Far East Journal of Applied Mathematics
Volume 72, Number 2, 2012, Pages 91-99
Published Online: December 2012
Available online at http://pphmj.com/journals/fjam.htm
Published by Pushpa Publishing House, Allahabad, INDIA

APPLICATION OF THE ADOMIAN DECOMPOSITION METHOD AND THE PERTURBATION METHOD TO SOLVING A SYSTEM OF PERTURBED EQUATIONS

Françis Bassono, Pare Youssouf, Gabriel Bissanga[*] and Blaise Some

Université de Ouagadougou
Burkina Faso
e-mail: sonobi2002@yahoo.fr; pareyoussouf@yahoo.fr
some@univ-ouaga.bf

[*]Université Marien Ngouabi
Congo
e-mail: bissanga1@yahoo.fr

Abstract

In this paper, the Adomian decomposition method and the perturbation method are used to construct the solution of the initial value problem of a system of differential equations.

1. Introduction

These last years, the Adomian decomposition method (ADM) is used a lot to get an approximation of a solution of several kinds of problems, and the perturbation method is too very useful to succeed to this same kind of objective. These two methods often drive us to a same result, better again they sometimes lead us toward the exact solution. Here, we use both methods to investigate a system of perturbed equations.

2010 Mathematics Subject Classification: 47H14, 34G20, 47J25, 65J15.

Keywords and phrases: Adomian decomposition method, perturbation method.

Received September 17, 2012

2. About Solution of a System of Perturbed Equations

Let us consider the following initial value problem of a system of differential equations:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \varepsilon \dfrac{\partial^2 v}{\partial x^2} \\ \dfrac{\partial v}{\partial t} = \varepsilon \dfrac{\partial^2 u}{\partial x^2} \\ u(x, 0) = \beta \sin \omega x \\ v(x, 0) = \mu \cos \omega x \end{cases} \tag{2.1}$$

with $0 < \varepsilon \ll 1$, where β, μ and ω are arbitrary constants independent of ε; and $u(x, t)$ the unknown function.

2.1. The Adomian decomposition method

General properties of the ADM and its applications can be found in [1-5]. Suppose that we need to solve the following equation:

$$Au = f \tag{2.2}$$

in a real Hilbert space H, where $A; H \to H$ is a linear or a nonlinear operator, $f \in H$ and u is the unknown. The principle of the ADM is based on the decomposition of the nonlinear operator A in the following form:

$$A = L + R + N, \tag{2.3}$$

where $L + R$ is linear, N nonlinear, L invertible with L^{-1} as inverse. Using that decomposition, equation (2.2) is equivalent to

$$u = \theta + L^{-1} f - L^{-1} Ru - L^{-1} Nu, \tag{2.4}$$

where θ verifies $L\theta = 0$. (2.4) is called the *Adomian's fundamental equation* or *Adomian's canonical form*. We look for the solution of (2.4) in a series expansion form $u = \sum_{n=0}^{+\infty} u_n$ and we consider $Nu = \sum_{n=0}^{+\infty} A_n$, where A_n are special polynomials of variables $u_0, u_1, \ldots, u_n$ called *Adomian polynomials* and defined by [1-4]:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{+\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \ldots,$$

where λ is a parameter used by "convenience". Thus (2.4) can be rewritten as follows:

$$\sum_{n=0}^{+\infty} u_n = \theta + L^{-1}f - L^{-1}R\left(\sum_{n=0}^{+\infty} u_n\right) - L^{-1}\left(\sum_{n=0}^{+\infty} A_n\right). \tag{2.5}$$

We suppose that the series $\sum_{n=0}^{+\infty} u_n$ and $\sum_{n=0}^{+\infty} A_n$ are convergent, and obtain by identification the Adomian algorithm:

$$\begin{cases} u_0 = \theta + L^{-1}f \\ u_1 = -L^{-1}(Ru_0) - L^{-1}A_0 \\ \vdots \\ u_{n+1} = -L^{-1}(Ru_n) - L^{-1}A_n. \end{cases} \tag{2.6}$$

In practice, it is often difficult to calculate all the terms of an Adomian series, so we approach the series solution by the truncated series: $u = \sum_{i=0}^{n} u_i$, where the choice of n depends on error requirements.

According to the Adomian decomposition method, we suppose that the solution (u, v) of (2.1) has the following form: $u = \sum_{i=0}^{n} u_i$ and $v = \sum_{i=0}^{n} v_i$. From (2.1), we have

$$\begin{cases} u(x, t) = u(x, 0) + \varepsilon\int_0^t \frac{\partial^2 v(x, s)}{\partial x^2}\,ds \\ v(x, t) = v(x, 0) + \varepsilon\int_0^t \frac{\partial^2 u(x, s)}{\partial x^2}\,ds \end{cases} \tag{2.7}$$

and we obtain the following Adomian algorithm:

$$\begin{cases} u_0(x, t) = u(x, 0) \\ u_{n+1}(x, t) = \varepsilon \int_0^t \frac{\partial^2 v_n(x, s)}{\partial x^2} ds, \quad n \geq 0 \\ v_0(x, t) = v(x, 0) \\ v_{n+1}(x, t) = \varepsilon \int_0^t \frac{\partial^2 u_n(x, s)}{\partial x^2} ds, \quad n \geq 0. \end{cases} \tag{2.8}$$

Finally, we get

$$\begin{cases} u_0(x, t) = \beta \sin \omega x \\ u_1(x, t) = -\mu \frac{(\varepsilon\omega^2 t)}{1!} \cos \omega x \\ u_2(x, t) = \beta \frac{(\varepsilon\omega^2 t)^2}{2!} \sin \omega x \\ u_3(x, t) = -\mu \frac{(\varepsilon\omega^2 t)^3}{3!} \cos \omega x \\ \dots \\ u_{2n}(x, t) = \beta \frac{(\varepsilon\omega^2 t)^{2n}}{(2n)!} \sin \omega x \\ u_{2n+1}(x, t) = -\mu \frac{(\varepsilon\omega^2 t)^{2n+1}}{(2n+1)!} \cos \omega x, \end{cases} \tag{2.9}$$

$$\begin{cases} v_0(x, t) = \mu \cos \omega x \\ v_1(x, t) = -\beta \frac{(\varepsilon\omega^2 t)}{1!} \sin \omega x \\ v_2(x, t) = \mu \frac{(\varepsilon\omega^2 t)^2}{2!} \cos \omega x \\ v_3(x, t) = -\beta \frac{(\varepsilon\omega^2 t)^3}{3!} \sin \omega x \\ \dots \\ v_{2n}(x, t) = \mu \frac{(\varepsilon\omega^2 t)^{2n}}{(2n)!} \cos \omega x \\ v_{2n+1}(x, t) = -\beta \frac{(\varepsilon\omega^2 t)^{2n+1}}{(2n+1)!} \sin \omega x. \end{cases} \tag{2.10}$$

Let us put

$$\begin{cases} \varphi_n^1(x, t) = \sum_{k=0}^{n} u_{2k}(x, t) \\ \varphi_n^2(x, t) = \sum_{k=0}^{n} u_{2k+1}(x, t) \\ \phi_n^1(x, t) = \sum_{k=0}^{n} v_{2k}(x, t) \\ \phi_n^2(x, t) = \sum_{k=0}^{n} v_{2k+1}(x, t). \end{cases} \tag{2.11}$$

Thus

$$\begin{cases} u(x, t) = \lim_{n\to+\infty} \varphi_n^1(x, t) + \lim_{n\to+\infty} \varphi_n^2(x, t) \\ v(x, t) = \lim_{n\to+\infty} \phi_n^1(x, t) + \lim_{n\to+\infty} \phi_n^2(x, t) \end{cases} \tag{2.12}$$

what is equivalent to

$$\begin{cases} u(x, t) = \beta \sin(\omega x) \cdot ch(t\varepsilon\omega^2) - \mu \cos(\omega x) \cdot sh(t\varepsilon\omega^2) \\ v(x, t) = \mu \cos(\omega x) \cdot ch(t\varepsilon\omega^2) - \beta \sin(\omega x) \cdot sh(t\varepsilon\omega^2). \end{cases} \tag{2.13}$$

2.2. The perturbation method

General theory of the perturbation method can be found in [6-8].

According to the perturbation theory, we suppose that the solution (u, v) of (2.1) has the following form:

$$u = \sum_{i=0}^{n} \varepsilon^i u_i; \quad v = \sum_{i=0}^{n} \varepsilon^i v_i. \tag{2.14}$$

Taking (2.14) into (2.1), and collecting equal powers of ε, we obtain a system of recurrent initial value problems for $u_n(x, t)$, $v_n(x, t)$, $n =$

0, 1, 2, ...,

$$\begin{cases} \dfrac{\partial u_0(x, t)}{\partial t} = 0 \\ u_0(x, 0) = \beta \sin \omega x \\ \dfrac{\partial v_0(x, t)}{\partial t} = 0 \\ v_0(x, 0) = \mu \cos \omega x \end{cases} \tag{2.15}$$

$$\begin{cases} \dfrac{\partial u_1(x, t)}{\partial t} = \dfrac{\partial^2 v_0(x, t)}{\partial x^2} \\ u_1(x, 0) = 0 \\ \dfrac{\partial v_1(x, t)}{\partial t} = \dfrac{\partial^2 u_0(x, t)}{\partial x^2} \\ v_1(x, 0) = 0 \end{cases} \tag{2.16}$$

$$\cdots$$

$$\begin{cases} \dfrac{\partial u_n(x, t)}{\partial t} = \dfrac{\partial^2 v_{n-1}(x, t)}{\partial x^2} \\ u_n(x, 0) = 0 \\ \dfrac{\partial v_n(x, t)}{\partial t} = \dfrac{\partial^2 u_{n-1}(x, t)}{\partial x^2} \\ v_n(x, 0) = 0 \\ n \geq 1. \end{cases} \tag{2.17}$$

We obtain:

$$\begin{cases} u_0(x, t) = \beta \sin \omega x \\ u_1(x, t) = -\mu \dfrac{(\omega^2 t)}{1!} \cos \omega x \\ u_2(x, t) = \beta \dfrac{(\omega^2 t)^2}{2!} \sin \omega x \\ u_3(x, t) = -\mu \dfrac{(\omega^2 t)^3}{3!} \cos \omega x \\ \cdots \\ u_{2n}(x, t) = \beta \dfrac{(\omega^2 t)^{2n}}{(2n)!} \sin \omega x \\ u_{2n+1}(x, t) = -\mu \dfrac{(\omega^2 t)^{2n+1}}{(2n+1)!} \cos \omega x, \end{cases} \tag{2.18}$$

$$\begin{cases} v_0(x,\, t) = \mu \cos \omega x \\ v_1(x,\, t) = -\beta \dfrac{(\omega^2 t)}{1!} \sin \omega x \\ v_2(x,\, t) = \mu \dfrac{(\omega^2 t)^2}{2!} \cos \omega x \\ v_3(x,\, t) = -\beta \dfrac{(\omega^2 t)^3}{3!} \sin \omega x \\ \dots \\ v_{2n}(x,\, t) = \mu \dfrac{(\omega^2 t)^{2n}}{(2n)!} \cos \omega x \\ v_{2n+1}(x,\, t) = -\beta \dfrac{(\omega^2 t)^{2n+1}}{(2n+1)!} \sin \omega x. \end{cases} \tag{2.19}$$

Thus we have

$$u = \sum_{i=0}^{n} \varepsilon^i u_i = \left(\beta \sin \omega x + \varepsilon^2 \beta \frac{(\omega^2 t)^2}{2!} \sin \omega x + \cdots + \varepsilon^{2n} \beta \frac{(\omega^2 t)^{2n}}{(2n)!} \sin \omega x \right)$$

$$- \left(\varepsilon \mu \frac{(\omega^2 t)}{1!} \cos \omega x + \varepsilon^3 \mu \frac{(\omega^2 t)^3}{3!} \cos \omega x + \cdots \right.$$

$$\left. + \varepsilon^{2n+1} \mu \frac{(\omega^2 t)^{2n+1}}{(2n+1)!} \cos \omega x \right)$$

$$= \beta \sin(\omega x) \cdot ch(t\varepsilon\omega^2) - \mu \cos(\omega x) \cdot sh(t\varepsilon\omega^2), \tag{2.20}$$

$$v = \sum_{i=0}^{n} \varepsilon^i v_i = \left(\mu \cos \omega x + \varepsilon^2 \mu \frac{(\omega^2 t)^2}{2!} \cos \omega x + \cdots + \varepsilon^{2n} \mu \frac{(\omega^2 t)^{2n}}{(2n)!} \cos \omega x \right)$$

$$- \left(\varepsilon \beta \frac{(\omega^2 t)}{1!} \sin \omega x + \varepsilon^3 \beta \frac{(\omega^2 t)^3}{3!} \sin \omega x + \cdots \right.$$

$$\left. + \varepsilon^{2n+1}\beta \frac{(\omega^2 t)^{2n+1}}{(2n+1)!} \sin \omega x \right)$$

$$= \mu \cos(\omega x) \cdot ch(t\varepsilon\omega^2) - \beta \sin(\omega x) \cdot sh(t\varepsilon\omega^2). \tag{2.21}$$

3. Conclusion

In this paper, we showed that using the both methods, we get the same solution. We have obtained the same results in [9-11].

References

[1] K. Abbaoui, Les fondements de la méthode décompositionnelle d'Adomian et application à la résolution de problèmes issus de la biologie et de la médécine, Thèse de Doctorat de l'Université Paris VI, 1995.

[2] K. Abbaoui and Y. Cherruault, Convergence of Adomian method applied to differential equations, Math. Comput. Modelling 28(5) (1994), 103-109.

[3] K. Abbaoui and Y. Cherruault, Convergence of Adomian method applied to non-linear equations, Math. Comput. Modelling 20(9) (1994), 60-73.

[4] K. Abbaoui and Y. Cherruault, The decomposition method applied to the Cauchy problem, Kybernetes 28(1) (1999), 68-74.

[5] N. Ngarhasta, B. Some, K. Abbaoui and Y. Cherruault, New numerical study of Adomian method applied to a diffusion model, Kybernetes 31(1) (2002), 61-75.

[6] A. H. Nafey, Perturbation Methods, Wiley, New York, 1973.

[7] R. E. O'Malley, Introduction to Singular Perturbations, Academic Press, New York, 1974.

[8] R. E. O'Malley, Singular Perturbation Methods for Ordinary Differential Equations, Springer, 1991.

[9] Gabriel Bissanga and A.-K. Nsemi, Application of Adomian decomposition method to solving the Van der Pol equation and comparison with the regular perturbation method, Proceedings of the Five International Workshop on Contemporary Problems in Mathematical Physics, Cotonou, Republic of Benin, J. Govaerts and M. N. Hounkounou, eds., International Chair in Mathematical Physics and Applications, ICMPA-UNESCO Chair, University of Abomey-Calavy, 072 BP 50 Cotonou, Republic of Benin, December 2008.

[10] Gabriel Bissanga, Application of Adomian decomposition method to solving the Duffing equation, Comparison with Perturbation Method, Proceedings of the Fourth International Workshop on Contemporary Problems in Mathematical Physics, Cotonou, Republic of Benin, November 2005, World Scientific Publishing Co. Pte. Ltd., 2006.

[11] Pierre Baki-Tangou and Gabriel Bissanga, Application of Adomian decomposition method to solving the Duffing-Van der Pol equation, Commun. Math. Anal. 4 (2008), 30-40.