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A MATHEMATICAL MODEL FOR THE DIMENSIONING OF CIRCULAR FOOTINGS

Arnulfo Luévanos Rojas

Facultad de Ingeniería, Ciencias y Arquitectura
Universidad Juárez del Estado de Durango
Av. Universidad S/N, Fracc. Filadelfia
CP 35010, Gómez Palacio, Durango, México
e-mail: arnulfol_2007@hotmail.com

Abstract

In the design of circular reinforced concrete footings subjected to axial load and flexure in a direction are presented different pressures throughout contact surface, such pressures are exerted by the soil on footings. In this paper, we develop a mathematical model to obtain the most economical dimension of the contact surface in circular footings, when is applied the load that must support said structural member. The classical model is developed by trial and error, i.e., it is proposed a dimension, and using the formulates of the unidirectional flexure to obtain the stress acting on the circular footing, which must meet the following conditions: (1) The minimum stress must be equal or greater than zero, because the soil is not capable of withstand tensile stresses. (2) The maximum stress must be equal or less than the allowable capacity that can withstand the ground. Therefore, normal practice to use the classic model will not be a recommended solution. Then, it is best to use the proposed model since it is more economic.

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Introduction

The foundation is the part of the structure responsible for transmitting the loads to the ground. Given that the strength and stiffness of the soil are, except in rare cases, much lower than those of the structure, the foundation has an area on the ground much greater than the sum of the areas of all supports and load-bearing walls. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems [1-3].

A superficial foundation is a structural element whose cross section is of large dimensions with respect to height and whose function is to transfer the loads of a building at depths relatively short, less than 4m approximately with respect to the level of the natural ground surface [1-3].

Superficial foundations, whose constructive systems generally do not present major difficulties, may be of various types, according to their functions:

- Cyclopean foundations
- Footings:
    - Isolated footings
    - Continuous footings
    - Combined footings
- Foundation slabs.

A footing is an extension of the base of a column or a wall that is to transmit the load to subsoil at a suitable pressure of soil properties. Footings that support a single column are called *individual footings* or *isolated*. The footing that is constructed under a wall is called *strip footing* or *continuous footing*. A footing that supports multiple columns is called *combination footing*. A special form of combined footings is used normally in case that supports a column and an exterior wall is called *cantilever footings* [2, 4-8].

The structural design of foundations, by itself, represents the union and the frontier of structural design and soil mechanics. As such, share the hypothesis, assumptions and models of both disciplines, which do not always coincide [1-3]. Structural analysis is usually done with the hypothesis that the building structure is fixed in the ground, i.e., supported by an undeformable material.

On the other hand, the engineer of soil mechanics, for calculating the conditions of service by soil settlement, despises the structure, whose model considers only forces as resulting from the reactions.

The reality is that neither the soil is undeformable nor the structure is flexible as its effects are not interrelated. After all, the system soil structure is continuous whose deformations depend on one another.

However, for ease in calculations, this dependence usually is ignored. The most recent case is used for the design of common footings. The normal procedure is almost universally accepted, that is, designed to transmit the same allowable pressure recommended by the soils engineer. Based on this value, which is the only League of Engineers of soils and structures, the footings are dimensioned for all sizes with common premise of the resistance of materials; pressures equal correspond equal deformations.

The classification of footings is very broad. According to the way of work, they can be classified as: isolated, combined, continuous and braced or attached. According to its form, classification will be rectangular, square, circular, annular or polygonal.

In the design of superficial foundations, the specific case of isolated footings is of three types in terms of the application of loads: (1) Footings subject to concentric axial load, (2) footings subject to axial load and moment in one direction (unidirectional flexure), and (3) footings subject to axial load and moment in two directions (bidirectional flexure). The hypothesis used in the classical model is developed by trial and error, i.e., it is proposed a dimension, and using the formulates of the bidirectional flexure to obtain the stress acting on the circular footing, which must meet the following conditions: (1) The minimum stress should be equal to or greater than zero, because the soil is not capable of withstand tensile stresses. (2) The maximum stress must be equal or less than the allowable capacity that can withstand the ground [2, 7].

In this paper, a mathematical model for the dimensioning of circular footings subjected to axial load and moment in one direction (unidirectional flexure), is developed, where there are two conditions: the first condition is that the minimum stress should be equal to zero, because the soil is not able to withstand tensile stresses, the second condition is that the maximum stress should be equal to the allowable capacity of the soil. This model may be applicable to the other two cases, e.g., for the first case where acts an axial load concentrically, pressures are identical and the third case where acts an axial load and moments in two directions (bidirectional flexure) when there are moments in two directions obtained a resultant moment because the moment of inertia in circular footings does not change in any direction.

Mathematical Development of Proposed Model

Figure 1 shows a circular footing due to real load subject to axial load and moment in two directions (bidirectional flexure), whereas Figure 2 presents a circular footing due to equivalent load that is subjected to axial load and moment in one direction (unidirectional flexure) where the pressures are different on the entire surface of contact, linearly varying along the contact area with the ground.

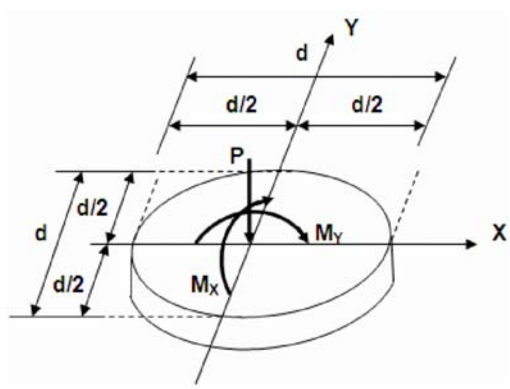

Figure 1. Circular footing due to real load.

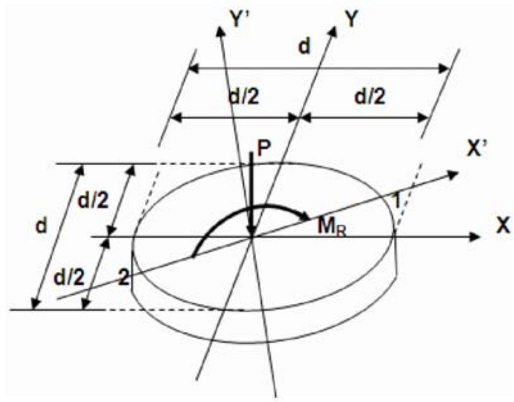

Figure 2. Circular footing due to equivalent load.

Expression general for bidirectional flexure is:

$$\sigma = \frac{P}{A} \pm \frac{M_X C_Y}{I_X} \pm \frac{M_Y C_X}{I_Y}, \tag{1}$$

where

σ is the stress at any point of the footing,

A is the contact area of the footing,

P is the axial load applied at the center of gravity of the footing,

M_X is the moment around the axis "X",

M_Y is the moment around the axis "Y",

C_X is the distance in the direction "X", measured from the axis "Y" to the fiber farthest,

C_Y is the distance in the direction "Y", measured from the axis "X" to the fiber farthest,

I_Y is the moment of inertia about axis "Y",

I_X is the moment of inertia about axis "X".

When moments are presented in two directions, a resultant moment is obtained because the moment of inertia in circular footings does not change in any direction [9-11].

Below the resultant moment is obtained as follows:

$$M_R = \sqrt{M_X^2 + M_Y^2}, \tag{2}$$

where M_R is the resultant moment of the vector sum of M_X and M_Y.

Using equation (1) to find the stresses at any point, circular footings are as follows:

$$\sigma = \frac{P}{A} \pm \frac{M_R C_{X'}}{I}, \tag{3}$$

where $C_{X'}$ is the distance in the direction "X'", measured from the axis "Y'" to the fiber farthest, $I = I_Y = I_X$.

Then, substituting the values of $A = \frac{\pi d^2}{4}$, $C_{X'} = \frac{d}{2}$ and $I = \frac{\pi d^4}{64}$ into equation (3), it is shown:

$$\sigma = \frac{P}{\frac{\pi d^2}{4}} \pm \frac{(M_R)\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}}, \tag{4}$$

where *d* is the diameter of the circular footing.

Simplifying equation (4), it is observed:

$$\sigma = \frac{4P}{\pi d^2} \pm \frac{32 M_R}{\pi d^3}. \tag{5}$$

Equation (5) is the expression general to find the maximum and minimum stresses of circular footings.

Using equation (5) to obtain the dimension more economic, these stresses acting on the circular footings must meet the following conditions: (1) the minimum stress must be equal to or greater than zero because the soil is not capable of withstand tensile stresses, (2) the maximum stress must be equal or less than the allowable capacity that can withstand the ground.

- First condition:

The minimum stress is zero:

$$\sigma_{\min} = \sigma_2 = 0. \tag{6}$$

Substituting equation (6) into equation (5), it is presented:

$$0 = \frac{4P}{\pi d^2} - \frac{32M_R}{\pi d^3}. \tag{7}$$

Equation (7) is simplified to find "*d*":

$$d = \frac{8M_R}{P}. \tag{8}$$

Through equation (8) is located the diameter of a circular footing, when the pressure is zero.

- Second condition:

The maximum stress is the loading capacity of the soil:

$$\sigma_{\max} = \sigma_1. \tag{9}$$

Substituting equation (9) into equation (5), it is presented:

$$\sigma_{\max} = \frac{4P}{\pi d^2} + \frac{32M_R}{\pi d^3}. \tag{10}$$

Equation (10) is simplified:

$$\sigma \max \pi d^3 = 4Pd + 32M_R. \tag{11}$$

Passing all the values of equation (11) of a side, we have:

$$\sigma \max \pi d^3 - 4Pd - 32M_R = 0. \qquad (12)$$

Then the solution is obtained from equation (12) for value of "*d*", and then this value is the diameter of a circular footing when the pressure is the loading capacity of the soil.

Therefore, the minimum diameter of a circular footing must satisfy the conditions two that is the diameter higher obtained by equations (8) and (12).

Application

Below, three cases of circular footings are shown, in each case have the same loads applied to the foundation and only is varied the load capacity of the soil. These examples were developed by the proposed model to obtain the diameter of the circular footing.

The diameter is obtained by means of equation (8), when this is subjected to the minimum pressure where this is zero and when is subjected to the maximum pressure such as the soil loading capacity is obtained by means of equation (12). The proposed diameter is the larger of the two above conditions.

Once that we have defined the diameter of the circular footing, we proceed to obtain the stresses generated by loads applied to the foundation, to verify that these stresses are within the established parameters, i.e., the maximum stress is equal or less than the load capacity of the soil and the minimum stress is equal to or greater than zero, since the ground not supports tensile stresses. These stresses are obtained by means of equation (5) and the results are presented in Table 1.

Table 1. Dimensioning of circular footings

Load capacity of the soil (ton/m^2)	Axial load in the footing P (ton)	Moments (ton-m)		Dimensions for zero minimum pressure (m)	Dimensions for the maximum pressure (m)	Dimensions proposed (m)	Stresses generated by the loads (ton/m^2)	
		M_Y	M_X	d	d	d	$\sigma_{máx}$	σ_{min}
Case 1								
25	70	7	10	1.40	2.38	2.40	24.47	6.48
20	70	7	10	1.40	2.61	2.65	19.37	6.01
15	70	7	10	1.40	2.96	3.00	14.51	5.30
10	70	7	10	1.40	3.53	3.55	9.85	4.29
Case 2								
25	50	7	10	1.95	2.19	2.20	24.83	1.48
20	50	7	10	1.95	2.40	2.45	19.06	2.15
15	50	7	10	1.95	2.70	2.75	14.40	2.44
10	50	7	10	1.95	3.20	3.25	9.65	2.41
Case 3								
25	50	10	15	2.88	2.37	2.90	15.10	0.04
20	50	10	15	2.88	2.59	2.90	15.10	0.04
15	50	10	15	2.88	2.91	2.95	14.47	0.16
10	50	10	15	2.88	3.42	3.45	9.82	0.88

Results and Discussion

Table 1 shows the results for the proposed model of the three cases of footings for 4 different types of load capacity of the soil.

In case 1 is presented that the second condition prevails. This means that the footing should be dimensioned on the basis of load capacity of the soil.

Also, in case 2, the second condition is dominant.

Finally, we analyze case 3, in which the first two types of load capacities of the soil are dominant the first condition. This means that the footing should be dimensioned on the basis of the minimum pressure where this is zero, because the soil cannot support tensile stresses. The types three and four are dominant in the second condition.

Conclusions

The foundation is the member of a structure which is the essential part thereof due to it will permit the transmission of loads from the structure to the soil. Then, the foundation helps to the soil to resist structure loads, for that the building will not suffer settlements and behaves according to the conditions to which it will be submitted. Therefore, the foundation comes to form the basis of the structure and of hence that the behavior of the building or civil works for withstand the loads that must support.

The great importance of the foundation is forced to meet with certain geometrical parameters, pressure, conformation that respond to the characteristics of soil and loads of the structure. Therefore, the design of a foundation is not something that is performed in an intuitive manner, but it must comply with a design methodology that evaluates from the form of the foundation up to the depth that is placed as member, as well as also the characteristics of soil.

This means that in terms of materials used (reinforcing steel and concrete) are obtained great savings for the fabrication of footings isolated under conditions mentioned above. Because that the principle of civil engineering in terms of structural conditions for any type of construction is that be safe and economical, and the latter is not met for traditional model for isolated footings form circular. Therefore, the practice of using the traditional model is not a recommended solution, because the materials are exceeded in some cases, with regard to design of these structural members.

The mathematical approach suggested in this paper produces results having tangible accuracy for all problems, under investigation for finding the solution more economical.

Then we recommend the proposed model for the structural design of isolated footings subjected to axial load, unidirectional flexure and bidirectional. Furthermore, this adheres more to the real conditions of the soil pressures that are applied to the foundation.

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