



NEW METHODS TO SOLVE FUZZY ASSIGNMENT PROBLEMS USING ORDERING BASED ON THE MAGNITUDE OF A FUZZY NUMBER

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Abstract

In this paper, the assignment problem with fuzzy costs or times \tilde{c}_{ij} is investigated. Here \tilde{c}_{ij} has been considered as the trapezoidal fuzzy number. Three different methods are proposed to solve the fuzzy assignment problem (FAP): (i) FAP is solved by using the fuzzy Hungarian method in which the magnitude of a fuzzy number is used to order the fuzzy numbers, (ii) FAP is defuzzified into the crisp assignment problem using the magnitude of a fuzzy number and optimal solution is obtained using Hungarian method and (iii) FAP is transformed into crisp linear programming problem using the magnitude of a fuzzy number and optimal solution is obtained by using TORA. Numerical example is provided to illustrate these three different methods and the solutions are compared.

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1. Introduction

Assignment problem is a particular case of the transportation problem in which the number of jobs (or origins or sources) is equal to the number of facilities (destinations or machines or persons and so on). The goal of a general assignment problem is to find an optimal assignment of machines (laborers) to jobs without assigning an agent more than once and ensuring that all jobs are completed. The objective might be to minimize the total time to complete a set of jobs or to maximize skill rating, maximize the total satisfaction of the group or minimize the cost of the assignment. Zadeh [7] introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life. In recent years, fuzzy transportation and fuzzy assignment problems have received much attention. Stephen Dinagar and Palanivel [6] proposed a fuzzy modified distribution method to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [4, 5] introduced new algorithms for finding a fuzzy optimal solution for fuzzy transportation problems. Mukherjee and Basu [3] introduced application of fuzzy ranking method for solving assignment problems with fuzzy costs.

In this paper, the assignment problem with fuzzy costs or times \tilde{c}_{ij} is investigated. Here \tilde{c}_{ij} has been considered as the trapezoidal fuzzy number. Three different methods are proposed to solve the fuzzy assignment problem (FAP): (i) FAP is solved by using the fuzzy Hungarian method in which the magnitude of a fuzzy number is used to order the fuzzy numbers, (ii) FAP is defuzzified into the crisp assignment problem using the magnitude of a fuzzy number and optimal solution is obtained using Hungarian method and (iii) FAP is transformed into crisp linear programming problem using the magnitude of a fuzzy number and optimal solution is obtained by using TORA. Numerical example is provided to illustrate these three different methods and the solutions are compared.

Here, in Section 2, some necessary concepts of fuzzy set theory, arithmetic operations between trapezoidal fuzzy numbers [2, 8] and

definitions of ordering on the set of the fuzzy numbers based on the magnitude of a fuzzy number are reviewed [1, 4, 5]. In Section 3, fuzzy assignment problem is defined and three different methods are presented using ordering based on the magnitude of a fuzzy number to solve it. In Section 4, numerical example is provided to illustrate these three different methods and the solutions are compared.

2. Preliminaries

Definition. Let X be a classical set of objects called the *universe* whose generic elements are denoted by x . The membership in a crisp subset of X is often viewed as characteristic function $\mu_A(x)$ from X to $\{0, 1\}$ such that

$$\begin{aligned}\mu_A(x) &= 1, \text{ if } x \in A \\ &= 0, \text{ otherwise,}\end{aligned}$$

where $\{0, 1\}$ is called *valuation set*.

If the valuation set is allowed to be the real interval $[0, 1]$, A is called a *fuzzy set*. $\mu_A(x)$ is the degree of membership of x in A . The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A . Therefore, A is completely characterized by the set of ordered pairs: $A = \{(x, \mu_A(x)) / x \in X\}$.

Definition. The support of a fuzzy set A is the crisp subset of X and is presented as:

$$\text{supp } A = \{x \in X / \mu_A(x) > 0\}.$$

Definition. The α level (α -cut) set of a fuzzy set A is a crisp subset of X and is denoted by

$$A_\alpha = \{x \in X / \mu_A(x) > \alpha\}.$$

Definition. A fuzzy set A in X is *convex* if $\mu_A(\lambda x + (1 - \lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}$, $x, y \in X$ and $\lambda \in [0, 1]$. Alternatively, a fuzzy set is convex if all α level sets are convex.

Definition. A fuzzy number \tilde{A} is a *convex normalized fuzzy set* on the real line R such that if

(1) it exists at least one $x_0 \in R$ with $\mu_A(x_0) = 1$.

(2) $\mu_A(x)$ is piecewise continuous.

Definition. We can define *trapezoidal fuzzy number* as $\tilde{A} = (a_1, a_2, a_3, a_4)$.

The membership function of this fuzzy number will be interpreted as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \frac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & x > a_4 \end{cases}$$

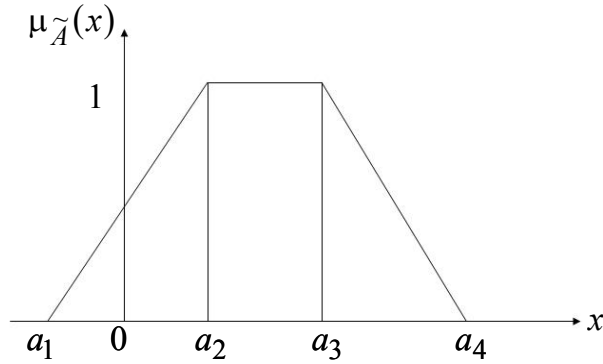


Figure 1

Arithmetic operations

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers.

$$\begin{aligned}\text{Addition: } \tilde{A}(+) \tilde{B} &= (a_1, a_2, a_3, a_4)(+)(b_1, b_2, b_3, b_4) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4);\end{aligned}$$

$$\begin{aligned}\text{Subtraction: } \tilde{A}(-) \tilde{B} &= (a_1, a_2, a_3, a_4)(-)(b_1, b_2, b_3, b_4) \\ &= (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1);\end{aligned}$$

$$\begin{aligned}\text{Scalar Multiplication: } x > 0, \quad x\tilde{A} &= (xa_1, xa_2, xa_3, xa_4) \\ x < 0, \quad x\tilde{A} &= (xa_4, xa_3, xa_2, xa_1).\end{aligned}$$

Definition. The *magnitude* of the trapezoidal fuzzy number $\tilde{u} = (x_0 - \sigma, x_0, y_0, y_0 + \beta)$ with parametric form $\tilde{u} = (\underline{u}(r), \bar{u}(r))$, where $\underline{u}(r) = x_0 - \sigma + \sigma r$ and $\bar{u}(r) = y_0 + \beta - \beta r$ is defined as $\text{Mag}(\tilde{u}) = \frac{1}{2} \left(\int_0^1 ([\underline{u}(r) + \bar{u}(r)] + x_0 + y_0) r dr \right)$, where $r \in [0, 1]$.

Remark. The magnitude of the trapezoidal fuzzy number $\tilde{u} = (a, b, c, d)$ is given by

$$\text{Mag}(\tilde{u}) = \frac{a + 5b + 5c + d}{12}.$$

Definition. Let \tilde{u} and \tilde{v} be two trapezoidal fuzzy numbers. The ranking of \tilde{u} and \tilde{v} by the $\text{Mag}(\cdot)$ on E , the set of trapezoidal fuzzy numbers is defined as follows:

- (i) $\text{Mag}(\tilde{u}) > \text{Mag}(\tilde{v})$ if and only if $\tilde{u} \succ \tilde{v}$;
- (ii) $\text{Mag}(\tilde{u}) < \text{Mag}(\tilde{v})$ if and only if $\tilde{u} \prec \tilde{v}$;
- (iii) $\text{Mag}(\tilde{u}) = \text{Mag}(\tilde{v})$ if and only if $\tilde{u} \approx \tilde{v}$.

Definition. The ordering \succeq and \preceq between any two trapezoidal fuzzy numbers \tilde{u} and \tilde{v} are defined as follows:

- (i) $\tilde{u} \succeq \tilde{v}$ if and only if $\tilde{u} \succ \tilde{v}$ or $\tilde{u} \approx \tilde{v}$ and
- (ii) $\tilde{u} \preceq \tilde{v}$ if and only if $\tilde{u} \prec \tilde{v}$ or $\tilde{u} \approx \tilde{v}$.

Note. (i) $\tilde{u} = (a, b, c, d) \approx \tilde{0}$ if and only if $\text{Mag}(\tilde{u}) = 0$;

(ii) $\tilde{u} = (a, b, c, d) \succeq \tilde{0}$ if and only if $\text{Mag}(\tilde{u}) \geq 0$ and

(iii) $\tilde{u} = (a, b, c, d) \preceq \tilde{0}$ if and only if $\text{Mag}(\tilde{u}) \leq 0$.

Definition. Let $\{\tilde{a}_i, i = 1, 2, \dots, n\}$ be a set of trapezoidal fuzzy numbers. If $\text{Mag}(\tilde{a}_k) \leq \text{Mag}(\tilde{a}_i)$, for all i . Then the fuzzy number \tilde{a}_k is the *minimum* of $\{\tilde{a}_i, i = 1, 2, \dots, n\}$.

Definition. Let $\{\tilde{a}_i, i = 1, 2, \dots, n\}$ be a set of trapezoidal fuzzy numbers. If $\text{Mag}(\tilde{a}_t) \geq \text{Mag}(\tilde{a}_i)$, for all i , then the fuzzy number \tilde{a}_t is the *maximum* of $\{\tilde{a}_i, i = 1, 2, \dots, n\}$.

3. Fuzzy Assignment Problem

The fuzzy assignment problem can be stated in the form of $n \times n$ fuzzy cost matrix $[\tilde{c}_{ij}]$ of real numbers as given in the following table:

Table 1

		Jobs				
		1	2	3	---j---	N
Persons	1	\tilde{c}_{11}	\tilde{c}_{12}	\tilde{c}_{13}	-- \tilde{c}_{1j} --	\tilde{c}_{1n}
	2	\tilde{c}_{21}	\tilde{c}_{22}	\tilde{c}_{23}	-- \tilde{c}_{2j} --	\tilde{c}_{2n}
	-	-	-	-	-	-
	i	\tilde{c}_{i1}	\tilde{c}_{i2}	\tilde{c}_{i3}	-- \tilde{c}_{ij} --	\tilde{c}_{in}
	-	-	-	-	-	-
N		\tilde{c}_{n1}	\tilde{c}_{n2}	\tilde{c}_{n3}	\tilde{c}_{nj}	\tilde{c}_{nn}

(1)

The cost $\tilde{c}_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$ is the cost of assigning the j th job to the i th person. The objective is to minimize the total cost of assigning all the jobs to the available persons (one job to one person).

3.1. Method 1

Fuzzy Hungarian method

In this section, a new method called the *fuzzy Hungarian method* using ordering based on the magnitude of a fuzzy number is proposed.

Step 1. Determine the fuzzy cost table from the given problem.

(i) If the number of sources is equal to the number of destinations, then go to Step 3.

(ii) If the number of sources is not equal to the number of destinations, then go to Step 2.

Step 2. Add a dummy source or dummy destination, so that the fuzzy cost table becomes a square matrix. The fuzzy cost entries of dummy source/destinations are always fuzzy zero.

Step 3. Select the row minimum by ordering based on the magnitude of a fuzzy number and then subtract it from each row entry of that row.

Step 4. Select the column minimum by ordering based on the magnitude of a fuzzy number and then subtract the column minimum of the resulting fuzzy assignment problem after using Step 3 from each column entry of that column. Each column and row now has at least one fuzzy zero.

Step 5. In the modified fuzzy assignment table obtained in Step 4, search for fuzzy optimal assignment as follows.

(a) Examine the rows successively until a row with a single fuzzy zero is found. Assign this fuzzy zero and cross off all other fuzzy zeros in its column. Continue this for all the rows.

(b) Repeat the procedure for each column of reduced fuzzy assignment table.

(c) If a row and/or column has two or more fuzzy zeros assign arbitrary any one of these fuzzy zeros and cross off all other fuzzy zeros of that row/column.

(d) Repeat (a) through (c) above successively until the chain of assigning or cross ends.

Step 6. If the number of assignments is equal to n , the order of the fuzzy cost matrix, fuzzy optimum solution is reached. If the number of assignments is less than n , the order of the fuzzy cost matrix, then go to Step 7.

Step 7. Draw the minimum number of horizontal and/or vertical lines to cover all the fuzzy zeros of the reduced fuzzy assignment matrix. This can be done by using the following:

- (i) Mark rows that do not have any assigned fuzzy zero.
- (ii) Mark columns that have crossed fuzzy zeros in the marked rows.
- (iii) Mark rows that do have assigned fuzzy zeros in the marked columns.
- (iv) Repeat (ii) and (iii) above until the chain of marking is completed.
- (v) Draw lines through all the unmarked rows and marked columns.

This gives the desired minimum number of lines.

Step 8. Develop the new revised reduced fuzzy cost matrix as follows:

(i) Find the smallest entry using ordering based on the magnitude of a fuzzy number of the reduced fuzzy cost matrix among the entries that not covered by any of the lines.

(ii) Subtract this entry from all the uncovered entries and add the same to all the entries lying at the intersection of any two lines.

Step 9. Repeat Step 6 to Step 8 until fuzzy optimal solution to the given fuzzy assignment problem is attained.

3.2. Method 2

The fuzzy assignment problem of form (1) can be transformed into the crisp assignment problem using the magnitude of the fuzzy number and then it can be solved by using Hungarian method.

3.3. Method 3

The fuzzy assignment problem of form (1) can be stated mathematically in the linear programming problem form as

$$\begin{aligned}
 &\text{Minimize } \tilde{z} = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}, \quad i = 1, 2, \dots, n; j = 1, 2, \dots, n \\
 &\text{Subject to } \sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n \\
 &\quad \sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, \dots, n \\
 &x_{ij} \in \{0, 1\}, \text{ where } x_{ij} = \begin{cases} 1, & \text{if the } i\text{th person is assigned the } j\text{th job} \\ 0, & \text{otherwise} \end{cases}
 \end{aligned} \tag{2}$$

is the decision variable denoting the assignment of the person i to job j . \tilde{c}_{ij} is the cost of assigning the j th job to the i th person. The objective is to minimize the total cost of assigning all the jobs to the available persons. \tilde{z} cannot be minimized directly. For solving the problem, the fuzzy cost coefficients are defuzzified into crisp ones by using the magnitude of the fuzzy number. Magnitude of the fuzzy number gives the representative value of the fuzzy number \tilde{c} . It satisfies the linearity and additivity property. Now the minimum objective value \tilde{z}^* can be obtained from the formulation.

$$\text{Mag}(\tilde{z}^*) = \text{Minimize } z = \sum_{i=1}^n \sum_{j=1}^n \text{mag}(\tilde{c}_{ij}) x_{ij}, \quad i = 1, 2, \dots, n; j = 1, 2, \dots, n$$

$$\text{Subject to: } \sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n$$

$$\sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, \dots, n \quad (3)$$

$$x_{ij} \in \{0, 1\}, \text{ where } x_{ij} = \begin{cases} 1, & \text{if the } i\text{th person is assigned the } j\text{th job} \\ 0, & \text{otherwise} \end{cases}$$

is the decision variable denoting the assignment of the person i to job j . \tilde{c}_{ij} is the cost of assigning the j th job to the i th person. The objective is to minimize the total cost of assigning all the jobs to the available persons. Since $\text{mag}(\tilde{c}_{ij})$ are crisp values, this problem (3) is obviously the crisp assignment problem of form (2) and this linear programming problem form of the problem can be solved by using TORA to obtain the fuzzy optimal solution. Knowing the optimal solution x^* , the optimal fuzzy objective value

$$\tilde{z}^* \text{ of the original problem can be calculated as } \tilde{z}^* = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}^*.$$

4. Numerical Example

Consider a fuzzy assignment problem with rows representing three persons P_1, P_2, P_3 and columns representing the three jobs J_1, J_2, J_3 . The cost matrix $[\tilde{C}_{ij}]$ is given whose elements are trapezoidal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

$$\begin{matrix} & \begin{matrix} J_1 & J_2 & J_3 \end{matrix} \\ \begin{matrix} P_1 \\ P_2 \\ P_3 \end{matrix} & \begin{pmatrix} (1, 2, 3, 4) & (1, 3, 4, 6) & (9, 11, 12, 14) \\ (0, 1, 2, 4) & (-1, 0, 1, 2) & (5, 6, 7, 8) \\ (3, 5, 6, 8) & (5, 8, 9, 12) & (12, 15, 16, 19) \end{pmatrix} \end{matrix}.$$

Method 1

The given problem is a balanced one. Now using Step 3 and Step 4 of the fuzzy Hungarian method, we have the following modified fuzzy assignment matrix.

$$\begin{matrix} & J_1 & J_2 & J_3 \\ \begin{matrix} P_1 \\ P_2 \\ P_3 \end{matrix} & \begin{pmatrix} \tilde{0} & (-3, 0, 2, 5) & (-4, 1, 5, 10) \\ (-2, 0, 2, 5) & \tilde{0} & \tilde{0} \\ \tilde{0} & (-3, 2, 4, 9) & (-5, 2, 6, 13) \end{pmatrix} \end{matrix}.$$

Now using Step 5 to Step 9 of the fuzzy Hungarian method, we have the following fuzzy optimal assignment matrix.

$$\begin{matrix} & J_1 & J_2 & J_3 \\ \begin{matrix} P_1 \\ P_2 \\ P_3 \end{matrix} & \begin{pmatrix} \tilde{\times} & \boxed{\tilde{0}} & (-9, -1, 5, 13) \\ (-5, 0, 4, 10) & \tilde{\times} & \boxed{\tilde{0}} \\ \boxed{\tilde{0}} & (-8, 0, 4, 12) & (-10, 0, 6, 16) \end{pmatrix} \end{matrix}.$$

Therefore, the fuzzy optimal assignment for the given fuzzy assignment problem is

$$P_1 \rightarrow J_2, P_2 \rightarrow J_3, P_3 \rightarrow J_1.$$

The fuzzy optimal total cost is calculated as

$$\begin{aligned} \tilde{c}_{12} + \tilde{c}_{23} + \tilde{c}_{31} &= (1, 3, 4, 6) + (5, 6, 7, 8) + (3, 5, 6, 8) \\ &= (9, 14, 17, 22) \text{ and } \text{Mag}(9, 14, 17, 22) = 186/12. \end{aligned}$$

Method 2

Using the magnitude of the trapezoidal fuzzy number, the given problem can be written as

$$\begin{matrix} & J_1 & J_2 & J_3 \\ P_1 & \left(30/12 & 42/12 & 138/12 \right) \\ P_2 & \left(19/12 & 6/12 & 78/12 \right) \\ P_3 & \left(66/12 & 102/12 & 186/12 \right) \end{matrix}$$

Solving this crisp assignment problem using Hungarian method, we have the following fuzzy optimal assignment matrix:

$$\begin{matrix} & J_1 & J_2 & J_3 \\ P_1 & \left(\text{X} & \boxed{0} & 2 \right) \\ P_2 & \left(25/12 & \text{X} & \boxed{0} \right) \\ P_3 & \left(\boxed{0} & 2 & 3 \right) \end{matrix}$$

Therefore, the optimal assignment is $P_1 \rightarrow J_2$, $P_2 \rightarrow J_3$, $P_3 \rightarrow J_1$.

And the optimal total cost is calculated as $c_{12} + c_{23} + c_{31} = 42/12 + 78/12 + 66/12 = 186/12$.

Method 3

The given fuzzy assignment problem can be formulated in the following mathematical programming form:

$$\begin{aligned} & \text{Min}[\text{Mag}(1, 2, 3, 4)x_{11} + \text{Mag}(1, 3, 4, 6)x_{12} + \text{Mag}(9, 11, 12, 14)x_{13} \\ & + \text{Mag}(0, 1, 2, 4)x_{21} + \text{Mag}(-1, 0, 1, 2)x_{22} + \text{Mag}(5, 6, 7, 8)x_{23} \\ & + \text{Mag}(3, 5, 6, 8)x_{31} + \text{Mag}(5, 8, 9, 12)x_{32} + \text{Mag}(12, 15, 16, 19)x_{33}] \end{aligned}$$

$$\text{Subject to: } x_{11} + x_{12} + x_{13} = 1, x_{11} + x_{21} + x_{31} = 1,$$

$$x_{21} + x_{22} + x_{23} = 1, x_{12} + x_{22} + x_{32} = 1,$$

$$x_{31} + x_{32} + x_{33} = 1, x_{13} + x_{23} + x_{33} = 1, x_{ij} \in \{0, 1\}.$$

Replacing the magnitude, the problem results in the crisp LPP form. Solving the above LPP using TORA we obtain the following optimal solution

$$\begin{aligned} x_{11}^* &= 0, x_{12}^* = 1, x_{13}^* = 0, x_{21}^* = 0, x_{22}^* = 0, x_{23}^* = 1, \\ x_{31}^* &= 1, x_{32}^* = 0, x_{33}^* = 0, \end{aligned}$$

with the optimal objective value $\text{Mag}(\tilde{z}^*) = 186/12 = 15.5$ which represents the optimal total cost. Therefore, the fuzzy optimal assignment for the given fuzzy assignment problem is

$$P_1 \rightarrow J_2, P_2 \rightarrow J_3, P_3 \rightarrow J_1.$$

The fuzzy optimal total cost is calculated as

$$\begin{aligned} \tilde{c}_{12} + \tilde{c}_{23} + \tilde{c}_{31} &= (1, 3, 4, 6) + (5, 6, 7, 8) + (3, 5, 6, 8) \\ &= (9, 14, 17, 22) \text{ and } \text{Mag}(9, 14, 17, 22) = 186/12. \end{aligned}$$

The fuzzy optimal total cost obtained in all the three methods is same.

5. Conclusion

Here, three different methods are proposed to solve the fuzzy assignment problem (FAP). The proposed three different methods: (i) FAP is solved by using the proposed fuzzy Hungarian method in which the magnitude of a fuzzy number is used to order the fuzzy numbers, (ii) FAP is defuzzified into the crisp assignment problem using the magnitude of a fuzzy number and optimal solution is obtained using Hungarian method and (iii) FAP is transformed into crisp linear programming problem using the magnitude of a fuzzy number and optimal solution is obtained by using TORA to solve the fuzzy assignment problem (FAP) are very effective. Finally, numerical example is provided to illustrate these three different methods and the solutions are found to be same. However, in Method 1, fuzzy numbers are handled without defuzzifying and so, the optimum fuzzy solution is obtained by using the proposed fuzzy Hungarian method in which the magnitude of a fuzzy number is used to order the fuzzy numbers. But in Method 2, fuzzy

numbers are defuzzified by using the magnitude of a fuzzy number and the optimum solution is obtained by using Hungarian method. This solution is same as the magnitude of the optimum fuzzy solution obtained in Method 1. In Method 3, fuzzy assignment problem is converted into crisp linear programming problem using magnitude of a fuzzy number and the optimum solution is obtained by using TORA. The optimum objective value obtained in this method coincides with the solutions of Method 1 and Method 2.

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