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A MATHEMATICAL MODEL FOR SOIL PRESSURES ACTING ON CIRCULAR FOOTINGS, FOR OBTAINING FORCE BY BIDIRECTIONAL SHEAR

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Abstract

In the design of circular reinforced concrete footings subject to axial load and flexure in a direction are presented different pressures throughout contact surface, which are exerted by the soil. In this paper, we develop a mathematical model to take into account the real pressure of the ground acting on the contact surface of the circular footing of shear force by penetration (shear force bidirectional), when applying the load that must support the said member of structure. The traditional model takes into account the maximum pressure of the ground to design the footings and is considered uniform at all the points of contact, i.e., that the entire surface has the same pressure. The proposed model takes into account the actual pressures, i.e., considers the pressures generated in the entire contact surface of the footing and generally are different, with a linear variation in the contact surface. Also, develops a comparison between the two models

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as shown in the result tables. According to the data obtained, it is shown that the traditional model is larger than the model proposed. Therefore, normal practice to use the traditional model will not be a recommended solution. When is best to use the proposed model, since it is more economic and also more attached to real conditions.

Introduction

The foundation is the part of the structure responsible for transmitting the loads to the ground. Since the strength and stiffness of the soil are, except in rare cases, much lower than those of the structure, the foundation has an area much greater than the sum of the areas of all the columns and load-bearing walls (vertical structure).

The foundation will be, therefore, generally, volume pieces considerable with respect to volume of the pieces of the structure. Those are constructed in reinforced concrete and the concrete that is generally employed is of relatively low quality as it is not economically interesting using of higher strength concrete.

In order to make a good foundation, prior knowledge of the soil on which is going to build the structure is required.

The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems [1, 2].

When level from the lower zone of the structure, the ground has adequate characteristics from the point of technical and economic terms to cement on it, the foundation is called *surface* or *direct*. The superficial foundations shall be constituted by: footings, beams and plates, or combinations of these elements.

These characteristics of the ground are fundamental during the election of foundation.

The influence of the type of building to execute is also important in the selection of the foundation.

The most important characteristics of the buildings at the time of the foundation include:

(a) Existence of basements.

(b) Light buildings of low-rise: shallow foundation will be used.

(c) Weighed buildings low-rise, floor slabs, piling.

(d) High-rise buildings: deep foundations or concrete slabs.

Before selection of the foundation, and as part previous to the drafting of the project, an additional geotechnical study of the land in its conclusions should recommend the most appropriate types of foundations.

As main characteristics, a footing must meet:

(a) Conduction of loads to ground through the structural elements.

(b) Uniform distribution of loads for that not exceeds the stresses in the surface of soil.

(c) Settlements of the structure should be limited to the maximum permissible for this, and also avoid differential settlement.

(d) The foundations must be hidden.

A footing is an expansion of the base of a column or wall that is to transmit the load to ground at a suitable pressure to soil properties. Footing that supports a single column called *individual footing* or *isolated footing*. The footing that is constructed under a wall is called *strip footing* or *continuous footing*. If a footing supports multiple columns it is called *combination footing*. A special form of combined footing that is normally used in case one of the columns supporting an exterior wall is the cantilever footing [3-6].

In the cold zones, the footings are placed to a depth not less that the penetration normal of the freezing. In warmer climates and especially in semiarid regions, the minimum depth of footings may depend on the greater depth to seasonal moisture changes that produce an expansion and contraction significant of the soil. The elevation at which is placed a footing depends on the type of subsoil, the load that should support and cost of the foundation. Generally the footing is placed at the maximum height in which can be a material that having of stably the adequate load capacity.

The excavation of a reinforced concrete footing must be kept dry, to placing reinforcement and hold it in correct position while concrete is dry. For doing this in soils containing water should be previously necessary to pump or install a drainage system.

The classification of footings is very broad. According to the way, their work can be classified as isolated, combined, continuous and braced or attached. According to its form, classification will be: rectangular, square, circular, annular or polygonal.

In the design of superficial foundations, in the specific case of isolated footings are of three types in terms of the application of loads: (1) the footings subject to concentric axial load, (2) the footings subject to axial load and moment in one direction (unidirectional flexure), (3) the footings subject to axial load and moment in two directions (bidirectional flexure). The hypothesis used is to consider the pressures uniforms for the design, i.e., the same pressure at all points of contact with the ground on the foundation; this design pressure is the maximum that is presented in a isolated footing [3, 6].

In this paper is developed a mathematical model of non-uniform pressures for circular footings subject to axial load and moment in a direction (unidirectional flexure) having a linear variation along all its contact area, which is as it really presents the pressures. This model may be applicable to the other two cases, for example for the first case where acts an axial load concentrically, pressures are identical and the last case where acts an axial load and moment in two directions (bidirectional flexure) when exists moments in both directions is obtained a resultant moment because the moment of inertia does not change in any direction. It also develops a comparison, in terms of shear force by penetration or bidirectional shear force, between the traditional model and the proposed model, to observe the differences.

Theoretical Development of New Model

Figure 1 shows a circular footing in plant, whereas Figure 2 shows in elevation, this footing is subject to axial load and moment in one direction (unidirectional flexure), where are presented different pressures on the entire surface of contact, linearly varying along the area of contact with the ground.

Figure 3 presents the differential element that intervenes in the analysis that developed below.

From Figure 2, by proportions the following is obtained:

$$\frac{\rho_1 - \rho_2}{2r} = \frac{\rho - \rho_2}{r + y}, \tag{1}$$

where

r is the radius of the footing,

ρ_1 is the maximum pressure exerted by the soil on the footing,

ρ_2 is the minimum pressure exerted by the soil on the footing and this is located at the point opposite to ρ_1,

ρ is the pressure at any point of soil on the footing and then be called $\rho(y)$.

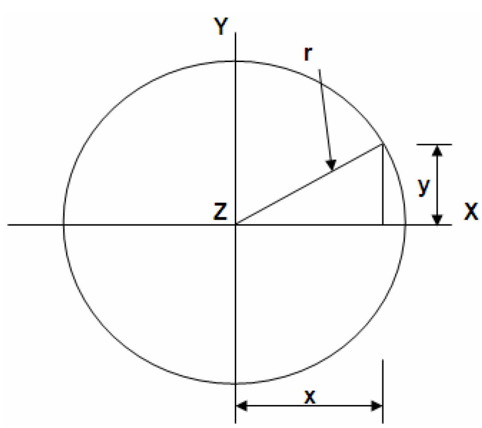

Figure 1. Circular footing typical top view.

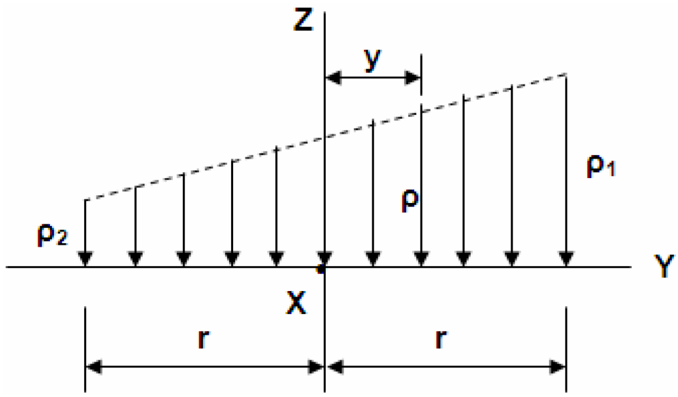

Figure 2. Soil pressures on a circular footing.

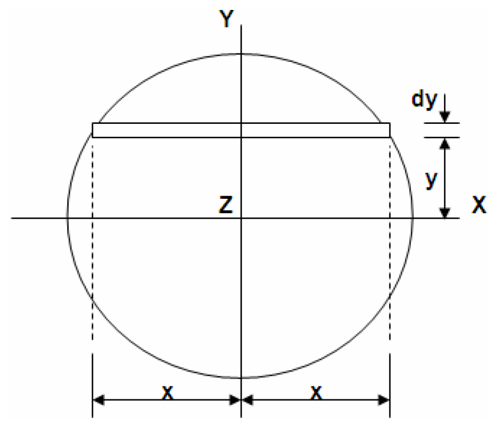

Figure 3. The element differential of the circumference.

From Figure 1, using the equation of the circle [7-10]:

$$r^2 = x^2 + y^2, \tag{2}$$

where

y is the variable of the vertical component of "*r*",

x is the variable of the horizontal component of "*r*".

From equation (2) is located "*x*":

$$x = \sqrt{r^2 - y^2}. \tag{3}$$

From equation (1) is found "ρ" as shown:

$$\rho(y) = \rho_2 + \frac{(\rho_1 - \rho_2)(r + y)}{2r}. \quad (4)$$

To find the resultant force "F_R" is calculated from the pressure volume as follows [7-10]:

$$F_R = \int_{-r}^{r} \rho(y) 2x dy. \quad (5)$$

Substituting equations (3) and (4) into equation (5) as below:

$$F_R = \int_{-r}^{r} \left[\rho_2 + \frac{(\rho_1 - \rho_2)(r + y)}{2r}\right](2\sqrt{r^2 - y^2})dy. \quad (6)$$

Realizing products of equation (6) as presented:

$$F_R = \int_{-r}^{r} \left[2\rho_2\sqrt{r^2 - y^2} + \frac{(\rho_1 - \rho_2)(r + y)\sqrt{r^2 - y^2}}{r}\right]dy. \quad (7)$$

Developing the corresponding operations of equation (7) as observed:

$$F_R = \int_{-r}^{r} \left[2\rho_2\sqrt{r^2 - y^2} + (\rho_1 - \rho_2)\sqrt{r^2 - y^2} + \frac{(\rho_1 - \rho_2)y\sqrt{r^2 - y^2}}{r}\right]dy. \quad (8)$$

Performing the simplification of equation (8) presents the following integral:

$$F_R = \int_{-r}^{r} [(\rho_1 + \rho_2)\sqrt{r^2 - y^2}]dy - \frac{1}{2}\int_{-r}^{r} \left[\frac{(\rho_1 - \rho_2)(-2y)\sqrt{r^2 - y^2}}{r}\right]dy. \quad (9)$$

A description of the integration of equation (9) is obtained:

$$F_R = \left[\left\{(\rho_1 + \rho_2)\left(\frac{y\sqrt{r^2 - y^2}}{2} + \frac{r^2}{2}\sin^{-1}\frac{y}{r}\right) - \frac{(\rho_1 - \rho_2)(r^2 - y^2)^{\frac{3}{2}}}{3r}\right\}\right]_{-r}^{r}. \quad (10)$$

Substituting the boundary conditions into equation (10) as:

$$F_R = \left\{(\rho_1 + \rho_2)\left[\frac{r^2}{2}\sin^{-1}\left(\frac{r}{r}\right)\right]\right\} - \left\{(\rho_1 + \rho_2)\left[\frac{r^2}{2}\sin^{-1}\left(\frac{-r}{r}\right)\right]\right\}. \quad (11)$$

The simplifications of equation (11) are performed:

$$F_R = \frac{(\rho_1 + \rho_2)\pi r^2}{2}. \quad (12)$$

Application

Below are the three types of circular footings and varying the minimum pressure. These examples are developed by the traditional model and the proposed model. Figure 4 shows the details of the footing:

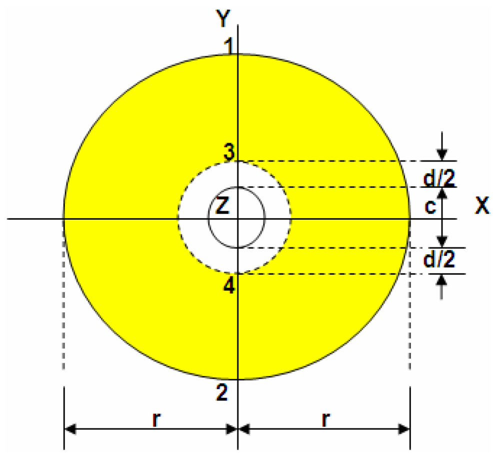

Figure 4. Circular footing isolated top view.

Shear force by penetration or shear force bidirectional "V_P" is obtained calculating the volume of pressures that is generated in the shaded area of yellow in Figure 4.

Traditional model

Shear force by penetration or shear force bidirectional, which is presented at a distance “$d/2$” to from the column and is obtained as follows:

$$V_P = \rho_1 \pi \left[r^2 - \left(\frac{c + d}{2} \right)^2 \right],$$

where

c is the diameter of the circular column,

d is the depth effective of the footing.

Proposed model

Where: $\rho_1 \neq \rho_2 \neq \rho_3 \neq \rho_4$.

ρ_2 is the pressure exerted by the ground on the footing at the point 3 according to Figure 4, this pressure can be obtained by means of equation (4), substituting “$(c + d)/2$” instead of “y”.

ρ_4 is the pressure exerted by the ground on the footing at the point 4 according to Figure 4, this pressure can be obtained by means of equation (4), substituting “$-(c + d)/2$” instead of “y”.

Shear force by penetration or shear force bidirectional, occurring at a distance “$d/2$” to from the column, using equation (12) is obtained as follows:

$$V_P = \frac{(\rho_1 + \rho_2)\pi r^2}{2} - \frac{(\rho_3 + \rho_4)\pi (c + d)^2}{8}.$$

Table 1 presents the results for the two models of the three types of footings.

Table 1. Comparison of results

Case	Footing dimensions and column (m)			Pressures soil on the contact area of the footing (ton/m^2)				Shear force bidirectional V_P (ton)	
	r	*c*	*d*	ρ_1	ρ_2	ρ_3	ρ_4	MT	MP
Footing 1									
1	1.00	0.40	0.20	20	15	18.25	16.75	57.1770	50.0299
2				20	10	16.50	13.50	57.1770	42.8827
3				20	5	14.75	10.25	57.1770	35.7356
4				20	0	13.00	7.00	57.1770	28.5885
Footing 2									
1	1.50	0.50	0.25	20	15	18.125	16.875	132.5359	115.9689
2				20	10	16.250	13.750	132.5359	99.4075
3				20	5	14.375	10.625	132.5359	82.8350
4				20	0	12.500	7.500	132.5359	66.2680
Footing 3									
1	2.00	0.60	0.30	20	15	18.0625	16.9375	238.6040	208.7785
2				20	10	16.1250	13.8750	238.6040	178.9530
3				20	5	14.1875	10.8125	238.6040	149.1275
4				20	0	12.2500	7.7500	238.6040	119.3020

MT = Traditional model

MP = Proposed model

Results and Discussion

Figures 5, 6 and 7 show the differences between the two models of the 3 types of footings for the 4 cases. In all cases, the proposed model is lower with respect to the traditional model.

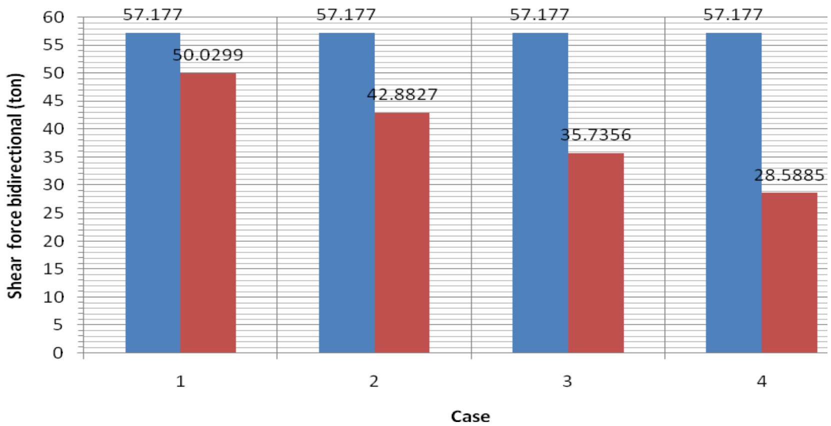

Figure 5. Footing type 1.

Figure 5 presents the footing 1 which shows that there are large differences in case 4. For example, the traditional model is a 100% largest than the model proposed.

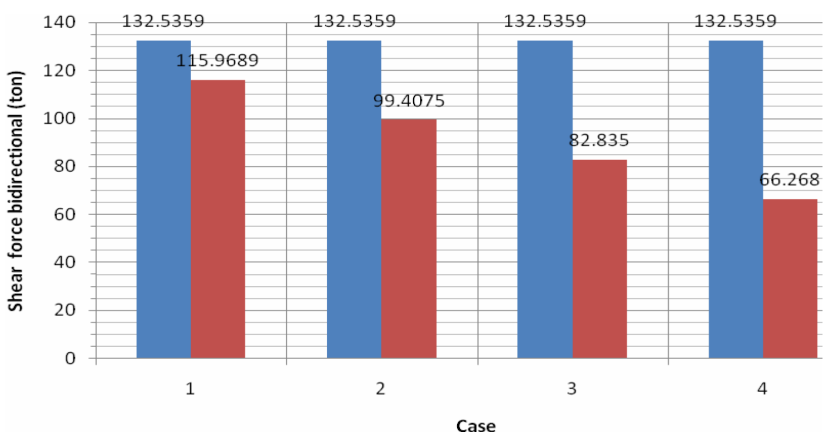

Figure 6. Footing type 2.

With respect to Figure 6 showing the footing 2, the difference being greater in the case 4. This difference is largest in the traditional model with respect to the proposed model of 100%.

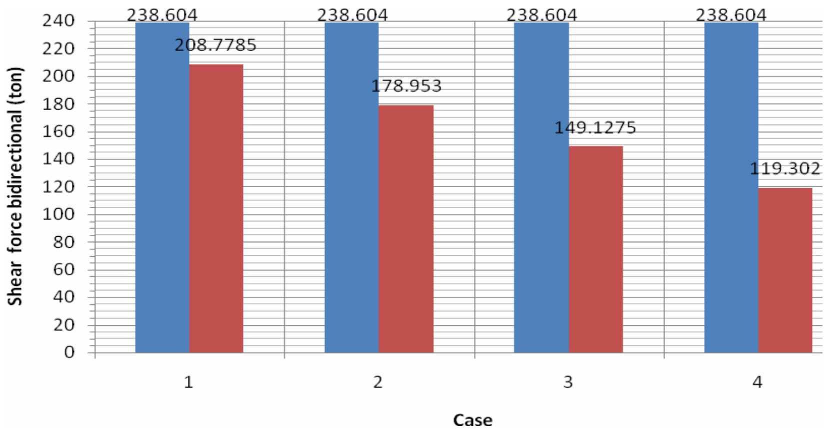

Figure 7. Footing type 3.

Finally, we examine Figure 7, which illustrates the footing 3, which shows the greatest difference also in the case 4. This difference is largest in a 100% for the traditional model with respect to the proposed model.

Conclusions

The results of the problem considered, through the application of two different models, are possible to conclude the following:

According to pressures, it is observed that when the difference between the maximum pressure and the minimum pressure is higher, presents a greater increase in traditional model with respect to the proposed model, in terms of shear force by penetration or shear force bidirectional. This is a logical situation, because in traditional model is retained its value, but in proposed model is reduced shear force by penetration or shear force bidirectional.

This means that can have great savings in terms of materials used (reinforcing steel and concrete) for the fabrication of footings isolated under conditions mentioned above. Since that the principle in civil engineering, in terms of structural conditions is that be safe and economical, and the latter is not met the traditional model.

Therefore, the practice of using the traditional model is not a recommended solution, because this very exceeded in some cases, with regard to design of these structural members.

Then it is proposed to use the model developed in this paper for structural design of circular footings isolated subjected to axial load and flexure unidirectional, to obtain shear force by penetration or shear force bidirectional. Moreover, this adheres more to the actual conditions of the soil pressures that are applied to the foundation.

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