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THE GENERALIZED Q MANDEL'S PARAMETER CANNOT REVEAL THE NONCLASSICALITY OF THE SQUEEZED COHERENT STATES

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Abstract

The Q parameter was introduced by Mandel [1] to characterize the photon number distribution of a given state, such that an arbitrary distribution is defined to be sub- or super-Poissonian depending on $Q < 0$, $Q > 0$, respectively. While a sub-Poissonian distribution implies photon antibunching, which is a manifestation of the non-classical characteristic of the radiation, super-Poissonian distribution does not require a complete quantum treatment to its explanation. Also, it is generally spoken of states whose Q Mandel's parameter is null as being "Poissonian" states, from which coherent states are the most popular example. Since $Q < 0$ is a sufficient, but not a necessary condition for the nonclassicality of a quantum state, in [2], a generalized $Q(k)$ parameter recovering the original Q for $k = 1$ was

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introduced. We show that whenever $Q(k)$ is negative for some k, it implies that the radiation is non-classical, such that, in principle, nonclassicality could be revealed by resorting to higher orders of $Q(k)$, what is corroborated by the examples analyzed by us. Here we carry out a detailed study of the $Q(k)$ evolving under a thermal reservoir and show, by an explicit counter-example, that the nonclassicality of squeezed coherent states is not revealed by $Q(k)$ regardless of its order.

I. Introduction

Several quantum effects in the properties of the quantized electromagnetic field have been investigated both theoretically and experimentally, as for example, sub-Poissonian statistics [1], detection of photon antibunching [3], squeezing of the quadratures [4, 5], oscillations in the photon statistics and interference in the phase space [6], among others. Photon number statistics is commonly studied through the Q parameter introduced by Mandel [1], and since then an arbitrary distribution is defined to be sub-Poissonian $(Q < 0)$, super-Poissonian $(Q > 0)$, or Poissonian $(Q = 0)$ [2, 7, 8]. The reason why states having $Q = 0$ is called "*Poissonian*" is due to coherent states, which always have $Q = 0$ and whose photon number distribution is Poissonian. It is to be noted, however, that some noncoherent states, pure and mixed, can have Poissonian distribution, although other statistical properties, as for example, squeezing of the quadratures, may reveal quantum properties usually not shown by coherent states [7, 8]. On the other hand, as we show in this paper, to name a state having $Q = 0$ by "Poissonian" is misleading, since there exist states having $Q = 0$ that do not possess Poissonian photon number distribution. Also, it is to be noted that once $Q \geq 0$ is not enough anymore to characterize the quantumness of a given state by analyzing only the photon number distribution, it is interesting to ask if a generalized version of the Q parameter [2] can be able to realize this task.

We focus our attention on squeezed coherent states (SCS) for which a proper choice of both displacement and squeezed parameters makes them to have either $Q < 0$ or $Q \geq 0$. Squeezed states of the electromagnetic field are specially interesting due to its potential applications in technology and also to investigate fundamental physics [3, 5, 9, 10]. We show that, despite being $Q = 0$, these SCS have quite different properties from both the Poissonian coherent state and the Poissonian states discussed in [7, 8]. It is worthwhile to mention that although statistical properties of both ideal (lossless) [11] and real (lossy) [12-15] squeezed coherent state superpositions were extensively studied, the engineering of arbitrary squeezed states into high-Q cavities became possible only after pioneering works of [16-20], thus allowing for squeezing arbitrary cavity field states in high-Q cavities.

This paper is organized as follows: in Section II, we present our model to study SCS in both ideal and realistic situations in the context of cavity QED. Our main results are presented in Sections III and IV, where we obtain analytically the characteristic function (Section III), from which we calculate both the $Q(t)$ parameter and the $Q(k, t)$ generalized parameter evolving in time (Section IV). Finally, in Section V, we present our conclusions.

II. The Model

For modeling our system, we will use, as usual, the following Hamiltonian:

$$H = \hbar\omega a^{\dagger}a + \sum_{k} \hbar\omega_k b_k^{\dagger} b_k + \sum_{k} \hbar(\lambda_k a^{\dagger} b_k + \lambda_k^* a b_k^{\dagger}), \tag{1}$$

where $a^{\dagger}$ and a are, respectively, the creation and annihilation operators for the cavity mode of frequency ω, $b_k^{\dagger}$ and b_k are the analogous operators for the kth reservoir oscillator mode, whose corresponding frequency and coupling with the cavity mode are ω_k and λ_k. A detailed treatment of how to engineer and teleport superpositions of SCS in lossy cavities scenario can be found in [12-14]. Here we will assume the SCS as prepared by the sequence of operations on the vacuum state $S(\xi)D(\alpha)|0\rangle = |\xi, \alpha\rangle$, where the displacement and squeezing operators are, respectively, given by $D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^{*}a)$ and $S(\xi) = \exp(\xi^{*}a^{2} - \xi a^{\dagger 2})$, and we will focus our attention on the characteristic function χ, to be introduced in the next section, from which we can obtain the Mandel's Q parameter evolving in time from a straightforward manner. The ideal case, i.e., when disregarding losses, can be obtained simply considering $t = 0$, or equivalently, $\{\lambda_k\} = 0$.

III. Q Parameter for SCS under a Thermal Reservoir

For solving the problem of obtaining the evolution of the statistical properties of the SCS taking into account the reservoir, particularly, the Q parameter, we use the characteristic χ function, which, in the normal order and in the Heisenberg picture, reads

$$\chi(\eta, \eta^{*}, t) = tr\{\rho_{AR}(0)e^{\eta a^{\dagger}(t)}e^{-\eta^{*}a(t)}\}, \tag{2}$$

where $\rho_{AR}(0)$ is the density operator for the whole system composed of the cavity mode field (system A) and the reservoir (R) at the instant $t = 0$, and tr indicates the trace on both mode and reservoir. Dynamic mean values then follow promptly from equation (2):

$$\langle (a^{\dagger}(t))^{j}\rangle = \left(\frac{\partial^{j}\chi(\eta, \eta^{*}, t)}{\partial\eta^{j}}\right)_{\eta=0}, \tag{3}$$

$$\langle (a(t))^{j}\rangle = \left[(-1)^{j}\,\frac{\partial^{j}\chi(\eta, \eta^{*}, t)}{\partial\eta^{*j}}\right]_{\eta=0}, \tag{4}$$

$$\langle a^{\dagger}(t)a(t)\rangle = \langle n(t)\rangle = \left(-\frac{\partial^{2}\chi(\eta, \eta^{*}, t)}{\partial\eta^{*}\partial\eta}\right)_{\eta=0}, \tag{5}$$

$$\langle n^{2}(t)\rangle - \langle n(t)\rangle = \left(\frac{\partial^{4}\chi(\eta, \eta^{*}, t)}{\partial\eta\partial\eta^{*}\partial\eta\partial\eta^{*}}\right)_{\eta=0}. \tag{6}$$

For the Hamiltonian model given by equation (1), the Heisenberg equation for $a(t)$ operator can be readily obtained [21, 22]:

$$a(t) = w(t)a(0) + \sum_k \vartheta_k(t) b_k(0), \tag{7}$$

where $w(t) = \exp\left[-\left(\frac{\Gamma}{2} + i\omega\right)t\right]$ and

$$\vartheta_k(t) = \frac{i\lambda \exp\left[-\left(\frac{\Gamma}{2} + i\omega\right)t\right] - \exp(\omega_k t)}{\frac{\Gamma}{2} + i(\omega_k - \omega_\ell)},$$

Γ being the damping rate of the cavity field. Once we have equation (7), we can obtain equation (2) for the SCS, whose initial density operator reads

$$\rho_A(0) = |\xi, \alpha\rangle\langle\xi, \alpha|, \tag{8}$$

where $\xi = r\exp(i\phi)$ and $\alpha = |\alpha|\exp(i\theta)$ are the corresponding parameters of the squeeze and displacement operators.

A. Characteristic function and dynamic Q parameter

When equation (8) is inserted in equation (2), the characteristic function χ will read:

$$\chi(\eta, \eta^*, t) = tr|\xi, \alpha\rangle\langle\xi, \alpha|\exp[\eta a^\dagger(t)]\exp[-\eta^* a(t)]. \tag{9}$$

If we now substitute $a(t)$ from equation (7) in equation (9) considering the reservoir at finite temperature, we obtain

$$\chi(\eta, \eta^*, t) = tr\rho_{AB}(0)e^{\eta\left(w^*(t)a^\dagger + \sum_k \vartheta_k^*(t)b_k^\dagger\right)}e^{-\eta^*\left(w(t)a + \sum_k \vartheta_k(t)b_k\right)}$$

$$= tr[\rho_A(0)e^{\eta w^*(t)a^\dagger}e^{-\eta^* w(t)a}]$$

$$\cdot\, tr\left[\prod_k \int d^2\beta_k \frac{e^{\frac{-|\beta_k|^2}{\langle n_k\rangle}}}{\pi\langle n_k\rangle}|\beta_k\rangle\langle\beta_k|e^{\eta\vartheta_k^*(t)b_k^\dagger}e^{-\eta^*\vartheta_k(t)b_k}\right]$$

$$= tr\left\{\rho_A(0)e^{\eta w^*(t)a^\dagger}e^{-\eta^* w(t)a}e^{-|\eta|^2\sum_k|\vartheta_k(t)|^2\langle n_k\rangle}\right\}, \tag{10}$$

where we have used $\rho_{AB}(0) = \rho_A(0)\rho_B(0)$ and the *P* Glauber-Sudarshan representation for the reservoir states [21]

$$\rho_B(0) = \prod_k \int d^2\beta_k \frac{e^{\frac{-|\beta_k|^2}{\langle n_k\rangle}}}{\pi\langle n_k\rangle}|\beta_k\rangle\langle\beta_k|,$$

with $\langle n_k\rangle$ being the mean occupation number characterizing the *k*th mode of the reservoir at temperature *T*. Using equation (8), we can write equation (10) as

$$\begin{aligned}\chi(\eta, \eta^*, t) &= e^{-\varepsilon(t)|\eta|^2}\langle\xi, \alpha|\exp(\eta w^*(t)a^\dagger)\exp(-\eta^* w(t)a)|\xi, \alpha\rangle \\ &= e^{-\varepsilon(t)|\eta|^2}\langle\alpha|\exp[\eta w^*(t)(\mu^* a^\dagger - \nu^* a)] \\ &\quad \cdot \exp[-\eta^* w(t)(\mu a - \nu a^\dagger)]|\alpha\rangle,\end{aligned} \tag{11}$$

where we have used $S(\xi)aS^{-1}(\xi) = \mu a - \nu a^\dagger$, with $\mu = \cosh(r)$, $\nu = \exp(i\phi)\sinh(r)$. We also put, for simplicity, $a(0) = a$ and $\sum_k|\vartheta_k(t)|^2\langle n_k\rangle = \varepsilon(t)$. The above equation allows us to apply the Baker-Campbell-Hausdorff formula $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A, B]} = e^B e^A e^{\frac{1}{2}[A, B]}$, with $[A, [A, B]] = 0$, thus resulting:

$$\begin{aligned}\chi(\eta, \eta^*, t) = \exp\Big[&-\frac{1}{2}\eta^2 w^*(t)^2\mu^*\nu^* - \frac{1}{2}\eta^{*2}w(t)^2\mu\nu \\ &+ \eta w^*(t)(\mu^*\alpha^* - \nu^*\alpha) - \eta^* w(t)(\mu\alpha - \nu\alpha^*) \\ &- |\eta|^2(|w(t)\nu|^2 + \varepsilon(t))\Big].\end{aligned} \tag{12}$$

Equation (12) will be the starting point to calculate the Q parameter as well as its generalized version evolving in time.

1. Dynamic Q parameter

The time evolution of the Q parameter can be readily calculated from (3)-(6) using the above χ function. For the average photon number, we obtain

$$\langle n(t)\rangle = [|\, w(t)\,|^2(\mu^*\alpha^* - \nu^*\alpha)(\mu\alpha - \alpha^*) + (|\, w(t)\nu\,|^2 + \varepsilon(t))] \tag{13}$$

while

$$\begin{aligned}&\langle \Delta n^2(t)\rangle - \langle n(t)\rangle \\ &= |\, w^2(t)\mu\nu\,|^2 - |\, w(t)\,|^4\mu^*\nu^*|\,\alpha\,|^2U^2 - |\, w(t)\,|^4\mu\nu|\,\alpha\,|^2U^{*2} \\ &\quad + 2|\, w(t)\,|^2|\,\alpha\,|^2UU^*(|\, w(t)\nu\,|^2 + \varepsilon(t)) + (|\, w(t)\nu\,|^2 + \varepsilon(t))^2,\end{aligned} \tag{14}$$

where $\Delta n^2(t)$ is the variance of the photon number, and we have written $U = \mu e^{i\theta} - \nu e^{-i\theta}$ and $U^* = \mu e^{-i\theta} - \nu^* e^{i\theta}$.

The dynamic Mandel's parameter

$$Q(t) = \frac{\langle \Delta n^2(t)\rangle - \langle n(t)\rangle}{\langle n(t)\rangle} \tag{15}$$

can be obtained by combining equation (14) with equation (13). The sub-Poissonian and Poissonian condition $\langle \Delta n^2(t)\rangle - \langle n(t)\rangle \le 0$ thus leads to

$$\begin{aligned}&|\,\alpha\,|^2[|\, w(t)\,|^4\mu^*\nu^*U^2 + |\, w(t)\,|^4\mu\nu U^{*2} - 2|\, w(t)\,|^2UU^*(|\, w(t)\nu\,|^2 + \varepsilon(t))] \\ &\ge |\, w^2(t)\mu\nu\,|^2 + (|\, w(t)\nu\,|^2 + \varepsilon(t))^2\end{aligned} \tag{16}$$

which can be solved for the displacement parameter as

$$|\,\alpha\,| \ge \sqrt{\frac{|\, w^2(t)\mu\nu\,|^2 + (|\, w(t)\nu\,|^2 + \varepsilon(t))^2}{|\, w(t)\,|^4\mu\nu^*U^2 + |\, w(t)\,|^4\mu\nu U^{*2} - 2|\, w(t)\,|^2UU^*(|\, w(t)\nu\,|^2 + \varepsilon(t))}}. \tag{17}$$

The above condition tells us that for finite temperature we have to vary $|\alpha|$ continuously in time to ensure $Q(t) \leq 0$, which results prohibitive, in practice. However, for zero temperature $(\varepsilon(t) = 0)$, this condition turns to be time independent:

$$|\alpha| \geq \sqrt{\frac{1}{2}\frac{|\mu\nu|^2 + |\nu|^4}{Re(\mu\nu^* U^2 + \mu\nu U^{*2}) - |U\nu|^2}}. \tag{18}$$

Equation (18) now tells us that at zero temperature we can always choose α, μ and ν to ensure the condition $Q \leq 0$ for all times. We have then the nice result that, at zero temperature, the SCS preserves the signal of its corresponding Q parameter, i.e., the photon distribution of the SCS does not change under the effect of a thermal reservoir at zero temperature [15]. This behavior resembles that of the coherent state, which at zero temperature, looses its excitation coherently. Particularly, for the case of $Q \geq 0$, the quantum character of the photon statistics of the SCS is hidden. We turn to this question in the next section.

IV. Generalized Q Parameter

We saw in Section III that the quantum nature of the photon number distribution of the SCS cannot be observed when both displacement and squeeze parameters are adjusted to ensure $Q \geq 0$, even when we can be sure that the state is really a quantum one, as is just the case of the SCS. In other words, focusing only in the photon number distribution, all the measures of the nature of the statistics, including the Mandel's parameter and the second-order correlation function for zero time delay $g^{(2)}(0) = 1 + Q/\langle n \rangle$ will take the coherent state values $(Q = 0,\ g^{(2)}(0) = 1)$ and thus it is impossible, resorting only to the Q parameter, to distinguish statistically between a Poissonian distribution due to coherent states and all other non-Poissonian states having $Q = 0$. Notwithstanding, as the photon number distribution is indeed different for true Poissonian states like those discussed in [7, 8], we could expect that experiments relying on photoncounts can reveal these differences. In fact, this is the case of the binomial states studied in [2], where, for certain parameters, Q is always positive, while the generalized parameter $Q(k)$, k integer, becomes negative for $k = 2$. Formally, the so-called generalized $Q(k, t)$ parameter in a given time t is given by [2]

$$Q(k, t) = \frac{[\langle n^{(k+1)}(t)\rangle - \langle n(t)\rangle^{(k)}\langle n(t)\rangle]}{\langle n(t)\rangle}, \quad k = 1, 2, 3, ..., \tag{19}$$

where $\langle n(t)\rangle^{(k)} = \sum_{n=k} \frac{n!}{(n-k)!} P(n, t)$ is the kth factorial moment of the photon-number distribution $P(n, t)$ at time t. The original Q parameter is recovered to $k = 1$. As discussed in [2], $Q(k, t) > 0$ for all k is expected for a classical radiation with positive definite Glauber-Sudarshan P function, while $Q(k, t) < 0$ for any value of k implies that the P function is not positive definite, and then we have a non-classical state. Besides, $Q(k, t)$ has an interesting physical meaning, since it can be identified with the change in the kth factorial moment of the photon-number distribution under the one-count process occurring at time t. The author in [2] also showed that for coherent states $Q(k, 0) = 0$ for all k, meaning that detection of one photon has no effect whatsoever on the photon number statistics of Poissonian states. On the other hand, thermal states and number states, which are expected to have classical and quantum photon number statistics, have $Q(k, 0) > 0$ and $Q(k, 0) < 0$, respectively, for all k.

As for coherent states $Q(k, 0) = 0$ for all k, this provides us with a criterion to distinguish true Poissonian states from other ones having $Q \equiv Q(1, 0) = 0$. Specifically, for the SCS, we can show, see Figure 1(a), that fixing the squeeze and displacement parameters that make it $Q(1, 0) = 0$, not all remaining $Q(k, 0)$, $k \neq 1$, can be simultaneously null. This implies, as we have pointed out in the introduction, that SCS with $Q = 0$ do not have Poissonian distribution. Also, as one would expect from the purpose of the generalized $Q(k, t)$, may be the quantumness of SCS photon number distribution can be revealed in higher orders of the generalized Q parameter. This will be investigated now, and we will show that, although $Q(k, t)$ can be useful to characterize Poissonian states, it is not always useful to reveal nonclassicality of a quantum state.

To find the generalized $Q(k, t)$ for the SCS evolving under a thermal reservoir, it is convenient to rewrite equation (19) using the characteristic function. For $k = 1, 2, 3, \ldots,$ we can write

$$\langle n(t)\rangle^{(k)} = \sum_{n=k} \frac{n!}{(n-k)!} P(n, t)$$

$$= \left((-1)^k \frac{\partial^{2k} \chi(\eta, \eta^*, t)}{\partial \eta^k \partial \eta^{*k}} \right)_{\eta=0} \tag{20}$$

and

$$\langle n(t)\rangle^{(k+1)} = \sum_{n=k+1} \frac{n!}{(n-k)!} P(n, t)$$

$$= \left[(-1)^{k+1} \frac{\partial^{2(k+1)} \chi(\eta, \eta^*, t)}{\partial \eta^{(k+1)} \partial \eta^{*(k+1)}} \right]_{\eta=0}, \tag{21}$$

and thus

$$Q(k, t) = \frac{[\langle n(t)\rangle^{(k+1)} - \langle n(t)\rangle^{(k)} \langle n(t)\rangle]}{\langle n(t)\rangle}$$

$$= - \frac{\left[(-1)^{k+1} \frac{\partial^{2(k+1)} \chi(\eta, \eta^*, t)}{\partial \eta^{(k+1)} \partial \eta^{*(k+1)}} \right]_{\eta=0} + \left[(-1)^k \frac{\partial^{2k} \chi(\eta, \eta^*, t)}{\partial \eta^k \partial \eta^{*k}} \right]_{\eta=0} \left[\frac{\partial^2 \chi(\eta, \eta^*, t)}{\partial \eta \partial \eta} \right]_{\eta=0}}{\left[\frac{\partial^2 \chi(\eta, \eta^*, t)}{\partial \eta \partial \eta} \right]_{\eta=0}}. \tag{22}$$

From equation (22), we can promptly calculate $Q(k, t)$; the calculus being straightforward when using equation (12).

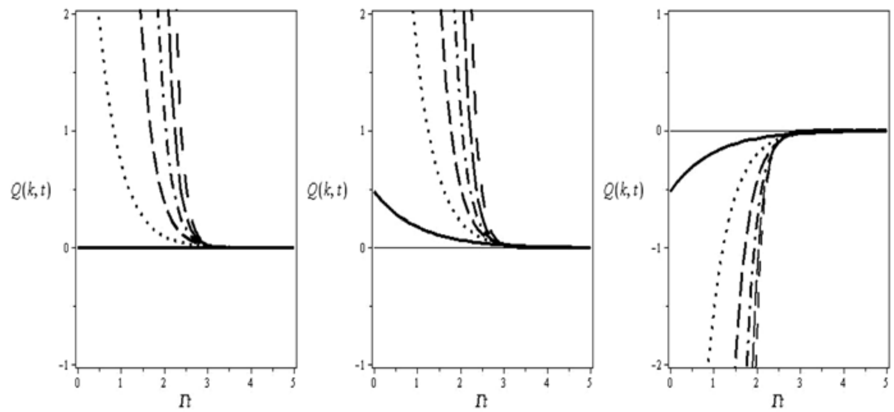

Figure 1. Generalized Mandel's $Q(k, t)$ function *versus* Γt at zero temperature for a set of parameters. $Q(1, 0) \equiv Q$ and higher orders of $Q(k, t)$ are shown by dot $(k = 2)$, dash $(k = 3)$, dash-dot $(k = 4)$, long dash $(k = 5)$, and short dash $(k = 6)$. (a) Starting from $Q = 0$, with $\alpha = 6.6635$, $\xi = 1.0$; (b) Starting from $Q > 0$, with $\alpha = 5.0$, $\xi = 1.0$ and (c) Starting from $Q < 0$ with $\alpha = 5.0$, $\xi = 0.4$. Orders ranging from $k = 7, ..., 12$, not shown in Figure 1(a), show that the slope of $Q(k, t)$ increases monotonically as k increases.

In Figures 1(a) and 1(c), we show the $Q(k, t)$ function of the SCS, evolving under a thermal reservoir at $T = 0$, for several k. In Figure 1(a), the squeeze $(\xi = 1.0)$ and displacement $(\alpha = 6.6635)$ parameters were chosen to ensure $Q(1, 0) = 0$, which is indicated by the straight (solid) line on the zero mark parallel to the Γt axis. Thus, as we anticipated, at zero temperature, a given state having $Q = 0$ remains as such at all times. Higher orders of $Q(k, t)$ are shown by dot $(k = 2)$, dash $(k = 3)$, dash-dot $(k = 4)$, long dash $(k = 5)$, and short dash $(k = 6)$. Orders ranging from

$k = 7, ..., 12,$ not shown in Figure 1(a), show that $Q(k, t)$ is always positive and increases its slope as k increases. From now on, we shall use the same notation (dot, dash, etc.) to indicate the higher orders of $Q(k, t)$. In Figure 1(b), the squeeze $(\xi = 1.0)$ and displacement $(\alpha = 5.0)$ parameters were chosen to start from $Q(k, 0) > 0,$ which is indicated by the solid line. Going to higher orders of k, essentially the same behavior seen in Figure 1(a), is reproduced. On the other hand, when the squeeze $(\xi = 0.4)$ and displacement $(\alpha = 5.0)$ parameters are chosen to start from $Q(1, 0) < 0,$ Figure 1(c), a reverse behavior occurs for $Q(k, t)$: with the solid line indicating the original Q parameter, we see that $Q(k, t)$ becomes more negative with increasing k.

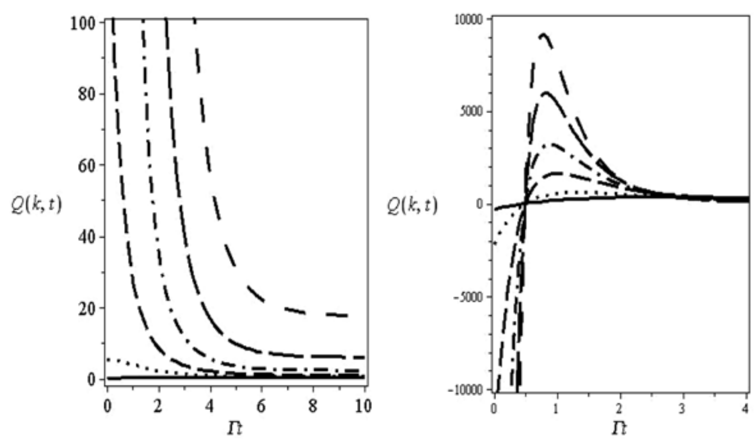

Figure 2. Generalized Mandel's $Q(k, t)$ function *versus* Γt at finite temperature corresponding to a mean thermal photon $\bar{n} = 0.4$ for a set of parameters. $Q(1, 0) \equiv Q$ and higher orders of $Q(k, t)$ are rescaled and shown by dot $(k = 2)$, dash $(k = 3)$, dash-dot $(k = 4)$, long dash $(k = 5)$, and short dash $(k = 6)$. (a) Starting from $Q = 0,$ with $\alpha = 6.6635,$ $\xi = 1.0$ and (b) Starting from $Q < 0,$ with $\alpha = 5.0, \xi = 0.4.$ Orders ranging from $k = 7, ..., 12,$ not shown in Figure 1(a), show that the slope of $Q(k, t)$ increases monotonically as k increases.

In Figures 2(a) and 2(b), we compute the effect of temperature on the generalized $Q(k, t)$. For the sake of clarity, we have rescaled the curves in Figure 2(b) by the factor $3000 \times Q(1, t)$; $800 \times Q(2, t)$; $200 \times Q(3, t)$; $40 \times Q(4, t)$; $7 \times Q(5, t)$; $1 \times Q(6, t)$. Note, from Figure 2(b), that the transition from sub- to super-Poissonian statistics occurs at the same point, no matter the order of the $Q(k, t)$. Also, our simulations reveal that this point moves to left when the temperature is increased. This is somewhat expected, since the role of the temperature is to accelerate the degradation of quantum properties, such that the nonclassicality of the SCS as a whole, including the photon statistics, is lost the faster the higher the temperature.

Anyway, it is remarkable that for each temperature, there exists only a single point indicating the transition from sub- to super-Poissonian statistics [15].

V. Conclusions

In this paper, we have studied the time evolution of the photon number statistics of squeezed coherent states (SCS) through both the Mandel's and the generalized Mandel's parameter under dissipation at zero and finite temperature. Once SCS can have its Q parameter greater than, lesser than, or equal to zero, we focus our attention to the case $Q \geq 0$, in which the photon statistics can be classically explained. On the other hand, it is well known that SCS is a quantum state, irrespective of the value of Q, notably in its squeeze properties. Since the quantumness of the photon number distribution of the SCS having displacement and squeezing parameters making its $Q \geq 0$ is hidden from the very definition of the Q parameter, we resort to the generalized $Q(k, t)$ parameter to try to detect non-classicality, as suggested in [2], in the photon statistics of the SCS. As pointed out in [2], $Q(k, t)$ has these nice properties: (i) it recovers the $Q(t)$ parameter when $k = 1$, (ii) if $Q(k, t) < 0$ for some integer k, then the state considered is nonclassical, (iii) to the coherent state, $Q(k, 0)$ is null for all k. When we calculate the generalized $Q(k, t)$ for the SCS with displacement and squeeze parameters adjusted to give $Q(1, 0) = 0$ or $Q(1, 0) > 0$, in both the cases, we found $Q(k, t) \geq 0$ for all $k \geq 2$. Therefore, different from what was expected, higher orders of $Q(k, t)$ cannot be neither simultaneously zero, thus indicating that SCS are different from true Poissonian states, nor negative for some $k \neq 1$, thus indicating that $Q(k, t)$ is not able to reveal nonclassicality in its higher orders.

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References

[1] L. Mandel, Opt. Lett. 4 (1979), 205.

[2] C. T. Lee, Quantum Opt. 6 (1994), 27.

[3] H. J. Kimble, M. Dagenais and L. Mandel, Phys. Rev. Lett. 39 (1977), 691.

[4] D. Stoler, Phys. Rev. Lett. 33 (1974), 1397.

[5] H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory 24 (1978), 657.

[6] W. Schleich and J. A. Wheeler, Nature 326 (1987), 574.

[7] C. M. A. Dantas and B. Baseia, Physica A 265 (1999), 185.

[8] J. M. C. Malbouisson et al., Physica A 285 (2000), 397.

[9] H. Paul, Rev. Mod. Phys. 54 (1982), 1061.

[10] D. J. Wineland, J. J. Bollinger, W. M. Itano and D. J. Heinzen, Phys. Rev. A 50 (1994), 67.

[11] Thomas F. George and Henke F. Arnouldos, eds., Horizons on World Physics, Vol. 243, Theoretical Physics 2002, Part 2, Nova Science Publishers, Inc., N.Y., 2002.

[12] J. S. Sales, L. F. da Silva and N. G. de Almeida, J. Phys. B: At. Mol. Opt. Phys. 43 (2010), 245504.

[13] J. S. Sales and N. G. de Almeida, Phys. Rev. A 83 (2011), 062121.

[14] J. S. Sales, L. F. da Silva and N. G. de Almeida, Phys. Rev. A 83 (2011), 034302.

[15] J. S. Sales and N. G. de Almeida, Dynamic statistical properties of the squeezed coherent state superposition, 2012, submitted.

[16] C. J. Villas-Boas, N. G. de Almeida, R. M. Serra and M. H. Y. Moussa, Phys. Rev. A 68 (2003), R061801.

[17] C. J. Villas-Boas, F. R. de Paula, R. M. Serra and M. H. Y. Moussa, Phys. Rev. A 68 (2003), 053808.

[18] N. G. de Almeida, R. M. Serra, C. J. Villas-Boas and M. H. Y. Moussa, Phys. Rev. A 69 (2004), 035802.

[19] R. M. Serra, C. J. Villas-Boas, N. G. de Almeida and M. H. Y. Moussa, Phys. Rev. A 71 (2005), 045802.

[20] F. O. Prado, N. G. de Almeida, M. H. Y. Moussa and C. J. Villas-Boas, Phys. Rev. A 73 (2006), 043803.

[21] M. O. Scully and M. S. Zubary, Quantum Optics, Cambridge Univ. Press, 1997.

[22] I. P. de Queirós, W. B. Cardoso and N. G. de Almeida, J. Phys. B: At. Mol. Opt. Phys. 40 (2007), 21.