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A MATHEMATICAL MODEL FOR SOIL PRESSURES ACTING ON CIRCULAR FOOTINGS, FOR OBTAINING MOMENTS AND SHEAR FORCES UNIDIRECTIONAL IN DIRECTION OF AXES "*X*" AND "*Y*"

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Abstract

In the design of circular reinforced concrete footings subject to axial load and flexure bidirectional are presented different pressures throughout contact surface, such pressures are exerted by the soil on footings. In this paper, we develop a mathematical model to take into account the real pressure of the ground acting on the contact surface of the circular footings, for obtaining moments and shear forces unidirectional in direction of main axes, when applying the load that must support the said structural member, this is to find the steel reinforcement in directions "*X*" and "*Y*". The traditional model takes into account the maximum pressure of the ground to design the footings and is considered uniform at all the points of contact, i.e., the

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entire surface has the same pressure. Also, a comparison is developed between the traditional model and proposed model as shown in the result tables. According to the data obtained, it is shown that the traditional model is larger with respect to the model proposed in terms of moments and shear forces unidirectional. Therefore, normal practice to use the traditional model will not be a recommended solution. Then it is best to use the proposed model, since it is more economic and also is more attached to the real conditions.

Introduction

The foundation is the part of the structure responsible for transmitting the loads to the ground. Given that the strength and stiffness of the soil are, except in rare cases, much lower than those of the structure, the foundation has an area on the ground much greater than the sum of the areas of all supports and load-bearing walls. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems [2, 5, 11].

A superficial foundation is a structural element whose cross section is of large dimensions with respect to height and whose function is to transfer the loads of a building at depths relatively short, less than 4m approximately with respect to the level of the natural ground surface [2, 5, 11].

Superficial foundations, whose constructive systems generally do not present major difficulties, may be of various types, according to their function:

- ♦ Cyclopean foundations
- ♦ Footings:
    - ❖ Isolated footings
    - ❖ Continuous footings
    - ❖ Combined footings
- ♦ Foundation slabs

A footing is an extension of the base of a column or a wall that is to transmit the load to subsoil at a suitable pressure of soil properties. Footings that support a single column are called *individual footings* or *isolated*. The footing that is constructed under a wall is called *strip footing* or *continuous footing*. A footing that supports multiple columns is called *combination footing*. A special form of combined footing is normally used in case one of the columns supporting an exterior wall is called *cantilever footing* [3, 6, 7, 10].

The structural design of foundations, by itself, represents the union and the frontier of structural design and soil mechanics. As such, share the hypothesis, assumptions and models of both the disciplines, which do not always coincide [2, 5, 11]. Structural analysis is usually done with the hypothesis that the building structure is fixed in the ground, i.e., supported by an undeformable material.

On the other hand, the engineer of soil mechanics, for calculating the conditions of service by soil settlement, despises the structure, whose model considers only forces as resulting from the reactions.

The reality is that neither the soil is undeformable nor the structure is flexible as its effects are not interrelated. After all, the system soil-structure is continuous whose deformations depend on one another.

However, for ease in calculations, usually this dependence is ignored. The most recent case is used for the design of common footings. The normal procedure is almost universally accepted that is designed to transmit the same allowable pressure recommended by the soils' engineer. Based on this value, which is by far the only League of Engineers of soils and structures, the footings are dimensioned for all sizes on common premise of the resistance of materials; pressures equally correspond to equal deformations.

The classification of footings is very broad. Accordingly, its function can be classified as: isolated, combined, continuous and braced or attached. According to its form, classification will be: rectangular, square, circular, annular or polygonal.

In the design of superficial foundations, the specific cases of isolated footings, there are three types in terms of the application of loads: (1) footings subject to concentric axial load, (2) footings subject to axial load and moment in one direction (flexure unidirectional) and (3) footings subject to axial load and moment in two directions (bidirectional flexure). The hypothesis used is to consider the pressure uniform for the design, i.e., the same pressure at all the points of contact with the ground on the foundation; this design pressure is the maximum value that is presented in an isolated footing [6, 10].

The author Luévanos Rojas developed a mathematical model for obtaining moments of design, and also can be applied to shear forces unidirectional, when is provided the reinforcing steel in radial form, considering the real pressure of the ground acting on the contact surface, as shown in Figure 1 [5].

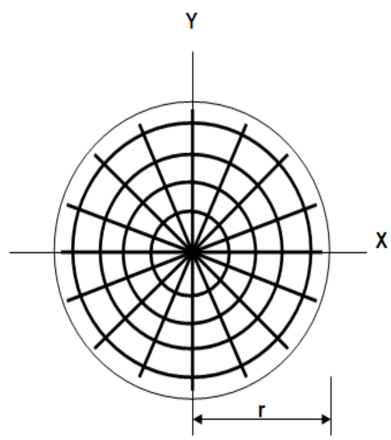

Figure 1. Plant of a circular footing with reinforcing steel in radial form.

In this paper, a mathematical model of non-uniform pressures for circular footings is developed subject to axial load and moments in two directions (bidirectional flexure), having a linear variation along all its contact area, which is as it really presents the pressures, for obtaining moments and shear forces unidirectional in the direction of main axes, when applying the load that must support the said structural member, this is to find the steel reinforcement in directions "*X*" and "*Y*" as shown in Figure 2. This model may be applicable to the other two cases, e.g., for the first case where acts an axial load concentrically, pressures are identical and, the second case where acts an axial load and moment in a direction (unidirectional flexure). It also develops a comparison in terms of moments and shear forces unidirectional between the traditional model and the proposed model to observe the differences.

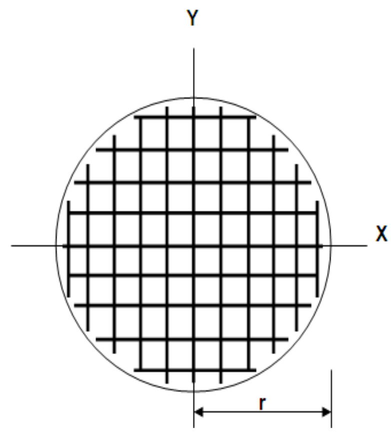

Figure 2. Plant of a circular footing with reinforcing steel in directions "*X*" and "*Y*".

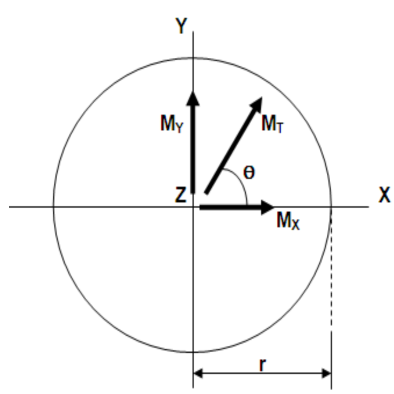

Figure 3. Plant of a typical circular footing.

Mathematical Development of Proposed Model

Figure 3 shows the moments acting on a circular footing in plant whereas Figure 4 presents the differential element involved in the analysis.

Figure 3 presents the moments acting on a circular footing in plant and can obtain the resultant moment as follows:

$$M_R = \sqrt{M_X^2 + M_Y^2}, \tag{1}$$

$$\tan\theta = \frac{M_Y}{M_X}, \tag{2}$$

where

M_Y is the moment around the axis "*X*" or in direction "*Y*"

M_X is the moment around the axis "*Y*" or in direction "*X*"

M_R is the resultant moment of the vector sum of "M_X" and "M_Y"

θ is the angle of inclination of the resultant moment "M_R" with respect to the axis "*X*".

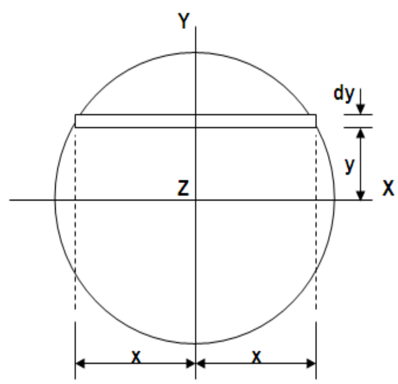

Figure 4. Element differential of the circumference.

Figure 4 shows the differential area "*dA*" as follows:

$$dA = 2xdy, \tag{3}$$

where

dy is the differential of length in direction "*Y*" of the circle

x is the horizontal distance of the circle.

Figure 5 presents the pressures in elevation of a circular footing that is subject to axial load and moment in two directions (bidirectional flexure) with different pressures on the entire surface of contact, linearly varying along the contact area with the ground.

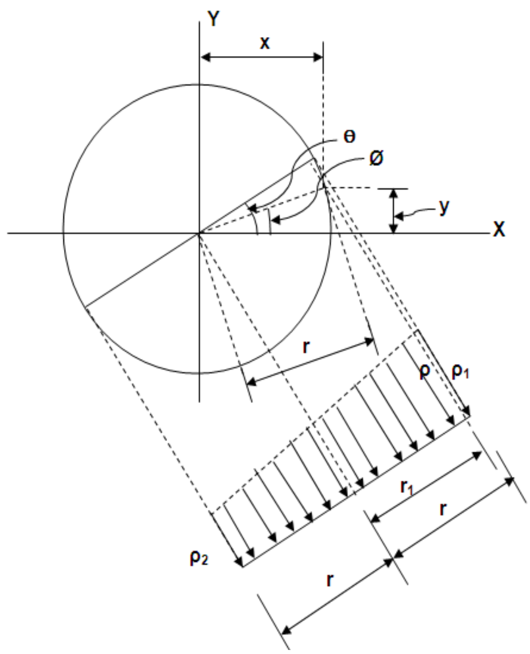

Figure 5. Soil pressures on a circular footing.

Figure 5, by proportions are obtained [1, 4, 8, 9]:

$$\frac{\rho_1 - \rho_2}{2r} = \frac{\rho - \rho_2}{r + r_1}, \tag{4}$$

where

r is the radius of the footing

r_1 is the component of "r" on the axis where the resultant moment is located

ρ_1 is the maximum pressure exerted by the soil of the footing

ρ_2 is the minimum pressure exerted by the soil of the footing

ρ is the pressure at any point of soil on the footing.

From Figure 5 using the equation for finding "r_1" in function of "r" and is presented:

$$r_1 = r\cos(\theta - \varnothing), \tag{5}$$

where $\varnothing$ is the angle formed by the triangle of sides "x" and "y".

Substituting a trigonometric identity in equation (5) to separate the angles, we obtain:

$$r_1 = r(\cos\theta\cos\varnothing + \sin\theta\sin\varnothing). \tag{6}$$

Also, we present trigonometric functions in terms of "x", "y" and "r"

$$\sin\varnothing = \frac{y}{r}, \tag{7}$$

$$\cos\varnothing = \frac{x}{r} = \frac{\sqrt{r^2 - y^2}}{r}. \tag{8}$$

Of equation (4) is presented "ρ" as follows:

$$\rho = \rho_2 + \frac{(\rho_1 - \rho_2)}{2r}[r + r_1]. \tag{9}$$

Substituting equation (6) into equation (9):

$$\rho = \rho_2 + \frac{(\rho_1 - \rho_2)}{2r}[r + r(\cos\theta\cos\varnothing + \sin\theta\sin\varnothing)]. \tag{10}$$

Simplifying equation (10) as observed:

$$\rho = \rho_2 + \frac{(\rho_1 - \rho_2)}{2r}[1 + \cos\theta\cos\varnothing + \sin\theta\sin\varnothing]. \tag{11}$$

Substituting equations (7) and (8) into equation (11) and simplifying as presented:

$$\rho(y) = \frac{(\rho_1 + \rho_2)}{2} + \frac{(\rho_1 - \rho_2)}{2}\left[\cos\theta\left(\frac{\sqrt{r^2 - y^2}}{r}\right) + \sin\theta\left(\frac{y}{r}\right)\right]. \tag{12}$$

To find the resultant force "F_R", it is the volume of pressures that is generated in the shaded area in Figure 6, and is presented as follows:

$$F_R = \int_b^r \rho(y) 2x dy. \tag{13}$$

Substituting equations (8) and (12) into equation (13):

$$F_R = \int_b^r \left\{\frac{(\rho_1 + \rho_2)}{2} + \frac{(\rho_1 - \rho_2)}{2}\left[\cos\theta\left(\frac{\sqrt{r^2 - y^2}}{r}\right) + \sin\theta\left(\frac{y}{r}\right)\right]\right\}(2\sqrt{r^2 - y^2})dy. \tag{14}$$

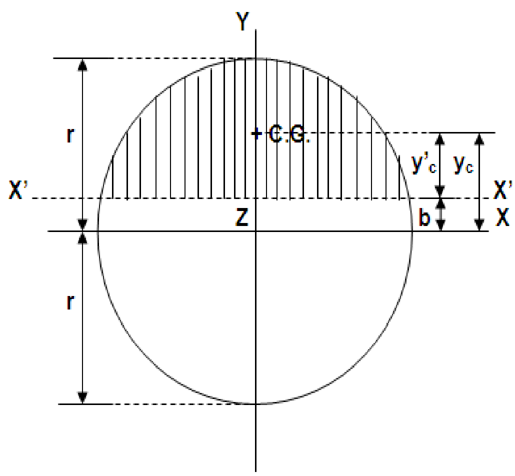

Figure 6. Circular footing isolated top view.

Developing the integration as presented:

$$F_R = (\rho_1 + \rho_2)\left[\left\{\frac{y\sqrt{r^2 - y^2}}{2} + \frac{r^2}{2}\sin^{-1}\left(\frac{y}{r}\right)\right\}\right.$$

$$\left.+ (\rho_1 - \rho_2)\left\{\cos\theta\left(\frac{r^2 y - y^3/3}{r}\right) + \sin\theta\left[-\frac{(r^2 - y^2)^{3/2}}{3r}\right]\right\}\right]_b^r. \quad (15)$$

Substituting the boundary conditions into equation (15) and simplifying as shown:

$$F_R = \left\{(\rho_1 + \rho_2)\left[\frac{\pi r^2}{4} - \frac{b\sqrt{r^2 - b^2}}{2} - \frac{r^2}{2}\sin^{-1}\left(\frac{b}{r}\right)\right]\right.$$

$$\left.+ (\rho_1 - \rho_2)\left[\left(\frac{2r^3 - 3r^2 b + b^3}{3r}\right)\cos\theta + \frac{(r^2 - b^2)^{3/2}}{3r}\sin\theta\right]\right\}. \quad (16)$$

Now the integral is developed to obtain the center of gravity "y_c" from the pressures of soil [1, 4, 8, 9]:

$$\bar{y} = \frac{\int_V y dV}{\int_V dV}. \quad (17)$$

The boundary conditions are substituted into equation (17):

$$\bar{y} = \frac{\int_b^r y\rho(y)2x dy}{\int_b^r \rho(y)2x dy}. \quad (18)$$

Substituting equation (12) into equation (18) as presented:

$$y_c = \frac{\int_b^r y\left\{\frac{(\rho_1+\rho_2)}{2} + \frac{(\rho_1-\rho_2)}{2}\left[\cos\theta\left(\frac{\sqrt{r^2-y^2}}{r}\right) + \sin\theta\left(\frac{y}{r}\right)\right]\right\}(2\sqrt{r^2-y^2})dy}{F_R}. \tag{19}$$

It develops the integration of equation (19) as presented:

$$y_c = \left[\left\{-(\rho_1+\rho_2)\frac{(r^2-y^2)^{3/2}}{3} + (\rho_1-\rho_2)\left[\cos\theta\left(\frac{r^2y^2/2 - y^4/4}{r}\right) + \sin\theta\left(-\frac{y(r^2-y^2)^{3/2}}{4r} + \frac{r^2y\sqrt{r^2-y^2}}{8r} + \frac{r^3}{8}\sin^{-1}\frac{y}{r}\right)\right]\right\}\right]_b^r / F_R. \tag{20}$$

Substituting the boundary conditions into equation (20) and simplifying as presented:

$$y_c = \left\{(\rho_1+\rho_2)\frac{(r^2-b^2)^{3/2}}{3} + (\rho_1-\rho_2)\left[\left(\frac{r^3}{4} - \frac{rb^2}{2} + \frac{b^4}{4r}\right)\cos\theta + \left(\frac{\pi r^3}{16} + \frac{b(r^2-b^2)^{3/2}}{4r} - \frac{rb\sqrt{r^2-b^2}}{8} - \frac{r^3}{8}\sin^{-1}\frac{b}{r}\right)\sin\theta\right]\right\} / F_R. \tag{21}$$

Substituting equation (16) into equation (21) and simplifying which is shown as follows:

$$y_c = \left\{(\rho_1+\rho_2)\frac{(r^2-b^2)^{3/2}}{3} + (\rho_1-\rho_2)\left[\left(\frac{r^4-2r^2b^2+b^4}{4r}\right)\cos\theta + \left(\frac{\pi r^3}{16} + \frac{(r^2-2b^2)b\sqrt{r^2-b^2}}{8r} - \frac{r^3}{8}\sin^{-1}\frac{b}{r}\right)\sin\theta\right]\right\} \Bigg/ \left\{(\rho_1+\rho_2)\left[\frac{\pi r^2}{4} - \frac{b\sqrt{r^2-b^2}}{2} - \frac{r^2}{2}\sin^{-1}\left(\frac{b}{r}\right)\right]\right.$$

$$+ (\rho_1 - \rho_2)\left[\left(\frac{2r^3 - 3r^2 b + b^3}{3r}\right)\cos\theta + \frac{(r^2 - b^2)^{3/2}}{3r}\sin\theta\right]\bigg\}. \quad (22)$$

The moment is presented around the axis X'-X' "$M_{X'X'}$", passing by the junction of column with footing, this means that $b = c/2$ in equations (16) and (22) obtained as follows:

$$M_{X'X'} = F_R(y_c - c/2). \quad (23)$$

The shear forces unidirectional are obtained in the axis X'-X' "V_S", passing to a distance *d* (thickness effective of the footing) to from junction of column with footing, this means that $b = c/2 + d$, in equation (16) presented as follows:

$$V_S = F_R. \quad (24)$$

Application

Below are presented three types of circular footings and varying the pressure in linear form on contact area, which is exerted by the ground on the structural element. These examples are developed by the traditional model and the proposed model. For all types are considered for $\theta = 0^\circ$, 45° and 90°. The design moments are obtained at the joint of the column with the footing and for the shear forces unidirectional is obtained in an axis that passes to a distance *d* (thickness effective of the footing) to from the joint of column with the footing. Figure 7 shows the details of the footing.

Traditional model

The resultant force "F_R" the footing is shown:

$$F_R = \rho\left\{\frac{\pi r^2}{2} - b\sqrt{r^2 - b^2} - r^2 \sin^{-1}\left(\frac{b}{r}\right)\right\},$$

where ρ is the maximum pressure exerted by the soil of the footing.

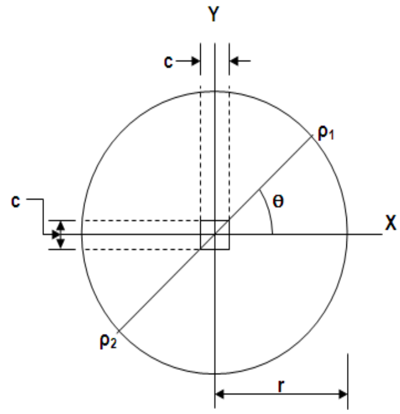

Figure 7. Plant of a circular footing isolated.

The center of gravity from the axis *X*-*X* is presented:

$$y_c = \frac{2(r^2 - b^2)^{3/2}}{3\left\{\frac{\pi r^2}{2} - b\sqrt{r^2 - b^2} - r^2 \sin^{-1}\left(\frac{b}{r}\right)\right\}},$$

where b is $c/2$.

The moment about the axis X'-X' "$M_{X'X'}$" is obtained:

$$M_{X'X'} = F_R(y_c - c/2).$$

The shear forces unidirectional in the axis X'-X' "V_S" using the same expression of "F_R" is presented as follows:

$$V_S = F_R,$$

where

b is $c/2 + d$

d is the thickness effective of the footing.

Proposed model

The resultant force "F_R" is found using equation (16) and to obtain the center of gravity "y_c" is used equation (22).

To find moment about the axis X'-X' "$M_{X'X'}$", is used equation (23).

To obtain shear forces unidirectional in the axis X'-X', "V_S" is employed equation (24).

Then Table 1 presents the results for the two models of the three types of footings for $\theta = 0^\circ$, in Table 2 are shown for $\theta = 45^\circ$ and in Table 3 appear for $\theta = 90^\circ$.

Table 1. Comparison of results for $\theta = 0^\circ$

Case	Footing dimensions and column (m)			Pressures (ton/m^2)		Resultant force F_R (ton)		Centroidal distance $y_c - c/2$ (m)		Moments about axis "$X'X'$" $M_{X'X'}$ (ton-m)		Shear forces unidirectional V_S (ton)	
	r	c	d	ρ_1	ρ_2	MT	MP	MT	MP	MT	MP	MT	MP
Footing 1													
1	1.00	0.20	0.30	20	15	27.423	26.830	0.379	0.374	10.393	10.034	15.853	15.312
2				20	10	27.423	26.237	0.379	0.369	10.393	9.681	15.853	14.770
3				20	5	27.423	25.644	0.379	0.363	10.393	9.309	15.853	14.228
4				20	0	27.423	25.051	0.379	0.358	10.393	8.968	15.853	13.687
Footing 2													
1	1.50	0.25	0.35	20	15	63.195	61.860	0.580	0.572	36.653	35.384	42.670	41.392
2				20	10	63.195	60.525	0.580	0.564	36.653	34.136	42.670	40.115
3				20	5	63.195	59.191	0.580	0.556	36.653	32.910	42.670	38.838
4				20	0	63.195	57.856	0.580	0.547	36.653	31.647	42.670	37.561
Footing 3													
1	2.00	0.30	0.40	20	15	113.675	111.302	0.780	0.770	88.666	85.703	82.225	79.919
2				20	10	113.675	108.929	0.780	0.760	88.666	82.786	82.225	77.613
3				20	5	113.675	106.555	0.780	0.749	88.666	79.810	82.225	75.306
4				20	0	113.675	104.182	0.780	0.737	88.666	76.782	82.225	73.000

MT = Traditional model

MP = Proposed model

Table 2. Comparison of results for $\theta = 45^{\circ}$

Case	Footing dimensions and column (m)			Pressures (ton/m^2)		Resultant force F_R (ton)		Centroidal distance $y_c - c/2$ (m)		Moments about axis "$X'X'$" $M_{X'X'}$ (ton-m)		Shear forces unidirectional V_S (ton)	
	r	c	d	ρ_1	ρ_2	MT	MP	MT	MP	MT	MP	MT	MP
Footing 1													
1	1.00	0.20	0.30	20	15	27.423	27.160	0.379	0.381	10.393	10.348	15.853	15.797
2				20	10	27.423	26.898	0.379	0.382	10.393	10.275	15.853	15.741
3				20	5	27.423	26.636	0.379	0.384	10.393	10.228	15.853	15.685
4				20	0	27.423	26.373	0.379	0.386	10.393	10.180	15.853	15.629
Footing 2													
1	1.50	0.25	0.35	20	15	63.195	62.561	0.580	0.582	36.653	36.411	42.670	42.467
2				20	10	63.195	61.928	0.580	0.585	36.653	36.228	42.670	42.265
3				20	5	63.195	61.295	0.580	0.588	36.653	36.041	42.670	42.063
4				20	0	63.195	60.661	0.580	0.590	36.653	35.790	42.670	41.860
Footing 3													
1	2.00	0.30	0.40	20	15	113.675	112.509	0.780	0.784	88.666	88.207	82.225	81.773
2				20	10	113.675	111.344	0.780	0.788	88.666	87.739	82.225	81.322
3				20	5	113.675	110.178	0.780	0.791	88.666	87.151	82.225	80.870
4				20	0	113.675	109.012	0.780	0.795	88.666	86.665	82.225	80.419

Results and Discussion

Figures 8, 9 and 10 show the differences between the two models of the 3 types of footings for $\theta = 90^{\circ}$, because this value is where there are more differences between both the models for the 4 cases by moments. In all cases, the proposed model is lower with respect to the traditional model.

Figure 8 presents the footing 1 which shows that there are large differences in case 4. For example, the traditional model is 22.91% higher than the model proposed.

Table 3. Comparison of results for $\theta = 90^{\circ}$

Case	Footing dimensions and column (m)			Pressures (ton/m^2)		Resultant force F_R (ton)		Centroidal distance $y_c - c/2$ (m)		Moments about axis " $X'X'$ " $M_{X'X'}$ (ton-m)		Shear forces unidirectional V_S (ton)	
	r	c	d	ρ_1	ρ_2	MT	MP	MT	MP	MT	MP	MT	MP
Footing 1													
1	1.00	0.20	0.30	20	15	27.423	25.637	0.379	0.386	10.393	9.896	15.853	15.155
2				20	10	27.423	23.850	0.379	0.395	10.393	9.421	15.853	14.456
3				20	5	27.423	22.064	0.379	0.405	10.393	8.936	15.853	13.758
4				20	0	27.423	20.278	0.379	0.417	10.393	8.456	15.853	13.059
Footing 2													
1	1.50	0.25	0.35	20	15	63.195	59.006	0.580	0.591	36.653	34.873	42.670	40.536
2				20	10	63.195	54.818	0.580	0.605	36.653	33.165	42.670	38.403
3				20	5	63.195	50.630	0.580	0.621	36.653	31.441	42.670	36.269
4				20	0	63.195	46.441	0.580	0.640	36.653	29.722	42.670	34.136
Footing 3													
1	2.00	0.30	0.40	20	15	113.675	106.076	0.780	0.796	88.666	84.436	82.225	77.872
2				20	10	113.675	98.477	0.780	0.815	88.666	80.259	82.225	73.518
3				20	5	113.675	90.878	0.780	0.836	88.666	75.974	82.225	69.165
4				20	0	113.675	83.279	0.780	0.862	88.666	71.786	82.225	64.812

With respect to Figure 9 is presented the footing 2, the difference being greater in the case 4. This difference is greater in the traditional model with respect to the proposed model of 23.32%.

Finally, we examine Figure 10 which illustrates the footing 3 which shows the greatest difference also in the case 4. This difference is greater in a 23.51% for the traditional model with respect to the proposed model.

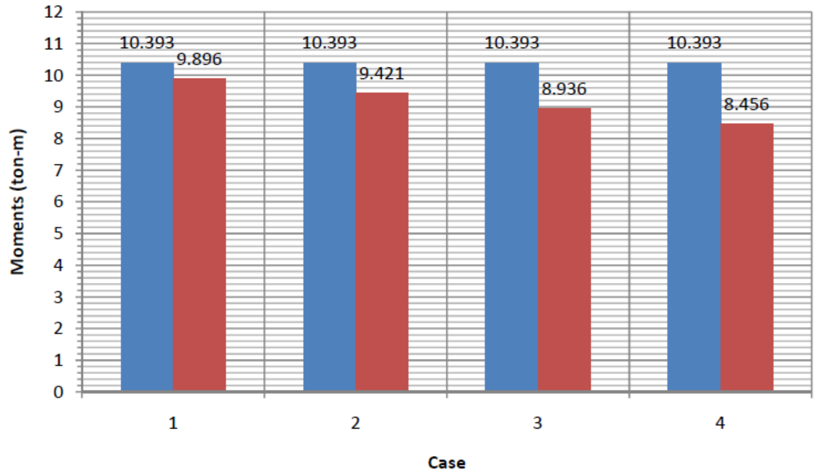

Figure 8. Footing type 1, for moments.

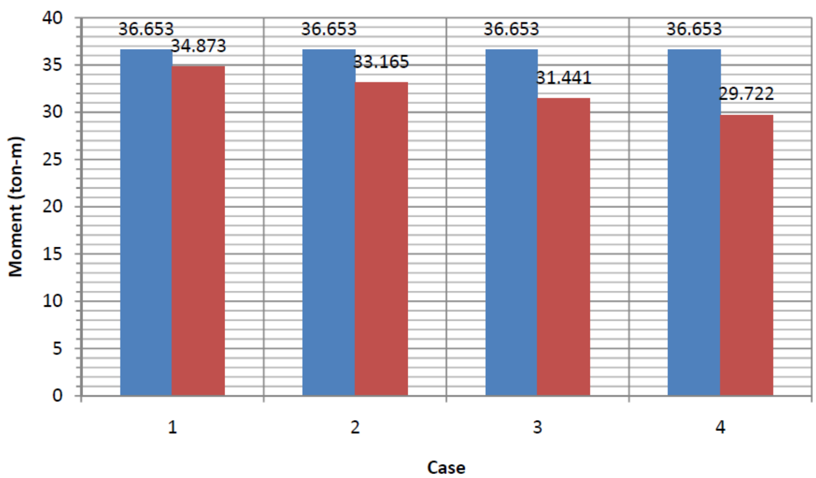

Figure 9. Footing type 2, for moments.

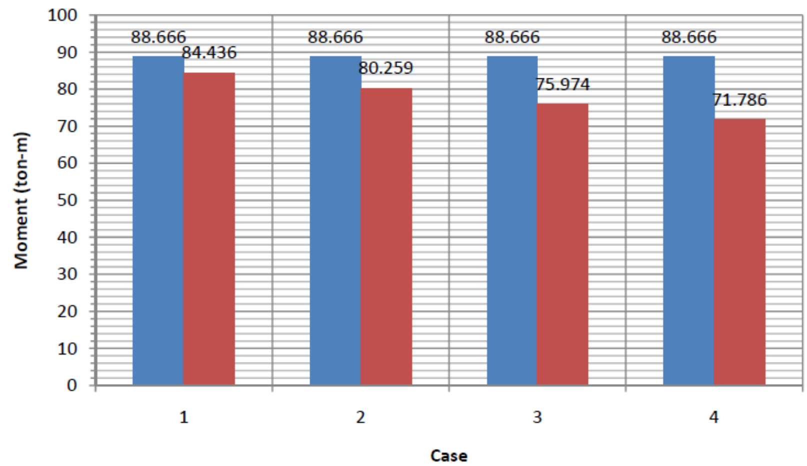

Figure 10. Footing type 3, for moments.

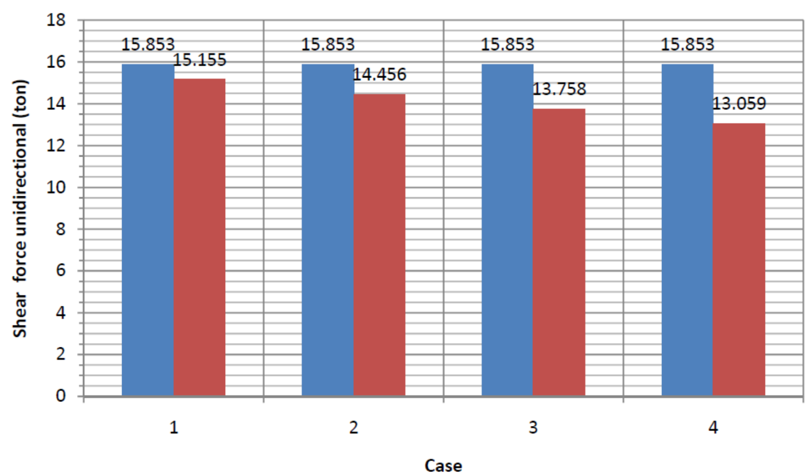

Figure 11. Footing type 1, for shear forces unidirectional.

Figures 11, 12 and 13 show the differences between the two models of the 3 types of footings for $\theta = 90^{\circ}$, because this value is where there are more differences between both the models for the 4 cases, by shear forces unidirectional. In all cases, the proposed model is lower with respect to the traditional model.

Figure 11 presents the footing 1 which shows that there are large differences in case 4. For example, the traditional model is 21.40% higher than the model proposed.

With respect to Figure 12 is presented the footing 2, the difference being greater in the case 4. This difference is greater in the traditional model with respect to the proposed model of 25.00%.

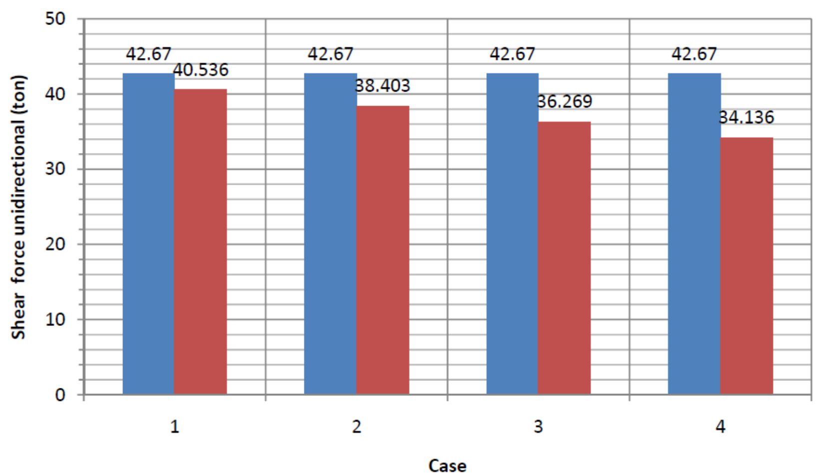

Figure 12. Footing type 2, for shear forces unidirectional.

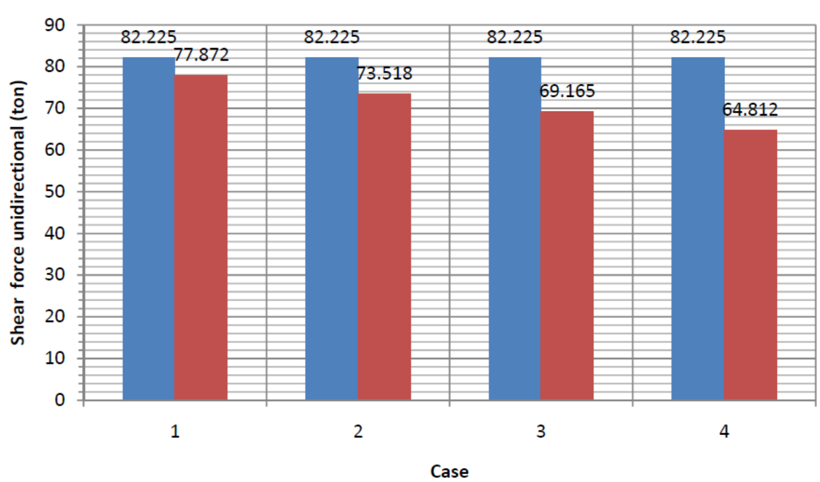

Figure 13. Footing type 3, for shear forces unidirectional.

Finally, we examine Figure 13 which illustrates the footing 3 which shows the greatest difference also in the case 4. This difference is greater in a 26.87% for the traditional model with respect to the proposed model.

Conclusions

The results of the problem considered, through the application of two different models, the conclusions are the following:

With respect to values of θ, it is shown that the difference is higher for $\theta = 90^{\circ}$, a greater increase in traditional model is presented with respect to the proposed model, in terms of moments and shear forces unidirectional.

According to pressures in all the types of footings and for all values of θ, it is observed that when the difference between the maximum pressure and minimum is higher, a greater increase in traditional model is presented with respect to the proposed model, in terms of moments and shear forces unidirectional. This is a logical situation, because in traditional model is retained its value, but in proposed model is reduced the resultant force.

This means that can have great savings in terms of materials used (reinforcing steel and concrete) for the fabrication of footings isolated under conditions mentioned above. Because that the principle of civil engineering in terms of structural conditions for any type of construction is that be safe and economical, and the latter is not met for traditional model for isolated footings form circular. Therefore, the practice of using the traditional model is not a recommended solution, because the materials in some cases are very exceeded, with regard to the design of these structural members.

Then it proposed to use the model developed in this paper for structural design of isolated footings subjected to axial load and flexure unidirectional, to find the steel reinforcement in directions "*X*" and "*Y*", because it is most economical. Moreover, this adheres more to the actual conditions of the soil pressures that are applied to the foundation.

References

[1] F. Ayres, Cálculo Diferencial e Integral, McGraw-Hill, Mexico, 1988.

[2] B. M. Das, E. Sordo-Zabay and R. Arrioja-Juarez, Principios de ingenieria de cimentaciones, Cengage Learning Latin America, México, 2006.

[3] M. L. Gambhir, Fundamentals of Reinforced Concrete Design, Prentice-Hall of India Private Limited, New Delhi, 2008.

[4] W. A. Granville, Cálculo Diferencial e Integral, Limusa, Mexico, 2009.

[5] A. Luévanos Rojas, A mathematical model for soil pressures acting on circular footings, for obtaining the moments of design, Far East J. Math. Sci. (FJMS) 69(2) (2012), 219-232.

[6] J. C. McCormac, Design of Reinforced Concrete, John Wiley & Sons, Inc., New York, U.S.A., 2008.

[7] W. H. Mosley, J. H. Bungey and R. Hulse, Reinforced Concrete Design, Palgrave, New York, U.S.A., 1999.

[8] A. Parker, Diseño Simplificado de Concreto Reforzado, Limusa Wiley, México, 2009.

[9] N. Piskunov, Cálculo Diferencial e Integral - Tomos 1 y 2, Limusa, Mexico, 2004.

[10] B. C. Punmia, Ashok Kumar Jain and Arun Kumar Jain, Limit State Design of Reinforced Concrete, Laxmi Publications (P) Ltd., India, 2007.

[11] C. Villalaz, Mecánica de suelos y cimentaciones, Limusa, México, 2009.