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CESÁRO STATISTICAL CORE OF DOUBLE SEQUENCES

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Abstract

In this paper, Cesáro statistical core of double sequences have been obtained. The concept of RH regularity of four dimensional matrices in Pringsheim's sense has been used to find the analogues of the results for statistical core of sequences.

1. Introduction and Preliminaries

The concept of statistical convergence was first introduced by Fast [2] and further studied by Sălat [13], Fridy [3] and many others. Mursaleen and Edely [9] and Moricz [8] introduced and studied the same concept for double sequences, separately in the same year. Many related concepts have been introduced and studied so far, for example, statistical limit points, statistical cluster point, statistical limitsuperior, statistical limit inferior and statistical core. Moricz [7] defined the concept of statistical $(C, 1)$-summability and studied some Tauberian theorems. Recently, Alotaibi [1] introduced $(C, 1)$-

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analogues of the above mentioned concepts and studied C_1-statistical core of complex sequences and established some results on C_1-statistical core. Here, we introduce $(C, 1.1)$ analogues of the above mentioned concepts and mainly study $C_{1.1}$ statistical core of double sequences and establish some results on $C_{1.1}$ statistical core.

Let $K \subseteq \mathbb{N} \times \mathbb{N}$ be a two dimensional set of positive integers and let $K(n, m)$ be the numbers of (i, j) in K such that $i \leq n, \quad j \leq m.$ Then the two dimensional natural density of K can be defined as follows.

The *lower asymptotic density* of a set $K \subseteq \mathbb{N} \times \mathbb{N}$ is defined as

$$\underline{\delta_2}(K) = \lim_{n, m} \inf \frac{K(n, m)}{nm}.$$

Similarly the *upper asymptotic density* of K is defined as

$$\overline{\delta_2}(K) = \lim_{n, m} \sup \frac{K(n, m)}{nm}.$$

In case, the sequence $\left(\frac{K(n, m)}{nm}\right)$ has a limit in the Pringsheim's sense, then we say K has *double natural density* and it is defined as

$$\lim_{n, m} \frac{K(n, m)}{nm} = \delta_2(k).$$

For example, let $K = \{(i^2, j^2) : i, j \in N\}.$ Then

$$\delta_2(k) = \lim_{n, m} \frac{K(n, m)}{nm} \leq \lim_{n, m} \frac{\sqrt{n}\sqrt{m}}{nm} = 0,$$

i.e., the set K has double density zero, while the set $\{(i, 2j) : i, j \in \mathbb{N}\}$ has double density $1/2$.

Definition 1.1. A real double sequence is said to be *statistically convergent* to the number l if for each $\varepsilon > 0,$ the set

$$\{(j, k), \; j \leq n \text{ and } k \leq m : | x_{jk} - l | \geq \varepsilon\}$$

has double natural density zero. In this case, we write $st_2 - \lim_{jk} x_{jk} = l$ and denote the set of all statistically convergent double sequences by st_2.

Define the first means $\sigma_{n,m}$ of a double sequence (x_{jk}) by setting

$$\sigma_{nm} = \frac{1}{nm}\sum_{j=1}^{n}\sum_{k=1}^{m} x_{jk},$$

we say that $x = (x_{jk})$ is *statistically summable* $(C, 1.1)$ to l, if the sequence $\sigma = (\sigma_{nm})$ is statistically convergent to l in Pringsheim's sense, that is, $st_2\text{-}\lim_{n,m} \sigma_{nm} = l.$ We denote by $C_{1.1}(st_2),$ the set of all double sequences which are statistically summable $(C, 1.1)$.

2. Known Results

We recall some concepts and results on the double sequences which are already known. These results will be used in this paper.

A double sequence x is bounded if there exists a positive number M such that $|x_{jk}| < M$ for all j and k, i.e., if

$$\| x \| = \sup_{j,k}|x_{jk}| < \infty.$$

Note that on contrast to the case for single sequences, a convergent double sequence need not be bounded.

Let $A = [a_{jk}^{mn}]_{j,k=0}^{\infty}$ be a doubly infinite matrix of real numbers for all $m, n = 0, 1, 2, \ldots.$ Forming the sum

$$y_{mn} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} a_{jk}^{mn} x_{jk}, \tag{2.1}$$

called the *A-means* of the double sequence x yields a method of summability. We say that a sequence x is *A-summable* to the limit s if the A-means exist for all $m, n = 0, 1, 2, \ldots,$ and converges in the sense of Pringsheim,

$$\lim_{p, q \to \infty} \sum_{j=0}^{p} \sum_{k=0}^{q} a_{jk}^{mn} x_{jk} = y_{mn}$$

and

$$\lim_{mn \to \infty} y_{mn} = s.$$

A two dimensional matrix transformation is said to be *regular* if it maps every convergent sequence into a convergent sequence with the same limit. In 1926, Robison [12] presented a four dimensional analogue of regularity for double sequences in which he added an additional assumption of boundedness. A four dimensional matrix A is said to be *bounded-regular* or *RH-regular* if it maps every bounded P-convergent sequence (or convergent in the Pringsheim sense) into a P-convergent sequence with same P-limit. The following is a four dimensional analogue of the well-known Silverman-Toeplitz theorem.

Theorem 2.1. *The four dimensional matrix A is bounded-regular or RH-regular if and only if*

(RH_1) $P\text{-}\lim_{m, n} a_{jk}^{mn} = 0 \ (j, k = 0, 1, \ldots)$

(RH_2) $P\text{-}\lim_{m, n} \sum_{j, k=0, 0}^{\infty, \infty} a_{jk}^{mn} = 1$

(RH_3) $P\text{-}\lim_{m, n} \sum_{j=0}^{\infty} |a_{jk}^{mn}| = 0 \ (k = 0, 1, \ldots)$

(RH_4) $P\text{-}\lim_{m, n} \sum_{k=0}^{\infty} |a_{jk}^{mn}| = 0 \ (j = 0, 1, \ldots)$

(RH_5) $\sum_{j, k=0, 0}^{\infty, \infty} a_{jk}^{mn}$ *is P-convergent, and*

(RH_6) *there exist positive integers A and B such that* $\sum_{j, k > B} |a_{jk}^{mn}| < A$

(*see Hamilton* [4] *and Robison* [12]).

The core (or K-core) of a real number sequence is defined to be the closed interval $[\liminf x, \limsup x]$. The well-known Knopp core theorem states as follows (see Knopp [5] and Maddox [6]).

Theorem 2.2. *In order that* $L(Ax) \leq L(x)$ *for every bounded sequence* $x = (x_k)$, *it is necessary and sufficient that* $A = (a_{nk})$ *should be regular and* $\sum |a_{nk}| = 1$.

Patterson [10] extended this idea for double sequences by defining the Pringsheim core as follows:

Let $P\text{-}C_n\{x\}$ be the least closed convex set that includes all points x_{jk} for $j, k > n$. Then the Pringsheim core of the double sequence $x = [x_{jk}]$ is the set

$$P\text{-}C\{x\} = \bigcap_{n=1}^{\infty}[P\text{-}C_n\{x\}].$$

Note that the Pringsheim core of a real valued bounded double sequence is the closed interval $[\liminf x, \limsup x]$.

In this regard, Patterson [11] proved the following:

Theorem 2.3. *If A is a four dimensional matrix, then for all real valued double sequences x,*

$$P\text{-}\limsup[Ax] \leq P\text{-}\limsup x \tag{2.2}$$

if and only if

(1) *A is an RH-regular summability matrix, and*

(2) $P\text{-}\lim_{m,n} \sum_{j,k}^{\infty,\infty} |a_{m,n,j,k}| = 1$.

Alotaibi [1] proved the following:

Theorem 2.4. *If the matrix A satisfies* $\|A\| < \infty$, *then K-core* $(Ax) \subseteq C_1(st)$ *for every* $x \in l_\infty$ *if and only if*

(i) *A is regular and* $\lim_m \sum_{K \in E} |a_{nk}| = 0$ *whenever* $\delta(E) = 0$ *for* $E \subseteq N$;

(ii) $\lim_k |a_{nk}| = 1$.

Patterson [10] proved the following:

Theorem 2.5. *If A is a non-negative RH-regular summability matrix, then $P - C\{Ax\} \subseteq P - C(x)$ for any bounded sequence $\{x\}$ for which $\{Ax\}$ exists.*

Lemma 2.1. *If $\{a_{m,n,k,l}\}_{k,l=0,0}^{\infty,\infty}$ is real or complex-valued four dimensional matrix such that (RH_1), (RH_3), (RH_4) and*

$$P\text{-}\limsup_{mn} \sum_{k,l=0,0}^{\infty,\infty} |a_{k,l}^{m,n}| = M$$

hold, then for any bounded double sequence $\{x\}$,

$$P\text{-}\limsup\{|Ax|\} \leq M(P - \limsup\{|x|\}),$$

where

$$y_{mn} = \sum_{k,l=0,0}^{\infty,\infty} a_{m,n,k,l} x_{k,l}.$$

In addition, there exists a real-valued double sequence $\{x\}$ such that if $a_{m,n,k,l}$ is real with $0 < P\text{-}\limsup\{|x|\} < \infty$, then

$$\limsup\{|y|\} = M(P\text{-}\limsup\{|x|\}).$$

3. Main Results

We first define the following:

Definitions 3.1. (i) A double sequence $x = (x_{jk})$ is said to be *lower $C_{1.1}$-statistically bounded* if there exists a constant M such that $\delta\{(j, k): \sigma_{jk} < M\} = 0$, or equivalently, we write $\delta_{C_{11}}\{(j, k): x_{jk} < M\} = 0$.

(ii) A double sequence $x = x_{jk}$ is said to be *upper $C_{1.1}$-statistically bounded* if there exists a constant N such that $\delta\{(j, k): \sigma_{jk} > N\} = 0$, or equivalently, we write $\delta_{C_{11}}\{(j, k): x_{jk} > N\} = 0$.

(iii) If $x = x_{jk}$ is both lower and upper $C_{1.1}$-statistically bounded, we say that $x = (x_{jk})$ is C_{11}-*statistically bounded*, equivalently written x is $c_{1.1}$(st)-bdd.

We denote the set of all c_{11} (st)-bdd sequences by $c_{1.1}(st_{2\infty})$.

Definition 3.2. For $M, N \in \mathbb{R}$, let

$$K_x = \{M : \delta(\{(j, k) : \sigma_{jk} < M\}) = 0\},$$

$$L_x = \{N : \delta(\{(j, k) : \sigma_{jk} > N\}) = 0\}.$$

Then

$$c_{1.1}(st_2)\text{-superior of } x = \inf L_x, \tag{3.1}$$

$$c_{1.1}(st_2)\text{-inferior of } x = \sup K_x. \tag{3.2}$$

Remark

Note that every bounded double sequence is Pringsheim bounded and every Pringsheim bounded double sequence is $C_{1.1}$ statistically bounded but not conversely, in general.

The following is an example of $x = (x_{jk})$ which is neither bounded above nor bounded below, but the Pringsheim limit superior and inferior are both finite numbers:

$$x_{jk} := \begin{cases} j, & \text{if } k = 0, \\ -k, & \text{if } j = 0, \\ (-1)^j, & \text{if } j = k, \\ 0, & \text{otherwise.} \end{cases}$$

Thus $P\text{-}\lim\inf[x] = -1$ and $P\text{-}\lim\sup[x] = 1.$

Now let us define the following notions:

Definition 3.3. The number λ is said to be a $(C, 1.1)$-*statistical limit point* of the double sequence $x = (x_{jk})$ provided that there is a nonthin subsequence $x = \{x_{jk(\tau)}\}_{\tau=1}^{\infty}$, that is, $(C, 1.1)$ summable to λ.

Definition 3.4. The number Γ is said to be a $(C, 1.1)$-*statistical cluster point* of the double sequence $x = (x_{jk})$ provided that for every $\varepsilon > 0$, the set $\{(j, k) \in \mathbb{N} \times \mathbb{N} : |\sigma_{jk} - \Gamma| < \varepsilon\}$ does not have density zero.

Now we adopt the notion: if x is a double sequence such that x_{jk} satisfies property P for all j, k except a set of natural density zero, then we say that x_{jk} satisfies P for "almost all jk" and we write "x_{jk} *satisfies P for a.a.j,k*".

Definition 3.5. For any double sequence $x = (x_{jk})$, let $C_{(xx)}$ denote the collection of all closed convex sets that contain σ_{jk} for almost all j, k. Then the $C_{1.1}$-*statistical core* of x is defined by

$$C_{11}(st_2)\text{-}core(x) = \bigcap_{G \in C_{(x,\, x)}} G.$$

Note that in defining $C_{1.1}(st_2)\text{-}core(x)$, we simply replaced x_{jk} by its $C_{1.1}$-mean in the definition of Pringsheim's Core in the same manner as Alotaibi [1] has defined $C_1(st)\text{-}core(x)$. Hence it follows

$$C_{1.1}(st_2)\text{-}core(x) \subset P\text{-}Core(x). \tag{3.3}$$

It is easy to see that for a bounded double sequence $x = (x_{jk})$,

$$C_{1.1}(st_2)\text{-}core(x) = [P\text{-}\liminf \sigma,\ P\text{-}\limsup \sigma],$$

where $\sigma = \sigma_{mn}$.

Now we prove the following theorems.

Theorem 3.1. *If A is a non-negative RH-regular summability matrix, then*

$$C_{1.1}(st_2)\text{-}core[A\sigma] \subseteq C_{1.1}(st_2)\text{-}core[\sigma]$$

for any bounded $C_{1.1}$*-summable sequence* (x) *for which* $(A\sigma)$ *exists.*

Proof. We have

$$\sigma = \sigma_{mn} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{k=1}^{m} x_{jk}, \tag{3.4}$$

$$A\sigma = A\sigma_{mn} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{k=1}^{m} a_{jk}^{mn} x_{jk}. \tag{3.5}$$

If $C_{1.1}(st_2)\text{-}core(\sigma)$ is the complex plane, then the result is trivial. Now we consider the cases when (x) is bounded or unbounded and establish the required result. In both cases, the result will be established by proving the following:

If there exists a q such that for $\omega \notin C_{1.1}(st_2)\text{-}core_q(\sigma)$, then there exists a p such that $\omega \notin C_{1.1}(st_2)\text{-}core_p(A\sigma)$. When (x) is bounded, $\omega \notin C_{1.1}(st_2)\text{-}core[\sigma]$ is not in the complex plane, thus there exists an $\omega \notin C_{1.1}(st_2)\text{-}core[\sigma]$. This implies that there exists a q for which $\notin C_{1.1}(st_2)\text{-}core_q(\sigma)$. Since ω is finite, we may assume that $\omega = 0$ by the linearity of A. Since we are also given that $C_{1.1}(st_2)\text{-}core_q(\sigma)$ is a convex set, we can rotate $C_{1.1}(st_2)\text{-}core_q(\sigma)$ so that the distance from zero to $C_{1.1}(st_2)\text{-}core_q(\sigma)$ is the minimum of $\{|\sigma| : \sigma \in C_{1.1}(st_2)\text{-}core_q(\sigma)\}$, and is on the positive real axis; say that this minimum is $3d$. Since $C_{1.1}(st_2)\text{-}core_q(\sigma)$ is convex, all points of $C_{1.1}(st_2)\text{-}core_q(\sigma)$ have a real part which is at least $3d$. Let $M = \max\left\{\frac{|x_{jk}|}{mn}\right\}$. By regularity conditions (RH_1) - (RH_4) and the assumption $a_{jk}^{mn} \geq 0$, there exists an N such that for $m, n > N$, the following hold:

$$\sum_{j,k\in B_1} a_{jk}^{mn} < \frac{d}{3M}, \qquad \sum_{j,k\in B_2} a_{jk}^{mn} < \frac{d}{3M},$$

$$\sum_{j,k\in B_3} a_{jk}^{mn} < \frac{d}{3M}, \qquad \sum_{j,k\in B_4} a_{jk}^{mn} < \frac{2}{3},$$

where

$$B_1 = \{(j, k): 0 \le j \le j_0 \text{ and } 0 \le k \le k_0\},$$

$$B_2 = \{(j, k): j_0 \le j < \infty \text{ and } 0 \le k \le k_0\},$$

$$B_3 = \{(j, k): 0 < j \le j_0 \text{ and } k_0 < k < \infty\},$$

$$B_4 = \{(j, k): j_0 < j < \infty \text{ and } k_0 < k < \infty\}.$$

Therefore for $m, n > N$,

$$R\left\{\frac{1}{mn}\sum_{j,k=0,0}^{\infty,\infty} a_{jk}^{mn} x_{jk}\right\}$$

$$= R\left\{\frac{1}{mn}\sum_{j,k\in B_1} a_{jk}^{mn} x_{jk}\right\} + R\left\{\frac{1}{mn}\sum_{j,k\in B_2} a_{jk}^{mn} x_{jk}\right\}$$

$$+ R\left\{\frac{1}{mn}\sum_{j,k\in B_3} a_{jk}^{mn} x_{jk}\right\} + R\left\{\frac{1}{mn}\sum_{j,k\in B_4} a_{jk}^{mn} x_{jk}\right\}$$

$$> -M\left\{\sum_{j,k\in B_1} a_{jk}^{mn}\right\} - M\left\{\sum_{j,k\in B_2} a_{jk}^{mn}\right\} - M\left\{\sum_{j,k\in B_3} a_{jk}^{mn}\right\}$$

$$+ 3d\left\{\sum_{j,k\in B_4} a_{jk}^{mn}\right\}$$

$$> -M\,\frac{3d}{3M} + 2d\,\frac{2}{3} = d.$$

Therefore, $R\{A\sigma\} > d$, which implies that there exists a p for which $\omega = 0$ is also outside $C_{11}(st_2)\text{-}core_p(A\sigma)$. Now suppose that $\{x\}$ is unbounded. Then ω may be the point at infinity or not. If ω is not the point at infinity, then choose N such that for $m, n > N$, the following hold:

$$\left\{\sum_{j,\,k\in B_1} a_{jk}^{mn}\right\} < \frac{d}{3M}, \qquad \sum_{j,\,k\in B_2\cup B_3\cup B_4} a_{jk}^{mn} > \frac{2}{3}.$$

In a manner similar to the first part, we obtain $R\{A\sigma\} > d$. In the case when ω is the point at infinity, $C_{1.1}(st_2)\text{-}core_q(\sigma)$ is bounded for all q, which implies that x_{jk} is bounded for $j, k > q$. We may assume that $[|x|] < B$ for some positive number B without loss of generality. Thus for m and n large, we obtain the following:

$$\left|\sum_{j,\,k=0,\,0}^{\infty,\,\infty} a_{jk}^{mn} x_{jk}\right| \leq \sum_{j,\,k=0,\,0}^{\infty,\,\infty} a_{jk}^{mn} |x_{jk}| \leq B \sum_{j,\,k=0,\,0}^{\infty,\,\infty} a_{jk}^{mn} < \infty.$$

Hence there exists a p such that the point at infinity is outside of $C_{1.1}(st_2)\text{-}core_p(A\sigma)$.

This completes the proof of the theorem.

The following lemma is an analogue of Lemma 2.1 above.

Lemma 3.1. *If* $\{a_{jk}^{mn}\}_{j,\,k=0,\,0}^{\infty,\,\infty}$ *is a four dimensional matrix, such that* (RH_1), (RH_3), (RH_4) *and*

$$P\text{-}\limsup_{mn} \sum_{j,\,k=0,\,0}^{\infty,\,\infty} |a_{jk}^{mn}| = M,$$

hold, then for any bounded $C_{1.1}$*-summable double sequence* $x = (x_{jk})$, *we obtain the following*:

$$P\text{-}\limsup\{A\sigma\} \leq M(P\text{-}\limsup\{\sigma\}),$$

where $\sigma = \sigma_{mn}$ *and* $A\sigma = A\sigma_{mn}$ *are given by* (3.4) *and* (3.5).

In addition, there exists a real valued $C_{1.1}$-summable sequence $\{x\}$ such that, if a_{jk}^{mn} is real with $0 < P\text{-}\limsup\{\sigma\} < \infty$, then

$$P\text{-}\limsup\{|A\sigma|\} = M(P\text{-}\limsup[|A\sigma|]).$$

We shall use the above lemma to prove the following.

Theorem 3.2. *If A is a four dimensional matrix, then the following are equivalent*:

(i) *for all real valued* $C_{1.1}$*-summable sequence* $[x]$, $P\text{-}\limsup\{A\sigma\} \leq P\text{-}\limsup[\sigma]$

(ii) *A is an RH-regular summability matrix with*

$$P\text{-}\lim_{m,n}\sum_{j,k=0,0}^{\infty,\infty}|a_{jk}^{mn}| = 1.$$

Proof. (i) $\Rightarrow$ (ii).

Let $[x]$ be a bounded P-convergent double sequence. Then

$$P\text{-}\liminf[\sigma] = P\text{-}\limsup[\sigma] = P\text{-}\lim[\sigma],$$

and also,

$$P\text{-}\limsup[|A(-\sigma)|] \leq -(P\text{-}\liminf[\sigma]).$$

These imply that

$$P\text{-}\liminf[\sigma] \leq P\text{-}\liminf[A\sigma] \leq P\text{-}\limsup[A\sigma] \leq P\text{-}\limsup[\sigma].$$

Hence $[A\sigma]$ is P-convergent and $P\text{-}\lim[A\sigma] = P\text{-}\lim[\sigma]$. Therefore, A is an *RH*-regular summability matrix. By Lemma 3.1, there exists a bounded $C_{1.1}$-summable double sequence $[\sigma]$ such that $\limsup[|\sigma|] = 1$ and $P\text{-}\limsup[A\sigma] = A$, where A is defined by (RH_6). This implies that

$$1 \leq P\text{-}\liminf_{m,n}\sum_{j,k=0,0}^{\infty,\infty} a_{jk}^{mn} \leq P\text{-}\limsup_{m,n}\sum_{j,k=0,0}^{\infty,\infty} a_{jk}^{mn} \leq 1,$$

whence

$$P\text{-}\lim_{m,n}\sum_{j,k=0,0}^{\infty,\infty}|a_{jk}^{mn}| = 1. \tag{3.6}$$

(ii) $\Rightarrow$ (i).

Here we show that if $\{\sigma\}$ is a P-convergent sequence and A is an RH-regular matrix satisfying (3.6). Then

$$P\text{-}\lim[A\sigma] \leq P\text{-}\lim\sup[\sigma].$$

For $p, q > 1$, we obtain the following:

$$A\sigma \leq \left| \frac{1}{mn} \sum_{j,k=0,0}^{\infty,\infty} a_{jk}^{mn} x_{jk} \right|$$

$$= \frac{1}{mn} \left| \sum_{j,k=0,0}^{\infty,\infty} \frac{|a_{jk}^{mn} x_{jk}| - a_{jk}^{mn} x_{jk}}{2} + \sum_{j,k=0,0}^{\infty,\infty} \frac{|a_{jk}^{mn} x_{jk}| + a_{jk}^{mn} x_{jk}}{2} \right|$$

$$\leq \frac{1}{mn} \sum_{j,k=0,0}^{\infty,\infty} |a_{jk}^{mn}||x_{jk}| + \frac{1}{mn} \sum_{j,k=0,0}^{\infty,\infty} (|a_{jk}^{mn}| - a_{jk}^{mn})|x_{jk}|$$

$$\leq \frac{\|x\|}{mn} \sum_{j,k=0,0}^{p,q} |a_{jk}^{mn}| + \frac{\|x\|}{mn} \sum_{\substack{p<j<\infty \\ 0\leq k\leq q}} |a_{jk}^{mn}| + \frac{\|x\|}{mn} \sum_{\substack{o\leq j<p \\ q<k<\infty}} |a_{jk}^{mn}|$$

$$+ \sup_{j,\, k>p,\, q} \frac{|x|}{mn} \sum_{j,\, k>p,\, q} |a_{jk}^{mn}| + \frac{|x|}{mn} \sum_{j,k=0,0}^{\infty,\infty} (|a_{jk}^{mn}| - a_{jk}^{mn}). \quad (3.7)$$

Using (RH_1)-(RH_4) and (3.6), we take the Pringsheim limit to get the required result.

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