem 2.1. *Assume that E is a separable real Banach space, X is a non-empty convex and closed set in E, and D is a bounded open set in X.*

Let $A, B : \Omega \times \overline{D} \to X$ *be random semi-closed* 1-*set contract operators,* $\theta \in D,$ *and real numbers* $\lambda > 0,$ $\mu \geq 1$ *be such that*

$$\| A(\omega, x) \|^6 \| A(\omega, x) + \mu x \| + \| B(\omega, x) - \lambda\mu x \|^7$$

$$\leq \| A(\omega, x) - \mu x \|^7 + \| \mu x \|^7, \quad \forall(\omega, x) \in \Omega \times \partial D. \tag{2.1}$$

Then the systems $A(\omega\ x) = \mu x,$ $B(\omega, x) = \lambda\mu x$ *have a common random solution in D.*

Proof. By Theorem A, assuming $m = 5, \beta = 1, \gamma = 1,$ we get

$$\| A(\omega, x) \|^6 \| A(\omega, x) + \mu x \| \leq \| A(\omega, x) - \mu x \|^7 + \| \mu x \|^7, \forall(\omega, x) \in \Omega \times \partial D.$$

By Lemma 1.1, this equation $A(\omega, x) = \mu x$ has a random solution x^* in D, that is, $A(\omega, x^*) = \mu x^*,$ and from (2.1), we have $\| B(\omega, x^*) - \lambda\mu x^* \|^7 \leq 0,$ that is, $B(\omega, x^*) = \lambda\mu x^*.$ Hence, we get that systems: $A(\omega, x) = \mu x,$ $B(\omega, x) = \lambda\mu x$ have a common random solution x^* in D.

Theorem 2.2. *Assume the hypothesis of Theorem* 2.1. *Substitute* (2.1) *in the following*:

$$\| A(\omega, x) - \mu x \|^{5(m+\beta+\gamma)+\beta_1} \geq \| A(\omega, x) \|^{5(m+\gamma)} \| A(\omega, x) + \mu x \|^{5\beta}$$

$$\cdot \| A(\omega, x) + 2\mu x \|^{\beta_1} - \| \mu x \|^{5(m+\beta+\gamma)+\beta_1},$$

$$\forall(\omega, x) \in \Omega \times \partial D, \tag{2.2}$$

where $m > 1, \beta \geq 0, \beta_1 \geq 0, \gamma \geq 0.$ *Then the equation* $A(\omega, x) = \mu x$ *has a random solution in D.*

Proof. Let $e_0 = \theta \in D$ in Lemma 1.1. Then we only prove that $A(\omega, x) \neq \alpha x,$ $(\omega, x) \in \Omega \times \partial D,$ $\alpha > \mu \geq 1.$

Assume to the contrary. Then for some $\alpha_0 > \mu \geq 1,$ $\omega_0 \in \Omega,$ $x_0 \in \partial D$, we have $A(\omega_0, x_0) = \alpha_0 x_0.$

Now, using $A(\omega_0, x_0) = \alpha_0 x_0$, on substitution in (2.2) and letting $\alpha_0 - \mu = \alpha > 0$, we have

$$l = 5(m + \beta + \gamma) + \beta_1,$$

$$(\alpha_0 - \mu)^l + \mu^l \geq (\alpha_0)^{5(m+\gamma)}(\alpha_0 + \mu)^{5\beta}(\alpha_0 + 2\mu)^{\beta_1}.$$

Further, letting $\alpha_0 - \mu = \alpha > 0$, we have

$$\alpha^l + \mu^l \geq (\alpha + \mu)^{5(m+\gamma)}(\alpha + 2\mu)^{5\beta}(\alpha + 3\mu)^{\beta_1} > (\alpha + \mu)^l.$$

This is a contradiction. Hence, by Lemma 1.1, $A(\omega, x) = \mu x$ has a random solution x^* in D such that $A(\omega, x^*) = x^*$.

Let $\varphi(u)$ be strictly increasing and continuous in $[0, \infty)$, $\varphi(0) = 0$. We will extend Theorem 3 in [11] in the following theorem for common solution of the system with random semi-closed 1-set contract operators.

Theorem 2.3. *Assume that E is a separable real Banach space, X is a non-empty convex and closed set in E, and D is a bounded open set in X.*

Let $A, B : \Omega \times \overline{D} \to X$ *be random semi-closed* 1-*set contract operators,* $\theta \in D$, *and real numbers* $\lambda > 0$, $\mu \geq 1$ *be such that*

$$\| A(\omega, x) - \mu x \|^{m+\gamma} \cdot \phi(\| A(\omega, x) + \mu x \|^{\beta})$$

$$\geq \| B(\omega, x) - \lambda\mu x \|^{m+\beta+\gamma} + \| A(\omega, x) \|^{m+\gamma} \phi(\| A(\omega, x) + \mu x \|)$$

$$- \| \mu x \|^{m+\gamma} \phi(\| \mu x \|) \quad (m > 1, \beta \geq 1, \gamma > 0, \forall (\omega, x) \in \Omega \times \partial\Omega). \quad (2.3)$$

Then the systems $A(\omega, x) = \mu x$, $B(\omega, x) = \lambda\mu x$ *have a common random solution in D.*

Proof. Letting $e_0 = \theta \in D$ in Lemma 1.1, we only prove that $A(\omega, x) \neq \alpha x$, $(\omega, x) \in \Omega \times \partial D$. By (2.3), we have

$$\| A(\omega, x) \|^{m+\gamma} \phi(\| A(\omega, x) + \mu x \|^{\beta}) - \| \mu x \|^{m+\gamma} \phi(\| \mu x \|^{\beta})$$

$$\leq \| A(\omega, x) - \mu x \|^{m+\gamma} \phi(\| A(\omega, x) + \mu x \|^{\beta}).$$

Assuming in contrary, there exist $\omega_0 \in \Omega$, $x_0 \in \partial D$, $\alpha_0 > \mu \geq 1$ such that $A(\omega_0\ x_0) = \alpha_0 x_0$. Substituting it into the above inequality, we have

$$(\alpha_0)^{m+\gamma} \phi(\| (\alpha_0 + \mu) x_0 \|^{\beta}) \leq (\alpha_0 - \mu)^{m+\gamma} \phi(\| (\alpha_0 + \mu) x_0 \|^{\beta})$$

$$+ (\mu)^{m+\gamma} \phi(\| \mu x_0 \|^{\beta}), \tag{2.4}$$

so

$$((\alpha_0 - \mu)^{m+\gamma} + (\mu)^{m+\gamma}) \phi((\alpha_0 + \mu)^{\beta} \| x_0 \|^{\beta}) \geq (\alpha_0)^{m+\gamma} \phi((\alpha_0 + \mu)^{\beta} \| x_0 \|^{\beta}).$$

That is,

$$(\alpha_0 - \mu)^{m+\gamma} + \mu^{m+\gamma} \geq \alpha_0^{m+\gamma} = ((\alpha_0 - \mu) + \mu)^{m+\gamma} > (\alpha - \mu)^{m+\gamma} + \mu^{m+\gamma}.$$

This is a contradiction. Hence, by Lemma 1.1, $A(\omega, x) = \mu x$ has a random solution x^* in D such that $A(\omega, x^*) = \mu x^*$. By (2.3), we have $\| B(\omega, x^*) - \lambda\mu x^* \|^{m+\gamma} \leq 0$. That is, $B(\omega, x^*) = \lambda\mu x^*$. Hence, $A(\omega, x^*) = \mu x^*$, $B(\omega, x^*) = \lambda\mu x^*$. Then the systems $A(\omega, x) = \mu x$, $B(\omega, x) = \lambda\mu x$ have a common random solution x^* in D.

Corollary 2.4. *Letting* $\lambda = 1$, $\gamma = 0$, $\beta_1 = 1$, *we get Theorem* 3 *in* [18].

Corollary 2.5. *Substituting* (2.3) *into*

$$\| A(\omega, x) - \mu x \|^{m+\gamma} \cdot \phi(\| A(\omega, x) - \mu x \|^{\beta})$$

$$\geq \| B(\omega, x) - \lambda\mu x \|^{m+\beta+\gamma} + \| A(\omega, x) \|^{m+\gamma} \cdot \phi(\| A(\omega, x) + \mu x \|^{\beta})$$

$$- \| \mu x \|^{m+\gamma} \phi(\| \mu x \|), \tag{2.5}$$

we have similar results.

Theorem 2.6. *Assume that E is a separable real Banach space, X is a non-empty convex and closed set in E, and D is a bounded open set in X.*

Let $A, B, C : \Omega \times \overline{D} \to X$ *be all random semi-closed* 1-*set contract operators*, $\theta \in D$, *and the real number* $\mu \geq 1$ *is such that*

$$\| A(\omega, x) \|^m \| A(\omega, x) + \mu x \|^\beta \varphi(\| A(\omega, x) + 2\mu x \|^\gamma)$$

$$+ \| \lambda^2 B(\omega, x) + \lambda C(\omega, x) - A(\omega, x) \|^{m+\beta+\gamma}$$

$$\leq \| A(\omega, x) - \mu x \|^{m+\beta} \varphi(\| \mu x \|^\gamma), \quad \forall (\omega, x) \in \Omega \times \partial D, \tag{2.6}$$

where $m \geq 1$, $\beta \geq 0$, $\gamma \geq 0$. *Then the systems* $\lambda^2 B(\omega, x) + \lambda C(\omega, x) = \mu x$, $A(\omega, x) = \mu x$ *have a common solution* x^* *in* $D(\lambda > 0)$.

Theorem 2.7. *Assume the hypothesis of Theorem* 2.1. *Substitute* (2.1) *in the following*:

$$\| C(\omega, x) - 3\mu x \|^3 \cdot \| B(\omega, x) - 2\mu x \|^3 + \| A(\omega, x) \|^5 \| A(\omega, x) + \mu x \|$$

$$\leq \| A(\omega, x) - \mu x \|^6 + \| \mu x \|^6, \quad \forall (\omega, x) \in \Omega \times \partial D. \tag{2.7}$$

Then the systems $A(\omega, x) = \mu x$, $B(\omega, x) = 2\mu x$ *have a common random solution* x^* *in D, or the* $A(\omega, x) = \mu x$, $C(\omega, x) = 3\mu x$ *have a common random solution* x^* *in D.*

Proof. It follows from Theorem A by taking $m = 5$, $\beta = 1$, $\gamma = 1$.

3. Altman Type Inequality

We can extend Altman's inequality into the type determinant form as follows:

$$\| A(\omega, x) - \mu x \|^2 \geq \| A(\omega, x) \|^2 - \| \mu x \|^2 = \begin{vmatrix} \| A(\omega, x) \|, & \| \mu x \| \\ \| \mu x \|, & \| A(\omega, x) \| \end{vmatrix} = D_2.$$

Hence, we easily get following Theorem 3.1.

Let n-order determinant

$$\begin{vmatrix} \| A(\omega, x) \|, \| \mu x \|, \ldots, \| \mu x \| \\ \| \mu x \|, \| A(\omega, x) \|, \ldots, \| \mu x \| \\ \vdots \cdots, \ddots, \ddots, \cdots, \ddots, \cdots, \vdots \\ \| \mu x \|, \| \mu x \|, \ldots, \| A(\omega, x) \| \end{vmatrix} = D_n \quad \text{(as symmetry form)}.$$

Clearly, $D_n = (\| A(\omega, x) \| + (n-1) \| \mu x \|)^{n-1}$, $D_{n+1} = |\| A(\omega, x) \| + (n) \| \mu x \||^n$ and $D_{n+2} = (\| A(\omega, x) \| + (n+1) \| \mu x \|)^{n+1}$.

Hence, we easily get the following results:

$$D_n = \begin{vmatrix} \| A(\omega, x) \|^3, \| \mu x \|^3, \ldots, \| \mu x \|^3 \\ \| \mu x \|^3, \| A(\omega, x) \|^3, \ldots, \| \mu x \|^3 \\ \vdots, \cdots, \ddots, \cdots, \ddots, \cdots, \ddots \vdots \\ \| \mu x \|^3, \| \mu x \|^3, \ldots, \| A(\omega, x) \|^3 \end{vmatrix}. \tag{$*$}$$

Simple calculation gives that

$$D_n = (\| A(\omega, x) \|^3 + (n-1) \| \mu x \|^3)(\| A(\omega, x) \|^3 - \| \mu x \|^3)^{n-1}.$$

Theorem 3.1. *Suppose that*

$$D_n D_{n+1} + D_{2n} \leq 2 \| A(\omega, x) - \mu x \|^{6n}. \tag{3.1}$$

Then the random operator equation $A(\omega, x) = \mu x$ *has a random solution* x^* *in D.*

Proof. Assume to the contrary. Then from $(*)$ and $D_n = (\| A(\omega, x) \|^3 + (n-1) \| \mu x \|^3)(\| A(\omega, x) \|^3 - \| \mu x \|^3)^{n-1}$, by Lemma 1.1, on considering $A(\omega_0, x_0) = \alpha_0 x_0$, substituting it in above (3.1) and by calculating that $\alpha_0 - \mu = \alpha > 0$, from $D_n D_{n+1}$, we have

$$(\alpha_0^3 + (n-1)\mu^3)(\alpha_0^3 + n\mu^3)(\alpha_0^3 - \mu^3)^{2n-1} > \alpha^{6n}.$$

And from D_{2n} corresponding, we have

$$(\alpha_0^3 + (2n-1)\mu^3)(\alpha_0^3 - \mu^3)^{2n-1} > \alpha^{6n}.$$

Thus, we substituting $A(\omega_0, x_0) = \alpha_0 x_0$ in (3.1) that holds larger $2\alpha^{6n}$, which is a contradiction. By Lemma 1.1, we obtain the random operator equation $A(\omega, x) = \mu x$ have a random solution x^* in D.

4. Some Remarks and Examples

Theorem 4.1. *Assume the hypothesis of Lemma* 1.1 *and substitute* (1.1) *in the following*:

$$(1-\varepsilon)\| A(\omega, x)\|^2$$

$$\le (1+7\varepsilon)\| \mu x + \delta A(\omega, x)\| \| \mu x - \delta A(\omega, x)\|, \ \forall (\omega, x) \in \Omega \times \partial D. \qquad (4.1)$$

Then $A(\omega, x) = \mu x$ *has a random solution in* D $(0 \le \delta < 1, \ 0 < \varepsilon < 1)$.

Proof. Similar to the proof of Theorem 2.1, considering $A(\omega_0, x_0) = \alpha_0 x_0$, and substituting it in (4.1), we get $(1-\varepsilon)\alpha_0^2 \le (1+7\varepsilon)(\mu^2 - (\delta\alpha_0)^2)$, $(1+7\varepsilon)\mu^2 - ((1-\varepsilon) + (1+7\varepsilon)\delta^2)\alpha_0^2 \ge 0$. Then the discriminant

$$\Delta = 4((1+7\varepsilon)^2\delta^2 + (1+6\varepsilon - 7\varepsilon^2))\alpha_0^2 > 0,$$

provides a contradiction. Hence, by Lemma 1.1, the equation $A(\omega, x) = \mu x$ has a random solution in D for $0 \le \delta < 1$, $0 < \varepsilon < 1$.

Corollary 4.2. *Letting* $\delta = 0, \ \varepsilon = 0,$ *from Theorem* 4.1,

$$\| A(\omega, x)\|^2 \le \| \mu x\|^2, \quad (\omega, x) \in \Omega \times \partial D. \qquad (4.2)$$

Corollary 4.3. *For random semi-closed* 1-*set contract operators* $A, B : \Omega \times \overline{D} \to X$ *that satisfy*:

$$\| A(\omega, x)\|^2 + \| B(\omega, x) - A(\omega, x)\|^2 \le \| \mu x\|^2, \qquad (4.3)$$

the systems $A(\omega, x) = \mu x$, $B(\omega, x) = \lambda\mu x$ *have a common random solution in* $x^* \in D$.

5. Some Random Operator Equations

Example 1. We consider the following equation similar to Example 4 in [7] or [18]:

$$\sin(x + 3\omega) + \frac{1}{3}\sin(x + \omega) - 2x = 0, \quad \omega \in \Omega = [0, 1] \tag{5.1}$$

which must have a random solution in $[-\pi, \pi]$.

In fact, we can take that $A(\omega, x) = \frac{1}{2}\sin(x + 3\omega) + \frac{1}{6}\sin(x + \omega)$ (where $\omega \in [0, 1] = \Omega$, $x \in [-\pi, \pi]$) is a random semi-closed 1-set contract operator (see Example 2 in [18]), and from Corollary 4.2 with $\mu = 1$, here it is at the boundary point in interval $[-\pi, \pi]$:

$$\| A(\omega, -\pi) \|^2 < \| -\pi \|^2 = \pi^2, \quad \| A(\omega, \pi) \|^2 < \| \pi \|^2 = \pi^2.$$

Hence, $A(\omega, x) = x$ has a random solution on $[-\pi, \pi]$.

Example 2. We consider the following equation:

$$a\sin(x + 3\omega) + \frac{a}{3}\sin(x + \omega) - 2x = 0, \;\; \omega \in [0, 1] = \Omega, \;\; 0 < a \le 1 \tag{5.2}$$

which has a random solution in $[-\pi, \pi]$. By $A(\omega, x) = \frac{a}{2}\sin(x + 3\omega) + \frac{a}{6}\sin(x + \omega)$, equation (5.2) must have a random solution in $[-\pi, \pi]$. Letting $a = 1$, we have Example 1.

6. Some Stability for the Solution of Stochastic Differential Equation

We consider stochastic differential equation:

$$dx(t) = x(t)(a(t)dt + b(t)dw(t)), \tag{6.1}$$

$w(t)$ is one-dimensional standard Brown motion in (6.1) and solution of (6.1) is given by:

$$x(t) = x_0 \exp\left\{\int_0^t (a(s) - S(s)/2)ds + \int_0^t b(s)dw(s)\right\}$$

(see [10, p. 60]). (6.2)

(i) Langevin equation: $dx(t) = -\alpha x(t)dt + \sigma dw(t)$, $x(t)$ expresses the velocity of particle with action, positive constant $\alpha, \sigma > 0$.

The solution:

$$x(t) = x_0 e^{-\alpha t} + \sigma \int_0^t e^{\alpha(t-s)} dw(s),$$

$$\sigma \int_0^t e^{\alpha(t-s)} dw(s) \sim N(0, (\sigma^2/2\alpha)(1 - e^{-2\alpha t})),$$

as $t \to \infty$, here $x(t)$ is asymptotic normal distribution $N(0, \sigma^2/2\alpha)$.

We have $Ex(t) = e^{-\alpha t} Ex_0$,

$$Var(x(t)) = e^{-2\alpha t} Var(x_0) + (\sigma^2/2\alpha)(1 - e^{-2\alpha t}).$$

(ii) By (see [10, p. 69]), we have $\sup\limits_{t \geq 0}(2a(t) + (p-1)S(t)) < 0$ with p-moment index stability, and $\inf\limits_{t \geq 0}(2a(t) + (p-1)S(t)) > 0$ with p-moment index without stability.

We consider stochastic differential equation: $dx(t) = x(t)(a(t)dt + b(t)dw(t))$, in which $w(t)$ is a one-dimensional standard Brown motion and solution of this equation is (6.2)-exact solution formula.

For $dx(t) = x(t)[a \sin(t)dt + bdw(t)]$, $a = 1$, $b = 1$, $x(0) = 1$, the zero solution is stable.

We use Milstein method and by MATLAB solve above stochastic differential equation with numerical test - figure as shown below (or may by using other as Euler-Taylor method, R-K method).

The red curve expresses orbit of exact solution, the black curve describes orbit of solution with numerical test in Figure 1. By Milstein method step large = 1/400.

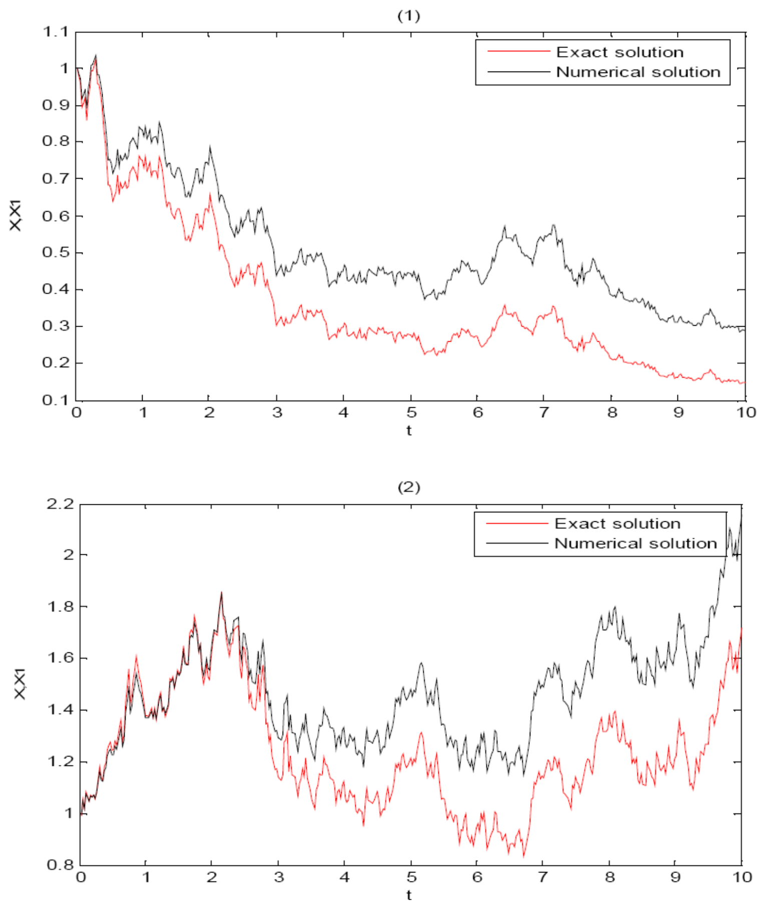

Figure 1

Using MATLAB, we find the solution mean square index-stability which is described as in the curve shown in the figure. Clearly, it belongs to the stability case.

Next, we consider

$$dx(t) = x(t)(a(\sin(t) - 2)dt + bdw(t)),$$

when $a = 1, b(t) = 1, b = 1, x(0) = 1, 0 < p < 3.$

Similar as above numerical test, Figure 2 shows the curve of solution for this equation with index p-exponential stability.

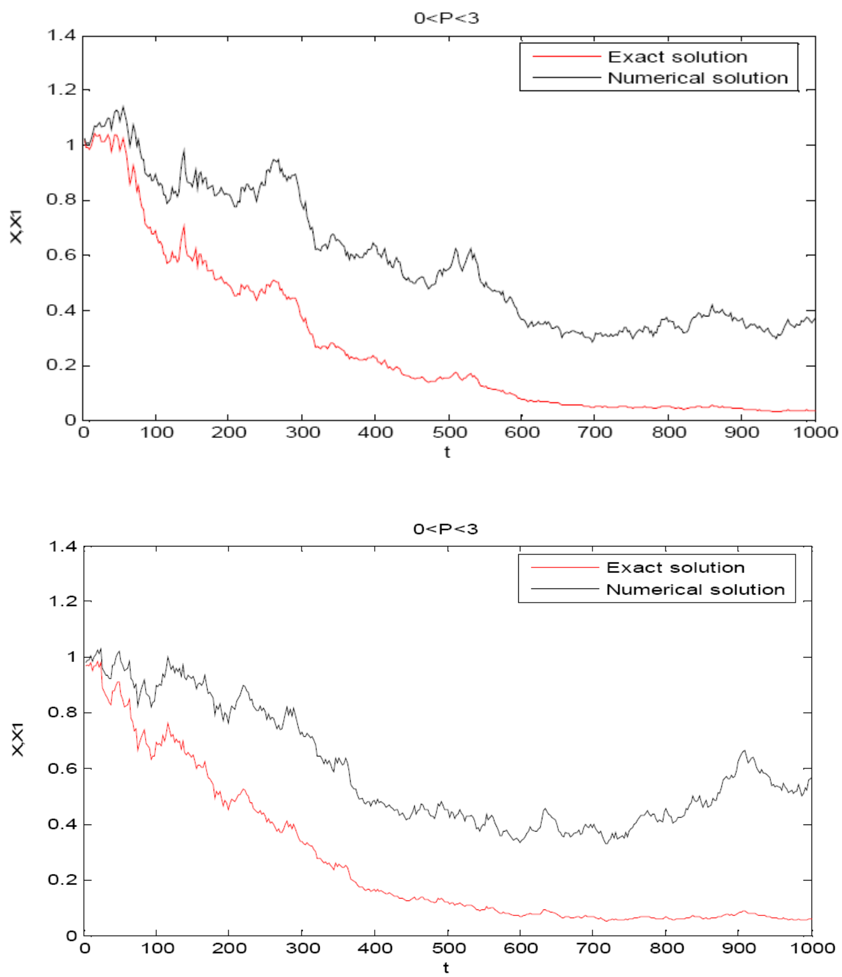

Figure 2

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