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INTUITIONISTIC FUZZY IRRESOLUTE AND CONTINUOUS MAPPINGS

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Abstract

The notions of intuitionistic irresolute mappings, intuitionistic pre-semi-open mappings, intuitionistic almost-open mappings, intuitionistic weakly continuous mappings, intuitionistic *H*. almost continuous mappings and intuitionistic *W*. almost open mappings are introduced, and their relations are investigated. A characterization of intuitionistic fuzzy irresolute mappings is also given.

1. Introduction

After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov is one among them. Using the notion of intuitionistic fuzzy sets, Çoker [5] introduced the notions of intuitionistic fuzzy topological spaces. In this paper, we introduce the notions of intuitionistic fuzzy irresolute mappings,

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intuitionistic fuzzy pre-semi-open mappings, intuitionistic fuzzy almost-open mappings, intuitionistic fuzzy weakly continuous mappings, intuitionistic fuzzy H. almost continuous mappings and intuitionistic fuzzy W. almost open mappings. We show that (1) every intuitionistic fuzzy irresolute mapping is intuitionistic fuzzy semi-continuous, (2) every intuitionistic fuzzy continuous mapping is an intuitionistic fuzzy weakly continuous mapping, and (3) every intuitionistic fuzzy pre-semi-open mapping is an intuitionistic fuzzy semi-open mapping. We provide a characterization of an intuitionistic fuzzy irresolute mapping and an intuitionistic fuzzy weakly continuous mapping. We give examples to show that an intuitionistic fuzzy H. almost continuous mapping is neither intuitionistic fuzzy weakly continuous nor intuitionistic fuzzy semi-continuous.

2. Preliminaries

Definition 2.1 [1]. An *intuitionistic fuzzy set* (IFS for short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\},$$

where the mappings $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of nonmembership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 [1]. Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle \mid x \in X\}$. Then

- $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
- $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle \mid x \in X\}$,
- $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle \mid x \in X\}$,
- $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, we use the notation $A = \langle X, \mu_A, \gamma_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$. A constant fuzzy set taking value $\alpha \in [0, 1]$ will be denoted by $\underline{\alpha}$. The IFSs $0_\sim$ and $1_\sim$ are defined to be $0_\sim = \langle X, \underline{0}, \underline{1}\rangle$ and $1_\sim = \langle X, \underline{1}, \underline{0}\rangle$, respectively. Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An *intuitionistic fuzzy point* (IFP for short), written $p_{(\alpha, \beta)}$, is defined to be an IFS in X given by

$$p_{(\alpha, \beta)}(x) := \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise,} \end{cases}$$

that is,

$$\mu_{p(\alpha, \beta)}(x) := \begin{cases} \alpha & \text{if } x = p, \\ 0 & \text{otherwise,} \end{cases} \qquad \gamma_{p(\alpha, \beta)}(x) := \begin{cases} \beta & \text{if } x = p, \\ 1 & \text{otherwise.} \end{cases}$$

If $A = \langle X, \mu_A, \gamma_A \rangle$ is an IFS in X and $p_{(\alpha, \beta)}$ is an IFP in X, then $p_{(\alpha, \beta)} \in A$ means $\alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$ for all $x \in X$.

Let f be a mapping from a set X to a set Y. If

$$B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$$

is an IFS in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$$

and the *image* of A under f, denoted by $f(A)$, is an IFS of Y defined by

$$f(A) = \langle Y, f(\mu_A), f(\gamma_A)\rangle,$$

where

$$f(\mu_A)(y) := \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$f(\gamma_A)(y) := \begin{cases} \inf\limits_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise,} \end{cases}$$

for each $y \in Y$. Çoker [5] generalized the concept of fuzzy topological space, first initiated by Chang [4], to the case of intuitionistic fuzzy sets as follows.

Definition 2.3 [5, Definition 3.1]**.** An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms:

- $0_{\sim}, 1_{\sim} \in \tau$,
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- $\bigcup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement $\overline{A}$ of an IFOS A in IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS for short) in X.

Definition 2.4 [5, Definition 3.13]**.** Let (X, τ) be an IFTS and $A = \langle X, \mu_A, \gamma_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* of A are defined by

$$\mathrm{int}(A) = \bigcup\{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$\mathrm{cl}(A) = \bigcap\{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have

$$\mathrm{cl}(\overline{A}) = \overline{\mathrm{int}(A)} \text{ and } \mathrm{int}(\overline{A}) = \overline{\mathrm{cl}(A)}.$$

Definition 2.5 [9]**.** Let f be a mapping from an IFTS (X, τ) to an IFTS (Y, κ). Then f is called

- an *intuitionistic fuzzy open mapping* if $f(A)$ is an IFOS in Y for every IFOS A in X.
- an *intuitionistic fuzzy semiopen mapping* if $f(A)$ is an IFSOS in Y for every IFOS A in X.

Definition 2.6 [6, 7]**.** Let f be a mapping from an IFTS (X, τ) to an IFTS (Y, κ). Then f is said to be

• *intuitionistic fuzzy continuous* if $f^{-1}(B)$ is an IFOS in X for every IFOS B in Y.

• *intuitionistic fuzzy semicontinuous* if $f^{-1}(B)$ is an IFSOS in X for every IFOS B in Y.

• *intuitionistic fuzzy almost continuous* if $f^{-1}(B)$ is an IFOS in X for every IFROS B in Y.

3. Intuitionistic Fuzzy Irresolute and Continuous Mappings

Definition 3.1. Let A be an IFS in an IFTS (X, τ). Then the *intuitionistic fuzzy semi-interior* and *intuitionistic fuzzy semi-closure* of A are defined by

$$\operatorname{int}_s(A) = \bigcup\{G \mid G \text{ is an IFSOS in } X \text{ and } G \subseteq A\},$$

$$\operatorname{cl}_s(A) = \bigcap\{K \mid K \text{ is an IFSCS in } X \text{ and } A \subseteq K\},$$

respectively.

Example 3.2. Let $X = \{x, y\}$ and $\tau = \{0_\sim, 1_\sim, G\}$, where

$$G = \left\langle X, \left(\frac{x}{0.2}, \frac{y}{0.3}\right), \left(\frac{x}{0.7}, \frac{y}{0.7}\right)\right\rangle.$$

Then (X, τ) is an IFTS and

$$\text{IFSOS}(X) = \{0_\sim, 1_\sim, A_{x,y}^{(l_1, m_1), (l_2, m_2)} \mid l_1, m_1 \in [0.2, 0.7],$$
$$l_2, m_2 \in [0.3, 0.7], l_i + m_i \leq 1, i = 1, 2\},$$

where $A_{x,y}^{(l_1, m_1), (l_2, m_2)} = \left\langle X, \left(\frac{x}{l_1}, \frac{y}{l_2}\right), \left(\frac{x}{m_1}, \frac{y}{m_2}\right)\right\rangle$. Taking an IFS

$$B = \left\langle X, \left(\frac{x}{0.4}, \frac{y}{0.7}\right), \left(\frac{x}{0.6}, \frac{y}{0.1}\right)\right\rangle$$

in X, we get

$$\operatorname{int}_s(B) = 0_\sim \cup \{A_{x,y}^{(\alpha_1, \beta_1), (\alpha_2, \beta_2)} \mid \alpha_1 \in [0.2, 0.4], \alpha_2 \in [0.3, 0.7],$$
$$\beta_1 \in [0.6, 0.7], \beta_2 \in [0.3, 0.7], \alpha_i + \beta_i \leq 1, i = 1, 2\}$$

$$= \left\langle X, \left(\frac{x}{0.4}, \frac{y}{0.7}\right), \left(\frac{x}{0.6}, \frac{y}{0.3}\right)\right\rangle \subseteq B.$$

Note that $1_\sim$ is the only IFSCS in X containing B. Thus $\mathrm{cl}_s(B) = 1_\sim$.

Obviously, $\mathrm{cl}_s(A)$ is the smallest IFSCS which contains A, and $\mathrm{int}_s(A)$ is the largest IFSOS which is contained in A. Furthermore, we have

$$A \subset \mathrm{cl}_s(A) \subset \mathrm{cl}(A) \text{ and } \mathrm{int}(A) \subset \mathrm{int}_s(A) \subset A.$$

If A and B are IFSs in an IFTS (X, τ), then

- A is intuitionistic fuzzy semi-open if and only if $A = \mathrm{int}_s(A)$.
- A is intuitionistic fuzzy semi-closed if and only if $A = \mathrm{cl}_s(A)$.
- $A \subset B \Rightarrow \mathrm{cl}_s(A) \subset \mathrm{cl}_s(B), \mathrm{int}_s(A) \subset \mathrm{int}_s(B)$.

Definition 3.3. Let f be a mapping from an IFTS (X, τ) to an IFTS (Y, κ). Then f is called

- an *intuitionistic fuzzy irresolute mapping* if $f^{-1}(B)$ is an IFSOS in X for every IFSOS B in Y.
- an *intuitionistic fuzzy pre-semi-open mapping* if $f(A)$ is an IFSOS in Y for every IFSOS A in X.
- an *intuitionistic fuzzy almost-open mapping* if $f(A)$ is an IFOS in Y for every IFROS A in X.

Example 3.4. (1) Let $X = \{x, y, z\}$, $Y = \{a, b\}$ and

$$G = \left\langle X, \left(\frac{x}{0.5}, \frac{y}{0.7}, \frac{z}{0.7}\right), \left(\frac{x}{0.5}, \frac{y}{0.1}, \frac{z}{0.1}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{a}{0.5}, \frac{b}{0.7}\right), \left(\frac{a}{0.5}, \frac{b}{0.1}\right)\right\rangle.$$

Then $\tau = \{0_\sim, 1_\sim, G\}$ and $\kappa = \{0_\sim, 1_\sim, H\}$ are IFTs on X and Y, respectively. It is easy to check that a mapping $f : X \to Y$ defined by $f(x) = a$ and $f(y) = f(z) = b$ is an intuitionistic fuzzy irresolute mapping.

(2) Let $X = \{x, y\}$, $Y = \{a, b\}$ and

$$G = \left\langle X, \left(\frac{x}{0.3}, \frac{y}{0.3}\right), \left(\frac{x}{0.7}, \frac{y}{0.7}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{a}{0.7}, \frac{b}{0.7}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}\right)\right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, G\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, H\}$ are IFTs on X and Y, respectively, and a mapping $f : X \to Y$ defined by $f(x) = a$ and $f(y) = b$ is not an intuitionistic fuzzy irresolute mapping. Because,

$$B = \left\langle Y, \left(\frac{a}{0.8}, \frac{b}{0.7}\right), \left(\frac{a}{0.1}, \frac{b}{0.1}\right)\right\rangle$$

is an IFSOS in Y, but

$$f^{-1}(B) = \left\langle X, \left(\frac{x}{0.8}, \frac{y}{0.7}\right), \left(\frac{x}{0.1}, \frac{y}{0.1}\right)\right\rangle \not\subseteq \overline{G} = \mathrm{cl}(\mathrm{int}(f^{-1}(B))).$$

(3) Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and

$$G = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.1}, \frac{c}{0.1}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{0}, \frac{y}{0.2}, \frac{z}{0.1}\right), \left(\frac{x}{1}, \frac{y}{0.1}, \frac{z}{0.1}\right)\right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, G\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, H\}$ are IFTs on X and Y, respectively. Let $f : X \to Y$ be defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$. Then f is an intuitionistic fuzzy pre-semi-open mapping.

(4) Let $X = \{a, b, c\}$, $Y = \{x, y\}$ and

$$G = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{0}, \frac{y}{0.4}\right), \left(\frac{x}{1}, \frac{y}{0.6}\right)\right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, G\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, H\}$ are IFTs on X and Y, respectively. It is routine to check that a mapping $f : X \to Y$ given by $f(a) = x$ and $f(b) = f(c) = y$ is an intuitionistic fuzzy almost-open mapping.

In general, an intuitionistic fuzzy irresolute mapping may not be intuitionistic fuzzy continuous. For example, the mapping f in Example 3.4 (2) is not intuitionistic fuzzy continuous. But, we have the following result.

Theorem 3.5. *Let* (X, τ) *and* (Y, κ) *be IFTSs. If a map* $f : X \to Y$ *is intuitionistic fuzzy irresolute, then* f *is intuitionistic fuzzy semi-continuous.*

Proof. Let G be an IFOS in Y. Then G is an IFSOS in Y. Since f is intuitionistic fuzzy irresolute, it follows that $f^{-1}(G)$ is an IFSOS in X. Hence f is intuitionistic fuzzy semi-continuous.

The converse of Theorem 3.5 is not true in general as is seen in the following example.

Example 3.6. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and

$$G = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{1}, \frac{y}{0.4}, \frac{z}{0.4}\right), \left(\frac{x}{0}, \frac{y}{0.6}, \frac{z}{0.6}\right)\right\rangle.$$

Then $\tau = \{0_\sim, 1_\sim, G\}$ and $\kappa = \{0_\sim, 1_\sim, H\}$ are IFTs on X and Y, respectively. Let $f : X \to Y$ be defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$. Then f is intuitionistic fuzzy semi-continuous, but it is not intuitionistic fuzzy irresolute since

$$V = \left\langle Y, \left(\frac{x}{1}, \frac{y}{0.7}, \frac{z}{0.7}\right), \left(\frac{x}{0}, \frac{y}{0.2}, \frac{z}{0.3}\right)\right\rangle \in \text{IFSOS}(Y),$$

but

$$f^{-1}(V) = \left\langle X, \left(\frac{a}{1}, \frac{b}{0.7}, \frac{c}{0.7}\right), \left(\frac{a}{0}, \frac{b}{0.2}, \frac{c}{0.3}\right)\right\rangle \notin \text{IFSOS}(X).$$

Lemma 3.7. *Let* A *be an IFS in an IFTS* (X, τ). *Then* A *is an intuitionistic fuzzy semi-open set if and only if for every IFP* $p_{(\alpha,\beta)}$ *which is contained in* A, *there exists* $V_{p_{(\alpha,\beta)}} \in IFSOS(X)$ *such that* $p_{(\alpha,\beta)} \in V_{p_{(\alpha,\beta)}} \subseteq A$.

Proof. If A is an intuitionistic fuzzy semi-open set, then we may take $V_{p_{(\alpha,\beta)}} = A$ for every $p_{(\alpha,\beta)} \in A$. Conversely, we have

$$A = \bigcup_{p_{(\alpha,\beta)} \in A} \{p_{(\alpha,\beta)}\} \subseteq \bigcup_{p_{(\alpha,\beta)} \in A} V_{p_{(\alpha,\beta)}} \subseteq A$$

and so $A = \bigcup_{p_{(\alpha,\beta)} \in A} V_{p_{(\alpha,\beta)}}$. This shows that A is an intuitionistic fuzzy semi-open set.

Theorem 3.8. *Let f be a mapping from an IFTS* (X, τ) *into an IFTS* (Y, κ). *Then f is intuitionistic fuzzy irresolute if and only if for every IFP* $p_{(\alpha,\beta)}$ *in X and for every* $V \in IFSOS(Y)$ *such that* $f(p_{(\alpha,\beta)}) \in V$, *there exists* $V^* \in IFSOS(X)$ *such that* $p_{(\alpha,\beta)} \in V^* \subseteq f^{-1}(V)$.

Proof. Assume that f is intuitionistic fuzzy irresolute. Let $p_{(\alpha,\beta)}$ be an IFP in X and let $V \in \text{IFSOS}(Y)$ such that $f(p_{(\alpha,\beta)}) \in V$. Since f is intuitionistic fuzzy irresolute, we have $f^{-1}(V) \in \text{IFSOS}(X)$ and $p_{(\alpha,\beta)} \in f^{-1}(V) = V^* \subseteq f^{-1}(V)$. Conversely, let $V \in \text{IFSOS}(Y)$ and $p_{(\alpha,\beta)} \in f^{-1}(V)$. Then $f(p_{(\alpha,\beta)}) \in f(f^{-1}(V)) \subseteq V$. It follows from the hypothesis that there exists $V^* \in \text{IFSOS}(X)$ such that $p_{(\alpha,\beta)} \in V^* \subseteq f^{-1}(V)$ and hence from Lemma 3.7 that $f^{-1}(V)$ is an intuitionistic fuzzy semi-open set. Thus f is an intuitionistic fuzzy irresolute mapping.

The following example shows that an intuitionistic fuzzy continuous mapping need not be an intuitionistic fuzzy irresolute mapping.

Example 3.9. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and

$$G = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.8}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{0}, \frac{y}{0.2}, \frac{z}{0.7}\right), \left(\frac{x}{0.8}, \frac{y}{0.6}, \frac{z}{0.2}\right)\right\rangle.$$

Consider IFTSs $\tau_1 = \{0_\sim, 1_\sim, G\}$ and $\kappa_1 = \{0_\sim, 1_\sim, H\}$ in X and Y respectively. It is easy to check that a mapping $f : X \to Y$ defined by $f(a) = x$ and $f(b) = f(c) = y$ is an intuitionistic fuzzy continuous mapping. But it is not an intuitionistic fuzzy irresolute mapping since

$$A = \left\langle Y, \left(\frac{x}{0.1}, \frac{y}{0.9}, \frac{z}{0.8}\right), \left(\frac{x}{0.7}, \frac{y}{0.1}, \frac{z}{0.1}\right)\right\rangle \in \mathrm{IFSOS}(Y),$$

$$f^{-1}(A) = \left\langle X, \left(\frac{a}{0.1}, \frac{b}{0.9}, \frac{c}{0.9}\right), \left(\frac{a}{0.7}, \frac{b}{0.1}, \frac{c}{0.1}\right)\right\rangle \notin \mathrm{IFSOS}(X).$$

The following example shows that an intuitionistic fuzzy irresolute mapping is neither an intuitionistic fuzzy weakly continuous mapping nor an intuitionistic fuzzy almost continuous mapping.

Example 3.10. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and

$$G = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{0}, \frac{y}{0.4}, \frac{z}{0.2}\right), \left(\frac{x}{1}, \frac{y}{0.6}, \frac{z}{0.6}\right)\right\rangle.$$

Consider IFTSs $\tau_1 = \{0_\sim, 1_\sim, G\}$ and $\kappa_1 = \{0_\sim, 1_\sim, H\}$ in X and Y respectively. Let $f : X \to Y$ be a mapping defined by $f(a) = x$, $f(b) = y$, and $f(c) = z$. Then f is an intuitionistic fuzzy irresolute mapping. But f is not an intuitionistic fuzzy weakly continuous mapping since

$$f^{-1}(H) = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle \not\subseteq \mathrm{int}(f^{-1}(\mathrm{cl}(H))).$$

Note that H is an IFROS in Y, but $f^{-1}(H)$ is not an IFOS in X. Hence f is not an intuitionistic fuzzy almost continuous mapping.

Definition 3.11. Let (X, τ) and (Y, κ) be IFTSs. A map $f : X \to Y$ is said to be *intuitionistic fuzzy weakly continuous* if it satisfies:

$$(\forall G \in \mathrm{IFOS}(Y))(f^{-1}(G) \subseteq \mathrm{int}(f^{-1}(\mathrm{cl}(G)))).$$

Example 3.12. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and

$$G = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{0}, \frac{y}{0.4}, \frac{z}{0.2}\right), \left(\frac{x}{1}, \frac{y}{0.6}, \frac{z}{0.7}\right)\right\rangle.$$

Then $\tau = \{0_\sim, 1_\sim, G\}$ and $\kappa = \{0_\sim, 1_\sim, H\}$ are IFTs on X and Y, respectively. Let $f : X \to Y$ be defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$. Then f is an intuitionistic fuzzy weakly continuous mapping.

Example 3.13. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and

$$G = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{0}, \frac{y}{0.4}, \frac{z}{0.2}\right), \left(\frac{x}{1}, \frac{y}{0.5}, \frac{z}{0.5}\right)\right\rangle.$$

Then $\tau = \{0_\sim, 1_\sim, G\}$ and $\kappa = \{0_\sim, 1_\sim, H\}$ are IFTs on X and Y, respectively. Define a mapping $f : X \to Y$ by $f(a) = x$, $f(b) = y$ and $f(c) = z$. Then f is intuitionistic fuzzy irresolute, but it is not an intuitionistic fuzzy weakly continuous mapping since $f^{-1}(H) \not\subseteq \operatorname{int}(f^{-1}(\operatorname{cl}(H)))$.

Theorem 3.14. *Let* (X, τ) *and* (Y, κ) *be IFTSs. A mapping* $f : X \to Y$ *is intuitionistic fuzzy weakly continuous if and only if it satisfies*:

$$(\forall B \in \mathrm{IFCS}(Y))(\operatorname{cl}(f^{-1}(\operatorname{int}(B))) \subseteq f^{-1}(B)).$$

Proof. Assume that $f : X \to Y$ is intuitionistic fuzzy weakly continuous and let $B \in \mathrm{IFCS}(Y)$. Then $\overline{B} \in \mathrm{IFOS}(Y)$, and so $f^{-1}(\overline{B}) \subseteq \operatorname{int}(f^{-1}(\operatorname{cl}(\overline{B})))$. It follows that

$$\overline{f^{-1}(B)} \subseteq \operatorname{int}\overline{(f^{-1}(\operatorname{int}(B)))} = \overline{\operatorname{cl}(f^{-1}(\operatorname{int}(B)))}$$

so that $\operatorname{cl}(f^{-1}(\operatorname{int}(B))) \subseteq f^{-1}(B)$. Similarly, we have the converse.

Theorem 3.15. *Every intuitionistic fuzzy continuous map is an intuitionistic fuzzy weakly continuous map.*

Proof. Let $f : X \to Y$ be an intuitionistic fuzzy continuous map and let G be an IFOS in Y. Then $f^{-1}(G)$ is an IFOS in X because f is intuitionistic fuzzy continuous, and $G \subseteq \mathrm{cl}(G)$. It follows that $f^{-1}(G) \subseteq f^{-1}(\mathrm{cl}(G))$ and $f^{-1}(G)$ is an IFOS in X so that $f^{-1}(G) \subseteq \mathrm{int}(f^{-1}(\mathrm{cl}(G)))$. Therefore f is an intuitionistic fuzzy weakly continuous map.

The converse of Theorem 3.15 is not true in general as is seen in the following example.

Example 3.16. The intuitionistic fuzzy weakly continuous mapping f in Example 3.12 is not an intuitionistic fuzzy continuous mapping since

$$f^{-1}(H) = \left\langle X, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.7}\right)\right\rangle \notin \mathrm{IFOS}(X).$$

Theorem 3.17. *Every intuitionistic fuzzy pre-semi-open map is an intuitionistic fuzzy semi-open map.*

Proof. Let $f : (X, \tau) \to (Y, \kappa)$ be an intuitionistic fuzzy pre-semi-open map and let G be an IFOS in X. Then G is an IFSOS in X (see [9]). Since f is an intuitionistic fuzzy pre-semi-open map, it follows that $f(G)$ is an IFSOS in Y so that f is an intuitionistic fuzzy semi-open map.

The following example shows that the converse of Theorem 3.17 may not be true.

Example 3.18. Let $X = \{x, y, z\}$, $Y = \{a, b, c\}$ and

$$G = \left\langle X, \left(\frac{x}{1}, \frac{y}{0.4}, \frac{z}{0.2}\right), \left(\frac{x}{0}, \frac{y}{0.6}, \frac{z}{0.6}\right)\right\rangle,$$
$$H = \left\langle Y, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.2}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle.$$

Consider IFTSs $\tau_1 = \{0_\sim, 1_\sim, G\}$ and $\kappa_1 = \{0_\sim, 1_\sim, H\}$ in X and Y respectively. Let $f : X \to Y$ be a mapping defined by $f(x) = a$, $f(y) = b$ and $f(z) = c$. Then f is an intuitionistic fuzzy semi-open mapping. Note that

$$\mathrm{IFSOS}(X) = \{0_\sim, 1_\sim, A_{x,y,z}^{(1,0),(l_1,m_1),(l_2,m_2)} \mid l_1 \in [0.4, 1], l_2 \in [0.2, 1],$$
$$m_1, m_2 \in [0, 0.6]\},$$

$$\mathrm{IFSOS}(Y) = \{0_\sim, 1_\sim, B_{a,b,c}^{(\alpha_1,\beta_1),(\alpha_2,\beta_2),(\alpha_3,\beta_3)} \mid \alpha_1, \beta_1 \in [0, 1],$$

$$\alpha_2, \beta_2 \in [0.4, 0.6], \alpha_3, \beta_3 \in [0.2, 0.6]\},$$

where

$$A_{x,y,z}^{(1,0),(l_1,m_1),(l_2,m_2)} = \left\langle X, \left(\frac{x}{1}, \frac{y}{l_1}, \frac{z}{l_2}\right), \left(\frac{x}{0}, \frac{y}{m_1}, \frac{z}{m_2}\right)\right\rangle,$$

$$B_{a,b,c}^{(\alpha_1,\beta_1),(\alpha_2,\beta_2),(\alpha_3,\beta_3)} = \left\langle Y, \left(\frac{a}{\alpha_1}, \frac{b}{\alpha_2}, \frac{c}{\alpha_3}\right), \left(\frac{a}{\beta_1}, \frac{b}{\beta_2}, \frac{c}{\beta_3}\right)\right\rangle.$$

For

$$V = \left\langle X, \left(\frac{x}{1}, \frac{y}{0.5}, \frac{z}{0.2}\right), \left(\frac{x}{0}, \frac{y}{0.3}, \frac{z}{0.2}\right)\right\rangle \in \mathrm{IFSOS}(X),$$

we have

$$f(V) = \left\langle Y, \left(\frac{a}{1}, \frac{b}{0.5}, \frac{c}{0.2}\right), \left(\frac{a}{0}, \frac{b}{0.3}, \frac{c}{0.2}\right)\right\rangle \notin \mathrm{IFSOS}(Y).$$

This shows that f is not an intuitionistic fuzzy pre-semi-open map.

Definition 3.19. Let (X, τ) and (Y, κ) be IFTSs. A mapping $f : X \to Y$ is said to be

- *intuitionistic fuzzy H. almost continuous* if it satisfies:

$$(\forall U \in \mathrm{IFOS}(Y))(f^{-1}(U) \subseteq \mathrm{int}(\mathrm{cl}(f^{-1}(U)))).$$

- *intuitionistic fuzzy W. almost open* if it satisfies:

$$(\forall U \in \mathrm{IFOS}(Y))(f^{-1}(\mathrm{cl}(U)) \subseteq \mathrm{cl}(f^{-1}(U))).$$

Note that every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy *H*. almost continuous. The following example shows that an intuitionistic fuzzy *H*. almost continuous mapping may not be intuitionistic fuzzy weakly continuous.

Example 3.20. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and

$$G = \left\langle X, \left(\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{x}{0.1}, \frac{y}{0.2}, \frac{z}{0.1}\right), \left(\frac{x}{0.6}, \frac{y}{0.5}, \frac{z}{0.4}\right)\right\rangle.$$

Then $\tau = \{0_\sim, 1_\sim, G\}$ and $\kappa = \{0_\sim, 1_\sim, H\}$ are IFTs on X and Y, respectively. If we define a mapping $f : X \to Y$ by $f(a) = x$, $f(b) = y$ and $f(c) = z$, then f is intuitionistic fuzzy H. almost continuous but not intuitionistic fuzzy weakly continuous since

$$f^{-1}(H) = \left\langle X, \left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.4}\right)\right\rangle \not\subseteq 0_\sim = \operatorname{int}(f^{-1}(\operatorname{cl}(H))).$$

Example 3.21. An intuitionistic fuzzy H. almost continuous mapping need not be an intuitionistic fuzzy semi-continuous mapping. In fact, the intuitionistic fuzzy H. almost continuous mapping f described in Example 3.20 is not intuitionistic fuzzy semi-continuous since

$$f^{-1}(H) = \left\langle X, \left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.4}\right)\right\rangle \notin \mathrm{IFSOS}(X).$$

Example 3.22. An intuitionistic fuzzy semi-continuous mapping need not be an intuitionistic fuzzy H. almost continuous mapping. In fact, let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$, and

$$G = \left\langle X, \left(\frac{x}{0.1}, \frac{y}{0.3}, \frac{z}{0.4}\right), \left(\frac{x}{0.7}, \frac{y}{0.6}, \frac{z}{0.6}\right)\right\rangle,$$

$$H = \left\langle Y, \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle.$$

Then $\tau = \{0_\sim, 1_\sim, G\}$ and $\kappa = \{0_\sim, 1_\sim, H\}$ are IFTs on X and Y, respectively. We know that

$$\mathrm{IFSOS}(X) = \{0_\sim, 1_\sim, A_{x,y,z}^{(l_1, m_1), (l_2, m_2), (l_3, m_3)} \mid l_1, m_1 \in [0.1, 0.7],$$
$$l_2, m_2 \in [0.3, 0.6], l_3, m_3 \in [0.4, 0.6]\},$$

where

$$A_{x,y,z}^{(l_1, m_1), (l_2, m_2), (l_3, m_3)} = \left\langle X, \left(\frac{x}{l_1}, \frac{y}{l_2}, \frac{z}{l_3}\right), \left(\frac{x}{m_1}, \frac{y}{m_2}, \frac{z}{m_3}\right)\right\rangle.$$

It is easily check that a mapping $f : X \to Y$ defined by $f(x) = a$, $f(y) = b$ and $f(z) = c$ is intuitionistic fuzzy semi-continuous, but not intuitionistic fuzzy H. almost continuous since

$$f^{-1}(H) = \left\langle X, \left(\frac{x}{0.2}, \frac{y}{0.4}, \frac{z}{0.4}\right), \left(\frac{x}{0.3}, \frac{y}{0.6}, \frac{z}{0.6}\right)\right\rangle \not\subseteq G = \operatorname{int}(\operatorname{cl}(f^{-1}(H))).$$

Examples 3.21 and 3.22 establish the following theorem.

Theorem 3.23. *Intuitionistic fuzzy semi-continuity and intuitionistic fuzzy H. almost continuity are independent notions.*

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