Advances and Applications in Statistics
Volume 28, Number 1, 2012, Pages 45-54
Published Online: July 2012
Available online at http://pphmj.com/journals/adas.htm
Published by Pushpa Publishing House, Allahabad, INDIA

RESERVE ESTIMATION

Sutawanir Darwis[1], Agus Yodi Gunawan[2], Asep K. Permadi[3] and Nina Fitriyati[4]

[1]Statistics Research Division
Faculty of Mathematics and Natural Sciences
Bandung Institute of Technology
Indonesia

[2]Industrial and Finance Research Division
Faculty of Mathematics and Natural Sciences
Bandung Institute of Technology
Indonesia

[3]Reservoir Engineering Research Division
Faculty of Mining and Petroleum Engineering
Bandung Institute of Technology
Indonesia

[4]Mathematics Department
Faculty of Sciences and Technology
UIN Syarif Hidayatullah Jakarta
Indonesia

Abstract

Recovery factor is an important aspect in reservoir reserve estimation study. Displacement efficiency studies of particular types of reservoir mechanism are used to determine the recovery factor. The study of

2010 Mathematics Subject Classification: 35A08, 60F05, 62P30.

Keywords and phrases: fluid displacement, efficiency, Bayesian methodology.

Received March 14, 2012

efficiency of water injection is based on fluid displacement front. Some indicators for injection efficiency are defined, i.e., water oil ratio, water cut, breakthrough time, etc. These indicators depend on reservoir parameters. Parameter estimation is an important issue in history matching. Improved forecast using available data, i.e., online updating, is an important issue in reservoir management. This paper aims to develop a Bayesian methodology for estimation of reservoir parameters and its application in reservoir efficiency. The proposed method consists of two steps: forecasting (prior) and updating (posterior). The forecasting step uses the frontal advance model and the updating step consists of weighted average of forecast and prediction error correction. The simulation study shows that the proposed method is able to estimate the parameters sequentially and is used for the prediction of injection efficiency.

Introduction

Reservoir reserve can be estimated from different methods: volumetric, material balance and reservoir simulation. Deterministic method uses a single value for each parameter, stochastic method uses a probability model for each parameter and a simulation is used to generate the reserve distribution. Volumetric oil in place (OIP) is given by

$$\mathrm{OIP} = \frac{7758A}{B_{oi}} \sum_{i=1}^{n} h_i \phi_i (1 - S_{wi}),$$

where OIP is oil in place (stb), A is drainage area (acres), B_{oi} is initial formation volume factor (rb/stb), h_i is individual zone thickness (ft), ϕ_i is porosity (%), and S_{wi} is water saturation (%). El-Khatib [2] used inverse Laplace transform to estimate the hydrocarbon in place. Reserve R is defined by $R = R_f \times N$, where R_f is recovery factor. Recovery factor is determined from displacement efficiency studies of particular types of reservoir mechanism. Reservoir simulation is the technique that applies modeling to analyze the reservoir performance. The outputs from the simulator are reserve estimates, depletion, production forecast, and strategy to optimize the recovery. Reserve estimation carries a lot of uncertainties even when calculated by most skilled estimators, e.g., heterogeneity and anisotropy. In reserve estimation, typical unknown that needed to be estimated are: reservoir volume, porosity, oil saturation, recovery factor, and formation volume factor. In sweep efficiency, the time to water breakthrough at the producer is a function of porosity. It is, therefore, important to develop a methodology to estimate the porosity based on frontal advance study. This paper aims to discuss the recovery factor modeling and porosity estimation for reserve estimation. The indicator is calculated using piston like model for immiscible and convection dispersion model for miscible fluid displacement. The parameters are estimated using Bayesian methodology using the analytical solution as forecast (prior) and observations to update (posterior) the estimate.

Methodology

Fluid displacement requires contact between the displacing fluid and the displaced fluid. The movement of the front between displacing and displaced fluids and the breakthrough time are indicators of efficiency (Fanchi [4]). One of the simplest methods of estimating the advance of a fluid is piston like model. The approach uses fractional flow theory and is based on the assumptions: flow is linear and horizontal, water is injected into an oil reservoir, oil and water are both incompressible, oil and water are immiscible, and gravity and capillarity effects are negligible. The theory of piston like displacement is a simplification of the problem of the displacement of one fluid by another. It assumes that there is no displaced fluid movement behind the front. In practical terms, this implies that the oil saturation behind the front is at its residual value S_{or}, and that the macroscopic displacement efficiency $E_d = (1 - S_{1i} - S_{or})/(1 - S_{1i})$. Consider a linear piston like displacement of a fluid 2 of mobility $\lambda_2 = k_2/\mu_2$ by a fluid 1 of mobility $\lambda_1 = k_1/\mu_1$, corresponding to a mobility ratio $M = \lambda_1/\lambda_2 = k_1\mu_2/k_2\mu_1$, through a porous medium of length X. The pressure difference Δp between the inlet and outlet faces of the medium is held constant. If M is not unity, then velocity of the front would be a function of its position.

Consider the time at which the front is located at a distance x from the inlet. The velocity of the fluids on either side of the front is identical

$$\left(\frac{dp}{dx}\right)_2 = M\left(\frac{dp}{dx}\right)_1.$$

If pressure losses in the zones behind and ahead of the front are Δp_1 and Δp_2, then

$$\Delta p = \Delta p_1 + \Delta p_2, \quad \Delta p_1 = x\left(\frac{dp}{dx}\right)_1, \quad \Delta p_2 = (X - x)\left(\frac{dp}{dx}\right)_2.$$

Solving for $\left(\frac{dp}{dx}\right)_1$,

$$\begin{aligned}\Delta p = \Delta p_1 + \Delta p_2 &= x\left(\frac{dp}{dx}\right)_1 + (X - x)\left(\frac{dp}{dx}\right)_2 \\ &= x\left(\frac{dp}{dx}\right)_1 + (X - x)M\left(\frac{dp}{dx}\right)_1 \\ &= \left(\frac{dp}{dx}\right)_1 (x(1 - M) + XM)\end{aligned}$$

yields

$$\left(\frac{dp}{dx}\right)_1 = \frac{\Delta p}{MX + x(1 - M)}.$$

From Darcy's law

$$v = \frac{dx}{dt} = \frac{u_1}{\phi_D} = -\frac{\lambda_1}{\phi_D}\left(\frac{dp}{dx}\right)_1 = \frac{-\lambda \Delta p}{\phi_D[MX + x(1 - M)]},$$

where $\phi_D = \phi(S_{1M} - S_{1m})$. By integrating, time t taken for the front to travel from 0 to x is

$$t = \frac{\phi_D\left(MXx + \frac{x^2}{2}(1 - M)\right)}{\lambda_1 \Delta p}.$$

Solving $\frac{(1-M)}{2}x^2 + MXx - \frac{\lambda_1 \Delta pt}{\varphi_D} = 0,$

$$x = \frac{-MX + \sqrt{(MX)^2 - 2\lambda_1 \Delta pt[(1-M)/\phi_D]}}{1-M}.$$

If v_0 is the initial velocity of the front (velocity at $x = 0$), then

$$\frac{v}{v_0} = \frac{1}{1 + \frac{x}{X}\left(\frac{1}{M} - 1\right)}.$$

If M is greater than 1, then the velocity of the front increases with x and vice versa. Displacing fluids tend to move faster in zone with higher permeability. Dimensionless position of the saturation front

$$x_{fd}(k, \phi, t) = \frac{\sqrt{m^2 + 2(1-m)\frac{k}{\phi} \times \text{const} \times t} - m}{1-m},$$

where $m = \lambda_w/\lambda_o$ and $\text{const} = \frac{dp \frac{k_{rw0}}{\mu_w}}{\text{len}^2((1-S_{or}) - S_{wr})}$. Effective relative mobility of a layer as a function of front is

$$\lambda_{eff}(xfd) = \begin{cases} \dfrac{1}{\dfrac{xfd}{\lambda_w} + \dfrac{1-xfd}{\lambda_o}} & xfd < 1, \\ \lambda_w & xfd \geq 1. \end{cases}$$

Oil and water rates at the outlet end are

$$q_o(xfd, k) = \begin{cases} \dfrac{k\lambda_{eff}(xfd)dp}{len} & xfd < 1, \\ 0 & xfd \geq 1, \end{cases}$$

$$q_w(xfd, k) = \begin{cases} k\lambda_w\left(\dfrac{dp}{len}\right) & xfd > 1, \\ 0 & xfd \leq 1. \end{cases}$$

Total oil and water rates

$$q_{oT}(t,\ k,\ \phi) = q_o(xfd(k,\ \phi,\ t),\ k), \quad q_{wT} = q_w(xfd(k,\ \phi,\ t),\ k),$$

$$q_{Tot} = q_{oT}(t,\ k,\ \phi) + q_{wT}(t,\ k,\ \phi).$$

Water cut is the ratio of water rate to total rate of production

$$wc(t,\ k,\ \phi) = \frac{q_{wT}(t,\ k,\ \phi)}{q_{Tot}(t,\ k,\ \phi)}.$$

In estimating the advance of a fluid displacement front in an immiscible process, the efficiency indicator breakthrough time is defined as

$$t_{bt} = \frac{\phi AL}{q(df_w/dS_w)},$$

where ϕ is the porosity, q is the injection rate, L is the distance from injection to production, and df_w/dS_w is the slope of tangent line. Piston like theory treats the displacement of one fluid by another under immiscible conditions. An immiscible displacement occurs when the displaced and displacing fluid do not mix. In a miscible displacement, the fluids mix and is described by a convection dispersion *CD* equation:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - \nu\frac{\partial C}{\partial x},$$

where C is the concentration of displacing fluid, D is the dispersion, and ν is the velocity. The solution of the one dimensional *CD* equation is

$$C(x,\ t) = \frac{1}{2}\left\{erfc\left(\frac{x - \nu t}{2\sqrt{Dt}}\right) + e^{\nu x/D}erfc\left(\frac{x + \nu t}{2\sqrt{Dt}}\right)\right\},$$

where *erfc* is the complementary error function $erfc(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}dz.$

Oil recovery efficiency is a function of time, pressure, oil rate, gas rate, cumulative oil produced, and cumulative water produced.

Bayesian estimation is an approach for fusing observations with prior knowledge of physical process to obtain an estimate of the distribution of the true state of a process (Wikle and Berliner [6], Evensen [1] and Grewal and Andrews [3]). Let X denote the state of a process and Y be the observation.

Bayes' rule gives $f(x \mid y) = f(x)\, f(y \mid x) / f(y)$, where $f(y \mid x)$ is the distribution of the observations given the state x, $f(x)$ is prior of the state, and $f(x \mid y)$ is the distribution of state given the observations.

One iteration of Bayesian estimation consists of two steps: forecast and updating. Forecast step used the reservoir model to predict the state from time $t-1$ to t; $X_{t,i}^{f} = f(X_{t-1,i}^{u})$, $i = 1, \ldots, N$, N is the ensemble size and f is the reservoir model. In updating step, the state X_t^f is updated by error between prediction and actual measurement. In this paper, the state in estimating permeability k and porosity ϕ based on observations of total flow rate q_{Tot} and water cut wc at the outlet consists of four quantities $X = (\log k \quad \phi \quad q_{Tot} \quad wc)^t$. Porosity usually has a normal distribution while the permeability has a lognormal distribution. At time $t = 0$, log permeability $\log k_0$ and porosity ϕ_0 are generated using sample size of N from normal distributions $N(\mu_{\log k_0}, \sigma^2_{\log k_0})$ and $N(\mu_{\phi_0}, \sigma^2_{\phi_0})$. At time $t_k = k\Delta t$, permeability and porosity follow a random walk process $k_k = k_{k-1}$ and $\phi_k = \phi_{k-1}$. Before reaching an observation time t_{obs}, total flow rate and water cut are calculated based on the formula as function of time, permeability and porosity. The total flow rate and water cut are updated as function of t, however, the log permeability and porosity are constant (not updated). At observation time t_{obs} all four quantities are updated. The state space representation for oil displacement is

$$\begin{cases} \begin{pmatrix} \log k_k \\ \phi_k \\ q_{Tot,k} \\ w_k \end{pmatrix} = \begin{pmatrix} \log k_{k-1} \\ \phi_{k-1} \\ q_{Tot}(t_k, k_k, \phi_k) \\ wc(t_k, k_k, \phi_k) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ w^1_{k-1} \\ w^2_{k-1} \end{pmatrix}, \\ \\ Y_k = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_{Tot,k} \\ wc_k \end{pmatrix} + v_k. \end{cases}$$

Start by defining a time interval from $t = 0$ until T (experiment time).

Inside this interval, simulate a set of equally spaced observations, given $\log k_{true}$ and ϕ_{true}, perturb the observations at observation time t_{obs} with some normal error. The sequential estimation will converge to the true permeability and porosity after a matching of observations (total flow rate and water cut).

Results and Discussion

The estimation of reserve recovery factor is studied through the estimation of porosity and permeability on the basis of a one layer fluid displacement. The model implementation is provided by the solution of the model: water front, flow rates, total flow rate and water cut. Consider a one dimensional displacement with a length of $X = 50\,\mathrm{cm}$. The true parameters are $k_{true} = .242D$ and $\phi_{true} = 26\%$. The cross sectional area is $A = 1\,\mathrm{cm}^2$, oil viscosity .20cp, water viscosity 1, endpoint relative permeability for water $k_{rw0} = .5$, endpoint relative permeability for oil $k_{ro0} = .5$, residual water saturation $s_{wr} = .25$, residual oil saturation $s_{or} = .15$, pressure drop $\Delta p = 10\,\mathrm{atm}$, and mobility $m = 20\%$. Bayesian estimation is used to solve the history matching total flow rate and water cut and obtaining the estimate of permeability and porosity. The algorithm is run from $t = 0$ to 500 days, and the ensemble size $N = 40$. Two observations: total flow rate (with normal error mean 0 and standard deviation .01) and water cut (with normal error mean 0 and standard deviation .1) are used in history matching of oil displacement (Figure 1). The first stage of estimation is to find the best matching for the total flow rate and water cut. In this study, it is easier to match the total flow rate compared to the water cut. The total flow rate is a continuous variable, and water cut is a binary variable. In order to reduce the sum squares of error in history matching total flow rate and water cut, a trial and error was performed on the initial distribution of parameters. The simulation shows that initial distribution contributes significantly on the stability of Bayesian estimation. Figure 2 shows the results of Bayesian total rate and water cut matching and permeability and porosity estimate. The match is good and the estimate tends to converge to the true values.

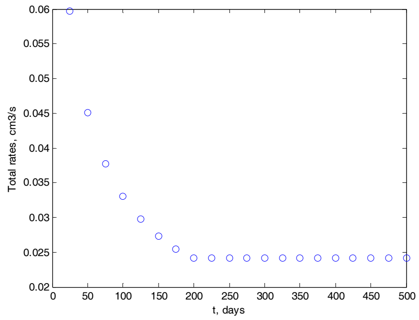

Figure 1. Observed total rate and water cut.

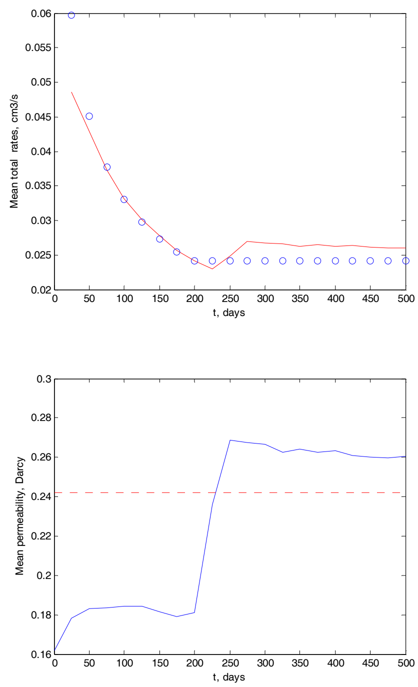

Figure 2. History matching permeability and porosity.

Conclusions

This paper discusses the Bayesian estimation methodology for reserve estimation through the estimation of recovery factor. Recovery was modeled using piston one layer fluid displacement model. The indicators of recovery are oil rate, water cut which is a function of porosity. This paper shows how to estimate the porosity using total rate and water cut measurements based on fluid displacement process. The study shows that the proposed method can be used in recovery factor estimation. Further study is needed to show the applicability of the proposed method in general fluid flow process and reserve estimation.

References

[1] G. Evensen, Data Assimilation: The Ensemble Kalman Filter, Springer, 2007.

[2] N. El-Khatib, Simultaneous estimation of aquifer parameters and original hydrocarbons in place from production data using numerical inversion of Laplace transform, SPE 104603 Middle East Oil and Gas Conference, Bahrain, 11-14 March, 2007.

[3] M. S. Grewal and A. P. Andrews, Kalman Filtering: Theory and Practice Using MATLAB, John Wiley & Sons, 2001.

[4] J. R. Fanchi, Principles of Reservoir Simulation, 2nd ed., Gulf Professional Publishing, 2001.

[5] M, Latil, C. Bardon, J. Burger and P. Sourieau, Enhanced Oil Recovery, Gulf Publishing Company, 1980

[6] C. K. Wikle and L. M. Berliner, A Bayesian tutorial for data assimilation, Physica D: Nonlinear Phenomena 230(1-2) (2007), 1-16.