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A STUDY ON THE SHEAR COMPATIBILITY EFFECT FOR REINFORCED PLATE STRUCTURE

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Abstract

For the development of the practical methods of the structural analysis of typical reinforced plate structures, idealized approaches with the shear compatibilities of the matching beam and plate elements are needed. Orthotropic plate theory or the grillage beam theory based modeling techniques can be employed. These are two limiting physical idealizations. Orthotropic plate theory lumps the stiffeners with the plating to form an equivalent homogeneous plate of uniform thickness, but with different rigidity properties in the orthogonal directions corresponding to stiffeners directions. Two-dimensional grillage modeling lumps the plating with the stiffeners in forming a planar grid work of orthogonally interconnecting beams. Two types of different approaches mentioned above are studied in the points of its applicability and efficiency for the inclusion of the shear compatibility. Structural idealization is performed to make overall structural analysis firsthand, and the structural behaviors of the model in the airplane landing simulation are evaluated. Throughout this

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study, it is found that the structural idealizations with proper assumptions using the orthotropic plate and the grillage modeling are proved to be adequate and the numerical analysis results for stiffened plated grillage structures give acceptable deformations in the simulations.

Introduction

For the analysis of reinforced plate structures, the structural integrity and corresponding structural analysis as well as the relevant modeling techniques are important. In this paper, the orthotropic plate theory, the 2-D grillage beam theory, and its applicability to the structural analysis and the adaptability are studied and analyzed.

For the verification of the possible adaptability of the orthotropic plate and grillage modeling for typical stiffened plated grillage structures, relevant structural modeling and necessary numerical analyses are carried out to provide the achievable application with the approaches of the orthotropic plate theory and the grillage beam theory.

Modeling Approaches

Adapting the orthotropic plate theory with condition-effective modeling methods and actual structural analyses can accomplish this idealization without severe difficulties anticipated as mentioned. For the typical stiffened plated grillage structures, this idealization can provide necessary calculations and back data through the orthotropic plate modeling and following structural analysis. Also, this kind of approach can be well proceeded through proper 2-D grillage modeling and their associated guided process [2, 4, 8].

When the typical stiffened plated grillage structures are characterized structurally with respect to x, y directions and corresponding modulus of elasticity E_x, E_y are properly defined, the orthotropic plate theory is applicable effectively to the analysis of the typical stiffened plated grillage structures. In this case, well-defined structural rigidity properties are prerequisite in the analysis. For the typical stiffened plated grillage structures 2-D and/or 3-D grillage beam modeling seems to be effective through selecting effective breath of the flange. The reliability of the structural analysis in this case is dependent on the precise calculation of the flange dimension used in the grillage beam modeling process [2, 3].

Orthotropic Plate Theory

The orthotropic plate modeling technique is based on the assumption that plate-stiffener combinations can be accurately replaced by the equivalent homogeneous orthotropic plate. Several assumptions needed are as follows and for the application of this theory to the typical stiffened plated grillage structures, the feasibility of the assumptions should be investigated [3, 7].

(1) Center plane of the plate should remain without deformation and obey the theory of Kirchhoff.

(2) Element normal to the center plane should remain normal after deformations as well as before deformations.

(3) Normal stress that acts through the lateral direction may be neglected.

Small vertical deflection *w* can be expressed with the corresponding bending moments and twisting moments through several steps with the assumptions mentioned above [11, 12],

$$M_x = -D_x\left(\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2}\right), \tag{1}$$

$$M_y = -D_y\left(\frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2}\right), \tag{2}$$

$$M_{xy} = -2D_{xy}\frac{\partial^2 w}{\partial x \partial y}, \tag{3}$$

where D_x, D_y are bending rigidities with respect to *x*, *y* directions respectively, and D_{xy} is twisting rigidity of the plate.

By using the above equations, we have following governing equations of the orthotropic plate finally:

$$D_x \frac{\partial^4 w}{\partial y^4} + (D_x \nu_y + D_y \nu_x + 4D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P(x,\ y). \qquad (4)$$

The equation can be applied for the analysis of the typical stiffened plated grillage structures, once the structure is characterized as the orthogonally stiffened plated structure and some conditions are met. For this purpose, precise and reliable structural properties should be obtained before applying the theory. The conditions are defined as shown below.

(1) Ratio between the plate boundary dimensions and stiffeners spacing should be all enough to guarantee the approximate homogeneity of stiffness.

(2) Stiffness of the structure should be distributed equally to the both directions. Bending and torsional stiffness should not depend on the boundary condition and vertical load distributions.

(3) Plate and stiffeners should be made of same materials.

(4) Plate and stiffeners are to be attached without any slippery.

Grillage Analysis

In the 2-D grillage approach to analyzing orthogonally stiffened plated structure, the system is converted into an equivalent grid-work of co-planar beams. This method is the converse of orthotropic plate theory where the stiffeners are lumped with the plate. The simplest approach to lumping the plating with the stiffeners in the grillage model is just to assume a width of the plating as the effective flange of each beam equal to the spacing of the respective stiffeners. This approach ignores both shear lag effects and the non-linearities associated with the coupling of plating in-plane and normal deformations. Both of these effects produce an attenuation of the in-plane plating stresses between the stiffeners. Allowance for this stress nonuniformity requires the concepts of "effective width" and "effective breadth" of plating for estimating the stiffener flanges [2, 3, 12].

For the modeling of the typical stiffened plated grillage structures by the 2-D grillage beam model, the stiffness of the original orthogonally stiffened plated structure, whether it is singly plated panel or doubt-plated panel, should be represented precisely by providing the corresponding equivalent sectional properties *I*, *J*, *A* for each beam element. Once the effective plating breadth for vertical bending is obtained, determination of the corresponding vertical bending moment of inertia *I* of the equivalent beam is obtained. However, the equivalent torsional rigidity constant of the equivalent beam is difficult to get precisely [6].

The membrane analogy introduced by L. Prandtl established relations between the deflection surface of a uniformly loaded membrane and the distribution of stresses in a twisted bar. A simple solution of the twisting of a narrow rectangular cross section is obtained. The torsional rigidity of the narrow rectangular cross section can be written as below [10, 11]:

$$\sum_{i=1}^{n} \frac{1}{3} ht^3, \tag{5}$$

where *h* is the depth of the section and *t* is its thickness. This is the torsional rigidity constant of the section with zero warping restraint. For the small deflection case, the outcome could be an approximate torsional rigidity, however, the total torsional rigidity of the equivalent beam in the large deformation case is to be calculated by the membrane analogy and the warping restraint condition which is based on the calculated effective breadth for the twisting. Grillage modeling procedures are defined as shown below.

(1) The system is converted into an equivalent grid-work of co-planar beams. The width of the effective flanges of the grillage beams that totally represent the plating cover can be determined by the rules of the thumb or by the current rules. Each associate is connected through its neutral axis and the member with very large stiffness can be treated as the rigid part.

(2) For the modeling of the typical stiffened plated grillage structures by the 2-D grillage beam model, the stiffness of the original orthogonally stiffened plated structure, whether it is single plated panel or doubt-plated panel, should be represented precisely by calculating the corresponding equivalent sectional properties *I*, *J*, *A* for each beam element.

(3) For the prevention of the rigid body mode and for the proper simulations, boundary conditions are to be forced for the modeled 2-D grillage structures.

Feasibility Study by Numerical Analysis

Based on the guidelines for the adaptability of the orthotropic plate theory and the grillage beam theory, numerical analysis is carried out for finding of the structural behaviors of typical stiffened plated grillage structures. This can be the basis for the usage of the orthotropic plate theory and 2-D grillage beam theory and associated modeling. The orthotropic plate modeling is chosen and the 2-D grillage modeling techniques are employed for the structural simulation of the typical reinforced structures when an airplane landed in to the structure. The maximum displacement of the structure is calculated as well as the deformation patterns of the typical stiffened plated grillage structures. In order to verify the validity of the methods employed here, the numerical results are compared with the same model analysis that has been analyzed by different people. The loading is taken as static vertical one of 675 tonf, which is equivalent to landing load [1, 5, 13].

Analysis by Plate Modeling

Orthotropic plate theory and its modeling are employed for the structural analysis of typical structure, and the specifications of the model are shown in Table 1.

Table 1. Specifications of the analyzed model

Material property	E_x	$2.9 \times 10^7 \text{lbf}/\text{in}^2$
	E_y	$2.9 \times 10^7 \text{lbf}/\text{in}^2$
	Density	$7.33 \times 10^{-4} \text{lbf} \cdot \text{sec}^2/\text{in}^4$
	Poisson's ratio	0.3
Structure dimensions	Equivalent plate thickness	30.71 inch
	Overall length	11811 inch
	Overall breadth	2362 inch

The model of typical stiffened plated grillage structures is chosen for the verification of the adaptability of the present modeling techniques [13]. This model has dimension of 300m length, 60m breadth, 2m depth. Dolphin Mooring system is employed and the boundary conditions are imposed by considering this dolphin mooring circumstance.

Figure 1 shows that the displacement at the landing point is to be maximum and it is 0.38944in, i.e., 0.989cm. Their results showed the maximum value of 1.1cm and the corresponding result by NASTRAN was 1.01cm. They are comparable and the values are very close to each other. It is said that the simple orthotropic plate modeling adopted in this paper is applicable for the early design of the typical stiffened plated grillage structures without tremendous efforts and computing times [1, 9].

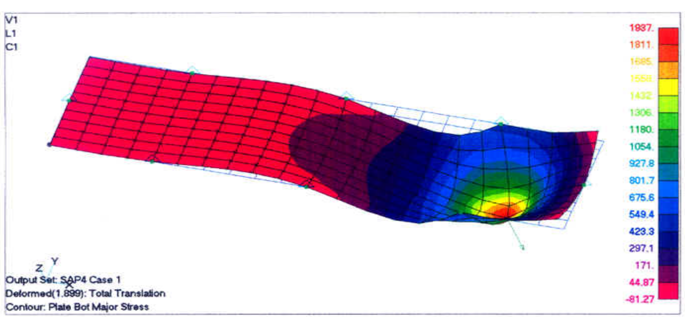

Figure 1. Analysis results around the landing point.

Analysis by Grillage Modeling

For the same structure, equivalent grillage beam properties are generated according to the guideline developed in this paper and the relevant structural analysis is carried out. Material and structural properties used in the analysis are shown in Table 2.

Table 2. Grillage beam modeling characteristics

Material property	E_x	$2.9 \times 10^7 \, \text{lbf}/\text{in}^2$
	E_y	$2.9 \times 10^7 \, \text{lbf}/\text{in}^2$
	Density	$7.33 \times 10^{-4} \, \text{lbf} \cdot \text{sec}^2/\text{in}^4$
	Poisson ratio	0.3
Structural properties	I_{xx}	$5.9832 \times 10^5 \, \text{in}^4$
	A_x	$432.76 \, \text{in}^2$
	J_{xx}	$89.6 \, \text{in}^4$
	I_{yy}	$58.0134 \times 10^5 \, \text{in}^4$
	A_y	$3964.0 \, \text{in}^2$
	J_{yy}	$400.0 \, \text{in}^4$

Typical configuration of the deformation is shown in Figure 2. It is well coincided with the deformation pattern of plate modeling analysis result.

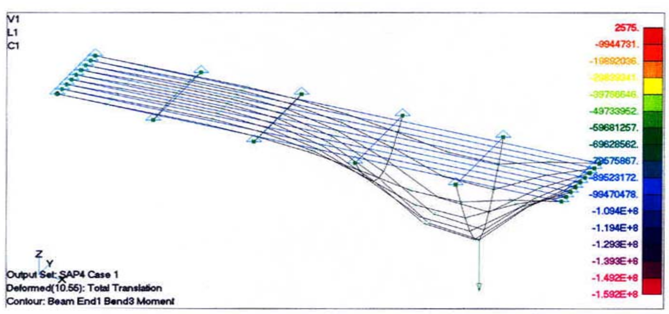

Figure 2. Grillage modeling and landing simulations.

Comparison Between Plate and Grillage Modeling

Structural analysis results by using the orthotropic plate and 2-D grillage modeling for the structural analyses are carried out and are compared with each other. It is found that these possible approaches give almost the same results in the matter of both displacement and the deformation pattern and therefore these are to be the reasonable results. It is notable that the analysis results by two approaches are almost same as the results by full 3-D modeling and associated structural analysis.

Comparison between the two methods gives some remarks. The displacement, 0.989cm, by the orthotropic modeling is a little smaller than expected and this can be explained by the inclusion of the in-plane and bending coupling effects thoroughly, which may give less stiffness in the actual structure. For larger displacements, as 1.151cm, simulated by the 2-D grillage modeling, it is explained that the torsional rigidities, which are taken lighter than the actual structure, are resulted as the membrane analogy with warping restraints and are not considered in the process.

Conclusions

For the design and relevant effective structural analysis, theoretical backgrounds and the feasibility of the adaptability were investigated, with the orthotropic plate and 2-D grillage modeling techniques and also some precautions and the guidelines for adapting these approaches in the analysis process are provided.

Two approaches mentioned above were studied in the points of its applicability and efficiency. For these purposes, structural idealization is performed to make overall structural analysis first, and the structural behaviors of the model in the airplane landing simulation are evaluated. Through this study, it is found that the structural idealizations using orthotropic plate and grillage modeling for design are proved to be adequate to include the shear compatibility and the numerical analysis results for real structure yields acceptable deformations in the simulations.

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