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INTUITIONISTIC FUZZY PRE SEMI EXTREMALLY DISCONNECTED SPACES

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Abstract

In this paper, a new class of intuitionistic fuzzy topological spaces called intuitionistic fuzzy pre semi extremally disconnected spaces is introduced and several other properties are discussed.

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [17], several researches were conducted on the generalizations of the notion of fuzzy set. The concept of "intuitionistic fuzzy sets" was published by Atanassov [2] and many works by the same author and his colleagues appeared in the literature [3-5]. Later this concept was generalized to "intuitionistic *L*-fuzzy sets" by Atanassov and Stoeva [6]. An introduction to intuitionistic fuzzy topological space was introduced by Coker [9]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces are defined by Coker [9]. The construction is based on the idea of intuitionistic fuzzy set developed

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by Atanassov [2, 3] and Atanassov and Stoeva [5]. In this paper, a new class of intuitionistic fuzzy topological spaces, namely, intuitionistic fuzzy pre semi extremally disconnected spaces is introduced by using the concepts of fuzzy extremally disconnected spaces [7], fuzzy pre open sets [8]. 'Intuitionistic fuzzy pre semi closed sets' was introduced by [1]. Tietze extension theorem for intuitionistic fuzzy pre semi extremally disconnected spaces has been discussed as in [15]. Some interesting properties and characterizations are studied.

2. Preliminaries

Definition 2.1 [4]**.** Let X be a non-empty fixed set. An intuitionistic fuzzy set (*IFS* for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\},$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of non-membership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Remark 2.1 [4]**.** For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$.

Definition 2.2 [4]**.** Let X be a non-empty set and the *IFSs* A and B be in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;

(d) $A \bigcap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \bigcup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\ ]A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\ \rangle A = \{\langle x, 1 - \gamma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.3 [9]**.** Let X be a non-empty set and let $\{A_i : i \in J\}$ be an arbitrary family of *IFSs* in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge \mu_{A_i}(x), \vee \gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee \mu_{A_i}(x), \wedge \gamma_{A_i}(x)\rangle : x \in X\}$.

Definition 2.4 [9]**.** Let X be a non-empty fixed set. Then $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.5 [9]**.** Let X and Y be two non-empty fixed sets and $f : X \to Y$ be a function. Then

(a) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an *IFS* in *Y*, then the pre image of *B* under *f*, denoted by $f^{-1}(B)$, is the *IFS* in *X* defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}.$$

(b) If $A = \{\langle x, \lambda_A(x), \nu_A(x)\rangle : x \in X\}$ is an *IFS* in *X*, then the image of *A* under *f*, denoted by $f(A)$, is the *IFS* in *Y* defined by

$$f(A) = \{\langle y, f(\lambda_A)(y), (1 - f(1 - \nu_A))(y)\rangle : y \in Y\},$$

where

$$f(\lambda_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \lambda_A(x), & \text{if } f^{-1}(y) \neq 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$(1 - f(1 - \nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x), & \text{if } f^{-1}(y) \neq 0, \\ 1, & \text{otherwise} \end{cases}$$

for the *IFS* $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.6 [9]**.** Let X be a non-empty set. An intuitionistic fuzzy topology (*IFT* for short) on a non-empty set X is a family τ of intuitionistic fuzzy sets (*IFSs* for short) in X satisfying the following axioms:

(T_1) $0_\sim, 1_\sim \in \tau$, (T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(T_3) $\bigcup\limits_{\subseteq\tau} G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\}$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (*IFTS* for short) and any *IFS* in τ is known as an intuitionistic fuzzy open set (*IFOS* for short) in X.

Definition 2.7 [9]**.** Let X be a non-empty set. Then the complement $\overline{A}$ of an *IFOS A* in an *IFTS* (X, τ) is called an *intuitionistic fuzzy closed set* (*IFCS* for short) in X.

Definition 2.8 [9]**.** Let (X, τ) be an *IFTS* and $A = \langle x, \mu_A, \gamma_A \rangle$ be an *IFS* in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$cl(A) = \bigcap\{K : K \text{ is an } IFCS \text{ in } X \text{ and } A \subseteq K\},$$

$$int(A) = \bigcup\{G : G \text{ is an } IFOS \text{ in } X \text{ and } G \subseteq A\}.$$

Remark 2.2 [9]**.** Let (X, τ) be an *IFTS*. $cl(A)$ is an *IFCS* and $int(A)$ is an *IFOS* in X, and

(a) A is an *IFCS* in X iff $cl(A) = A$; (b) A is an *IFOS* in X iff $int(A) = A$.

Proposition 2.1 [9]**.** *Let* (X, τ) *be an IFTS. For any IFS A in* (X, τ), *we have*

(a) $cl(\overline{A}) = \overline{int(A)}$, (b) $int(\overline{A}) = \overline{cl(A)}$.

Definition 2.9 [9]**.** Let (X, τ) and (Y, φ) be two *IFTSs* and let $f : X \to Y$ be a function. Then f is said to be *fuzzy continuous* iff the pre image of each *IFS* in ϕ is an *IFS* in τ.

Definition 2.10 [15]**.** Let (X, τ) and (Y, φ) be two *IFTSs* and let $f : X \to Y$ be a function. Then *f* is said to be *fuzzy open* (resp. *closed*) iff the image of each *IFS* in τ (resp. $(1 - \tau)$) is an *IFS* in ϕ (resp. $(1 - \phi)$).

An *IFTS* (X, T) represent intuitionistic fuzzy topological spaces and for a subset *A* of a space (X, T), $IFcl(A)$, $IFint(A)$, $IFPScl(A)$, $IFPSint(A)$ and $\overline{A}$ denote an intuitionistic fuzzy closure of *A*, an intuitionistic fuzzy interior of *A*, intuitionistic fuzzy pre semi closure of *A*, an intuitionistic fuzzy pre semi interior of *A* and the complement of *A* in *X*, respectively.

Definition 2.11 [12]**.** A subset *A* of an *IFTS* (X, T) is called an *IF semi pre open set* if $A \subseteq IFcl(IFint(IFcl(A)))$ and an *IF semi pre closed set* if $IFint(IFcl(IFint(A))) \subseteq A$.

Definition 2.12 [14]**.** A subset *A* of an *IFTS* (X, T) is called an *IF generalized closed* (briefly *IF g-closed*) *set* if $IFcl(A) \subseteq U$ whenever $A \subseteq U$ and *U* is *IF* open in (X, T). The complement of an *IF g*-closed set is called an *IF g-open set*.

Definition 2.13 [1]**.** A subset *A* of an *IFTS* (X, T) is called *intuitionistic fuzzy pre semi closed* (*IF pre semi closed* for short) if $IF\,spcl(A) \subseteq U$ whenever $A \subseteq U$ and *U* is *IF g*-open in (X, T).

Definition 2.14 [1]**.** A subset *A* of an *IFTS* (X, T) is called *intuitionistic fuzzy pre semi open* (*IF pre semi open* for short) if $\overline{A}$ is *IF* pre semi closed.

Definition 2.15 [1]**.** A function $f : (X, T) \to (Y, S)$ is called *intuitionistic fuzzy pre semi continuous* (*IF pre semi continuous* for short) if $f^{-1}(V)$ is an *IF* pre semi closed set of (X, T) for every *IF* closed set *V* of (Y, S).

3. Intuitionistic Fuzzy Pre Semi Extremally Disconnected Spaces

Definition 3.1. Let (X, T) be an *IFTS*. Let *A* be any *IF* pre semi open set in (X, T). If *IF* pre semi closure of *A* is *IF* pre semi open, then (X, T) is said to be *intuitionistic fuzzy pre semi extremally disconnected* (for short *IF pre semi extremally disconnected*).

Proposition 3.1. *For an IFTS* (X, T), *the following are equivalent*:

(a) (X, T) *is IF pre semi extremally disconnected.*

(b) *For each IF pre semi closed set A, IFPSintA is IF pre semi closed.*

(c) *For each IF pre semi open set A, we have* $IFPScl(IFPSint(\overline{A})) = \overline{IFPScl(A)}$.

(d) *For each pair of IF pre semi open sets A and B in* (X, T) *with* $\overline{IFPSclA} = B$, *we have* $IFPSclA = \overline{IFPSclB}$.

Proposition 3.2. *Let* (X, T) *be an IFTS. Then* (X, T) *is an IF pre semi extremally disconnected space if and only if for IF pre semi open set A and IF pre semi closed set B such that* $A \subseteq B$, *we have* $IFPSclA \subseteq IFPSintB$.

Notation. An *IFS* which is both *IF* pre semi open set and *IF* pre semi closed set is called *IF pre semi clopen set*.

Remark 3.1. Let (X, T) be an *IF* pre semi extremally disconnected space. Let $\{A_i, \overline{B}_i / i \in N\}$ be a collection such that A_i's are *IF* pre semi open sets, B_i's are *IF* pre semi closed sets and let A, $\overline{B}$ be *IF* pre semi clopen sets, respectively. If $A_i \subseteq A \subseteq B_j$ and $A_i \subseteq B \subseteq B_j$ for all $i, j \in N$, then there exists an *IF* pre semi clopen set C such that $IFPSclA_i \subseteq C \subseteq IFPSintB_j$ for all $i, j \in N$. By Proposition 3.2,

$$IFPSclA_i \subseteq IFPSclA \cap IFPSintB \subseteq IFPSintB_j \ (i, j \in N).$$

Put $C = IFPSclA \cap IFPSintB$. Now, C satisfies our required condition.

Proposition 3.3. *Let* (X, T) *be an IF pre semi extremally disconnected space. Let* $(A_q)_{q \in Q}$ *and* $(B_q)_{q \in Q}$ *be the monotone increasing collections of IF pre semi open sets and IF pre semi closed sets of* (X, T), *respectively,* *and suppose that* $A_{q_1} \subseteq B_{q_2}$ *whenever* $q_1 < q_2$ (Q *is the set of rational numbers*). *Then there exists a monotone increasing collection* $\{C_q\}_{q \in Q}$ *of IF pre semi clopen sets of* (X, T) *such that* $IFPSclA_{q_1} \subseteq C_{q_2}$ *and* $C_{q_1} \subseteq IFPSintB_{q_2}$ *whenever* $q_1 < q_2$.

4. Properties and Characterizations of Intuitionistic Fuzzy Pre Semi Extremally Disconnected Spaces

In this section, various properties and characterizations of intuitionistic fuzzy pre semi extremally disconnected spaces are discussed.

Definition 4.1. An *IF* real line $\mathbb{R}_{\mathbb{I}}(I)$ is the set of all monotone decreasing *IF* sets $A \in \zeta^{\mathbb{R}}$ satisfying $\bigcup \{A(t) : t \in \mathbb{R}\} = 1^{\sim}$ and $\bigcap \{A(t) : t \in \mathbb{R}\} = 0^{\sim}$ after the identification of *IF* sets $A, B \in \mathbb{R}_{\mathbb{I}}(I)$ if and only if $A(t-) = B(t-)$ and $A(t+) = B(t+)$ for all $t \in R$, where

$$A(t-) = \bigcap\{A(s) : s < t\} \quad \text{and} \quad A(t+) = \bigcup\{A(s) : s > t\}.$$

The *IF* unit interval $\mathbb{I}_{\mathbb{I}}(I)$ is a subset of $\mathbb{R}_{\mathbb{I}}(I)$ such that $[A] \in \mathbb{I}_{\mathbb{I}}(I)$ if the membership and non-membership of A are defined by

$$\mu_A(t) = \begin{cases} 1, & t < 0, \\ 0, & t > 1 \end{cases} \quad \text{and} \quad \gamma_A(t) = \begin{cases} 1, & t < 0, \\ 0, & t > 1, \end{cases}$$

respectively.

The natural *IF* topology on $\mathbb{R}_{\mathbb{I}}(I)$ is generated from the subbasis $\{L_t^{\mathbb{I}}, R_t^{\mathbb{I}}, t \in \mathbb{R}\}$, where $L_t^{\mathbb{I}}, R_t^{\mathbb{I}} : \mathbb{R}_{\mathbb{I}}(I) \to \mathbb{I}_{\mathbb{I}}(I)$ are given by $L_t^{\mathbb{I}}(A) = \overline{A(t-)}$ and $R_t^{\mathbb{I}}(A) = A(t+)$, respectively.

Definition 4.2. Let (X, T) be an *IFTS*. Then a mapping $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ is called *lower* (resp. *upper*) *IF pre semi continuous*, if $f^{-1}(R_t^{\mathbb{I}})$ (resp.

$f^{-1}(L_t^{\mathbb{I}}))$ is an *IF* pre semi open set (resp. *IF* pre semi open/*IF* pre semi closed) for each $t \in R$.

Notation. Let X be any nonempty set and $A \in \zeta^X$. Then for $x \in X$, $\langle \mu_A(x), \gamma_A(x) \rangle$ is denoted by $A^{\sim}$.

Proposition 4.1. *Let (X, T) be an IFTS. Let $A \in \zeta^X$, and let $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ be such that*

$$f(x)(t) = \begin{cases} 1^{\sim}, & \text{if } t < 0, \\ A^{\sim}, & \text{if } 0 \leq t \leq 1, \\ 0^{\sim}, & \text{if } t > 1, \end{cases}$$

for all $x \in X$ and $t \in R$. Then f is lower (*resp. upper*) *IF pre semi continuous iff A is an IF pre semi open* (*resp. IF pre semi open/IF pre semi closed*) *set.*

Definition 4.3. Let (X, T) be an *IFTS*. The characteristic function of *IFS* *A* in *X* is the function $\psi_A : X \to \mathbb{I}_{\mathbb{I}}(I)$ defined by $\psi_A(x) = A^{\sim}$, $x \in X$.

Proposition 4.2. *Let (X, T) be an IFTS, and let $A \in \zeta^X$. Then ψ_A is lower* (*resp. upper*) *IF pre semi continuous iff A is an IF pre semi open* (*resp. IF pre semi open/IF pre semi closed*) *set.*

Proof. The proof follows from Proposition 4.1.

Definition 4.4. Let (X, T) and (Y, S) be *IFTSs*. Then a mapping $f : (X, T) \to (Y, S)$ is called *intuitionistic fuzzy strongly pre semi continuous* (for short *IF strongly pre semi continuous*) if $f^{-1}(A)$ is *IF* pre semi clopen in (X, T) for every *IF* pre semi open set in (Y, S).

Proposition 4.3. *Let (X, T) be an IFTS. Then the following are equivalent*:

(a) (X, T) *is IF pre semi extremally disconnected.*

(b) *If* $g, h : X \to \mathbb{R}_{\mathbb{I}}(I)$, *g is lower IF pre semi continuous*, *h is upper IF pre semi continuous and* $g \subseteq h$, *then there exists an IF strongly pre semi continuous function*, $f : (X, T) \to \mathbb{R}_{\mathbb{I}}(I)$ *such that* $g \subseteq f \subseteq h$.

(c) *If* $\overline{A}$ *and B are IF pre semi open sets such that* $B \subseteq A$, *then there exists an IF strongly pre semi continuous function* $f : (X, T) \to \mathbb{I}_{\mathbb{I}}(I)$ *such that* $B \subseteq \overline{L_1^{\mathbb{I}} f} \subseteq R_0^{\mathbb{I}} f \subseteq A$.

5. Tietze Extension Theorem for *IF* Pre Semi Extremally Disconnected Spaces

In this section, Tietze extension theorem for *IF* pre semi extremally disconnected spaces is studied.

Notation. Let (X, T) be an *IFTS* and let $A \subset X$. Then an *IFS* χ_A^* is of the form $\langle x, \chi_A(x), 1 - \chi_A(x)\rangle$.

Proposition 5.1. *Let* (X, T) *be an upper IF pre semi extremally disconnected space and let* $A \subset X$ *be such that* χ_A^* *is an IF pre semi open set in* (X, T). *Let* $f : (A, T/A) \to \mathbb{I}_{\mathbb{I}}(I)$ *be an IF strongly pre semi continuous function*. *Then f has an IF strongly pre semi continuous extension over* (X, T).

Proof. Let $g, h : X \to \mathbb{I}_{\mathbb{I}}(I)$ be such that $g = f = h$ on A, and $g(x) = 0^{\sim}$, $h(x) = 1^{\sim}$ if $x \notin A$.

We now have

$$R_t^{\mathbb{I}} g = \begin{cases} B_t \cap \chi_A^*, & \text{if } t \geq 0, \\ 1_{\sim}, & \text{if } t < 0, \end{cases}$$

where B_t is an *IF* pre semi open set such that $B_t / A = R_t^{\mathbb{I}} f$ and

$$L_t^{\mathbb{I}} h = \begin{cases} A_t \cap \chi_A^*, & \text{if } t \leq 1, \\ 1_{\sim}, & \text{if } t > 1, \end{cases}$$

where A_t is *IF* pre semi open such that $A_t / A = L_t^{\mathbb{I}} f$. Thus, g is lower *IF* pre semi continuous, h is upper *IF* pre semi continuous and $g \subseteq h$. By Proposition 4.2, there is an *IF* strongly pre semi continuous function $F : (X, T) \to \mathbb{I}_{\mathbb{I}}(I)$ such that $g \subseteq F \subseteq h$; hence $F \equiv f$ on A.

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