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# TRANSIENT THERMAL BEHAVIOR OF A THIN SLAB UNDER THE EFFECT OF DUAL-PHASE-LAG CONDUCTION MODEL

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## Abstract

The transient thermal behavior under the effect of the dual-phase-lag heat conduction model using Laplace transformation technique is investigated semi-analytically. The heat transfer mechanism during rapid heating of the slab from a macroscopic point of view using the dual-phase-lag heat conduction model is studied. The slab consists of thin metal layer. The effects of the phase-lag in heat flux vector and temperature gradient on the thermal behavior of the slab are investigated. It is clear that all the three models (Fourier, hyperbolic, and dual-phase-lag) give almost the same predictions since the values of $\tau_q$ and $\tau_T$ are relatively small.

## Nomenclature

| | |
|---|---|
| $L$ | Width of domain, m |
| $c$ | Specific heat capacity, $\mathrm{J.kg^{-1}.K^{-1}}$ |



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| | |
|---|---|
| $k$ | Thermal conductivity, $\text{W.m}^{-1}.\text{K}^{-1}$ |
| $q_0$ | Reference conduction heat flux, $k\Delta T/L$ |
| $S$ | Laplacian domain |
| $t$ | Time, s |
| $t_0$ | Reference time, $\Delta T\rho cL/q_0$ |
| $T$ | Temperature domain 1, K |
| $T_\infty$ | Ambient temperature, K |
| $T_w$ | Wall temperature, K |
| $W$ | Laplace transformation of the dimensionless temperature |
| $x$ | $x$-coordinate |
| $y$ | $y$-coordinate |

**Greek symbols**

| | |
|---|---|
| $\alpha$ | Thermal diffusivity |
| $\eta$ | Dimensionless time, $\frac{t}{t_0}$ |
| $\theta$ | Dimensionless temperature, $\frac{T - T_\infty}{T_w - T_\infty}$ |
| $\xi$ | Dimensionless $x$-coordinate, $\frac{x}{L}$ |
| $\rho$ | Density, $\text{kg.m}^{-3}$ |
| $\tilde{\tau}$ | Thermal relaxation time, s |
| $\tilde{\tau}_q$ | Phase-lag in heat flux vector, s |
| $\tilde{\tau}_T$ | Phase-lag in temperature gradient, s |
| $\tau$ | Dimensionless thermal relaxation time, $\frac{\tilde{\tau}\alpha}{L^2}$ |

## 1. Introduction

Although Fourier's law is appropriate in describing heat conduction in most common engineering situations, however, it breaks down in situations involving very short times, high heat fluxes, and at very low temperatures [1]. The anomaly of this classical theory is from the assumption that the heat flux vector and the temperature gradient across a material volume occur at the same instant of time. Such an immediate response results in an infinite speed of heat propagation. In order to associate a finite heat propagation speed, Cattaneo [2] and Vernotte [3] modified Fourier's law by including a relaxation model that, in parallel to Fourier's law, can be written as [4]

$$q(\xi, \eta + \tau_q) = -k\nabla\theta(\xi, \eta). \tag{1}$$

This equation shows that the temperature gradient $\nabla\theta$ established at a position $\xi$ at time $\eta$ results in a heat flux to flow at the same position but at a different instant of time $\eta + \tau_q$. Physically, $\tau_q$ represents the relaxation time or the phase-lag time between the temperature gradient and the commencement of heat flow in a medium. This modified Fourier's law incorporating with the conservation of energy leads to the wave-based hyperbolic heat conduction equation (HHCE). For some initial or boundary conditions, the HHCE will introduce a sharp wave front in the history of wave propagation, resulting in several physical phenomena which cannot be depicted by diffusion. Comprehensive literature surveys of heat waves until the eighties can be found in the review articles by Joseph and Preziosi [5, 6] and more recently by Ozisik and Tzou [7]. Although the HHCE can solve the paradox of instantaneous response of thermal disturbance, it also introduces some unusual behaviors [8] and physically impossible solutions [9, 10]. Instead of the precedence assumption in equation (1), assuming the lead of the temperature gradient to the heat flux, a more general model, the dual-phase-lag (DPL) model, was proposed by Tzou [4, 11, 12]. This model allows either the temperature gradient to precede the heat flux or the heat flux to precede the temperature gradient. Mathematically, the constitutive law for DPL is represented by

$$q(\xi,\ \eta + \tau_q) = -k\nabla\theta(\xi,\ \eta + \tau_T), \tag{2}$$

where $\tau_T$ is the phase-lag of the temperature gradient. Ever since its agreement with experimental results was shown [13], the DPL model has attracted a considerable interest in the fundamental transport process of heat and mass including, for example, thermal stresses of thin plate [14], lagging behavior of heat transport in amorphous materials [15], semi-infinite slab with surface heat flux [16], non-equilibrium entropy production [17, 18], thermalization and relaxation during short-time transient in microscale [19, 20], temperature-dependent thermal lagging under ultrafast laser heating [21], and, more recently, the growth of interfacial phase compound in metal matrix composites as well as in thin films [22-24]. In this exposition, we shall study the propagation of an ultrashort pulsed energy across the solid-solid interface of dissimilar material layers. With the advent of modern laser with ultrashort pulse duration, picosecond or femtosecond, the ultrafast heat transport process has become an important problem with practical importance [25]. Many interesting phenomena and unusual results regarding energy transport at interface of dissimilar materials have been explored [26-29]. Most of them, however, were within the framework of HHCE, that is, attentions were mainly on the effect of $\tau_q$. No researches, to authors' best knowledge, had focused on the effect of $\tau_T$ on the energy disposition at layer interface. Mathematically, the DPL model introduces additional high-order, mixed spatial and time derivative terms in the governing equation as well as in boundary condition at the layer. This study, therefore, mainly focuses on the effect of $\tau_T$ on the fundamental nature of heat transfer at a thin slab layer within the DPL heat conduction model.

## 2. Case Study

Consider a slab that consists of $0 \le x \le L$ as illustrated in Figure 1. Let $k$ be the thermal conductivity, and $\alpha$ be the thermal diffusivity for the layer. Knowledge of the transient heat conduction in a layer composite thin slab is of importance in a number of different applications such as coating, cladding, foils forming, fabricating of the p-n junctions, semi-conductors and electric chips. Initially, the temperature was 0°C. For $t > 0$, the boundary surface at $x = 0$ is kept insulated and the boundary surface at $x = L$ is kept at $T_w$. The thickness of the layer is assumed to be very small relative to the height of the slab, so it is reasonable to assume that the conducted heat is transferred in the $x$-direction only. In this case, the energy equations are written as

$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial q}{\partial x}, \tag{3}$$

$$q + \tilde{\tau}_q \frac{\partial q_1}{\partial t} = -\kappa \frac{\partial T_1}{\partial x} - \kappa \tilde{\tau}_T \frac{\partial^2 T}{\partial t \partial x}. \tag{4}$$

Combining equations (3) and (4) yields

$$\frac{\partial^2 T}{\partial x^2} + \tilde{\tau}_T \frac{\partial^3 T}{\partial t \partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} + \frac{\tilde{\tau}_q}{\alpha} \frac{\partial^2 T}{\partial t^2}. \tag{5}$$

Initial and boundary conditions:

$$T(0,\ x) = 0,$$

$$\frac{\partial T}{\partial t}(0,\ x) = 0,$$

$$T(t,\ L) = T_w,$$

$$\frac{\partial T}{\partial x}(t,\ 0) = 0$$

using these dimensionless parameters,

$$\xi = \frac{x}{L}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \frac{t}{t_0}, \quad \tau = \frac{\tilde{\tau}\alpha}{L^2}, \quad t_0 = \Delta T \rho c L / q_0.$$

Equations (3) and (4) in dimensionless form become

$$\frac{\partial \theta}{\partial \eta} = -\frac{\partial Q}{\partial \xi} \tag{6}$$

$$Q + \tau_q \frac{\partial Q}{\partial \eta} = -\frac{\partial \theta}{\partial \xi} - \tau_T \frac{\partial \theta}{\partial \eta \partial \xi^2}. \tag{7}$$

Equation (5) is reduced to

$$\frac{\partial^2\theta}{\partial\xi^2} + \tau_T \frac{\partial^3\theta}{\partial\eta\partial\xi^2} = \frac{\partial\theta}{\partial\eta} + \tau_q \frac{\partial^2\theta}{\partial\eta^2}. \tag{8}$$

Initial and boundary conditions:

$$\theta(0, \xi) = 0, \quad \frac{\partial\theta}{\partial\eta}(0, \xi) = 0,$$

$$\theta(\eta, 1) = 1,$$

$$\frac{\partial\theta}{\partial\xi}(\eta, 0) = 0. \tag{9}$$

Now, with the notation that $W(\xi, S) = L\{\theta(\xi, \eta)\}$, Laplace transformation of equations (8) and (9) yields

$$\frac{d^2W}{d\xi^2} + \tau_T\left(S\frac{d^2W}{d\xi^2}\right) = SW + \tau_q S^2 W,$$

$$\frac{d^2W}{d\xi^2} - \left(\frac{S + \tau_q S^2}{1 + \tau_T S}\right)W = 0. \tag{10}$$

Equation (10) assumes the following solution:

$$W = C_1 e^{\xi\lambda} + C_2 e^{-\xi\lambda}, \tag{11}$$

where $\lambda = \sqrt{\left(\frac{S + \tau_q S^2}{1 + \tau_T S}\right)}$.

Also, Laplace transformation of the boundary conditions yields

$$W(S, 1) = 1/S,$$

$$\frac{dW}{\partial\xi}(S, 0) = 0. \tag{12}$$

Insert equation (11) into (12) and solve for $C_1$ and $C_2$ to yield

$$C_1 = C_2 = \frac{\left(\frac{1}{S}\right)}{e^{\lambda} + e^{-\lambda}}.$$

Equation (11) is inverted using a computer program based on Riemann-sum approximation as

$$\theta_k(\xi, \eta) \cong \frac{e^{\gamma\eta}}{\eta}\left[\frac{1}{2}W_k(\xi, \gamma) + \mathrm{Re}\sum_{n=1}^{N} W_k\left(\xi, \gamma + \frac{in\pi}{\eta}\right)(-1)^n\right], \qquad (13)$$

where $k = 1$ for domain (1) and $k = 2$ for domain (2). In equation (13), Re represents the real part of the summation and $i = \sqrt{-1}$. $\gamma\eta = 4.7$ gives the most satisfactory results Tzou [11-13]. Equation (13) yields the exact temperature distribution in the domain for the dual phase heat conduction model case.

## 3. Results and Discussion

Figure 2 shows spatial temperature distribution within the domain using the parabolic and hyperbolic heat conduction models. It is clear from the figure that both models give almost the same predictions since the values of $\tau_q$ are relatively small. This implies that the phase-lag concept has insignificant effect on the predictions of the hyperbolic heat conduction model when $\tau_q$ are relatively small. It is clear that as $\tau_q$ increases, the deviation between both models increases. Spatial temperature distribution within the domain at different phase-lag in temperature gradient using the dual phase heat conduction model is shown in Figure 3. Figure 4 shows transient temperature distribution for parabolic and dual-phase-lag perfect contact heat conduction model in two domains. It is clear that the parabolic and dual-phase-lag gives almost the same predictions for small values of $\tau_T$, also, the deviation between the two models increases as $\tau_T$ increases.

## 4. Conclusion

The thermal behavior of a thin layer under the effect of the dual-phase-lag heat conduction model is investigated. It is clear that all the three models (Fourier, hyperbolic, and dual-phase-lag) give almost the same predictions since the values of $\tau_q$ and $\tau_T$ are relatively small.

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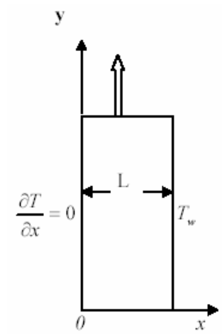


**Figure 1.** Schematic diagram of the problem under consideration.

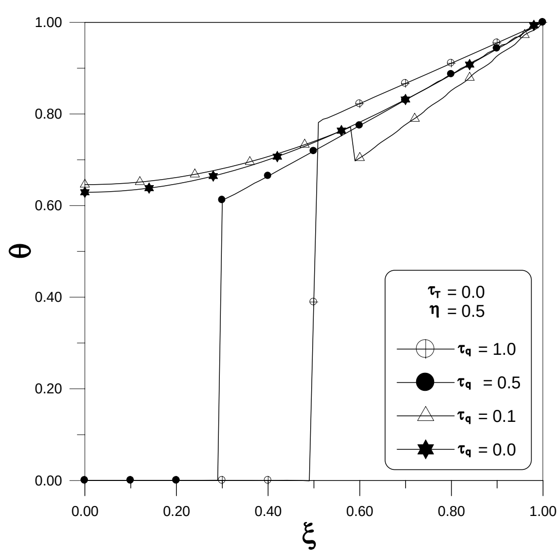


**Figure 2.** Spatial temperature distribution within the domain using the parabolic and hyperbolic heat conduction models.

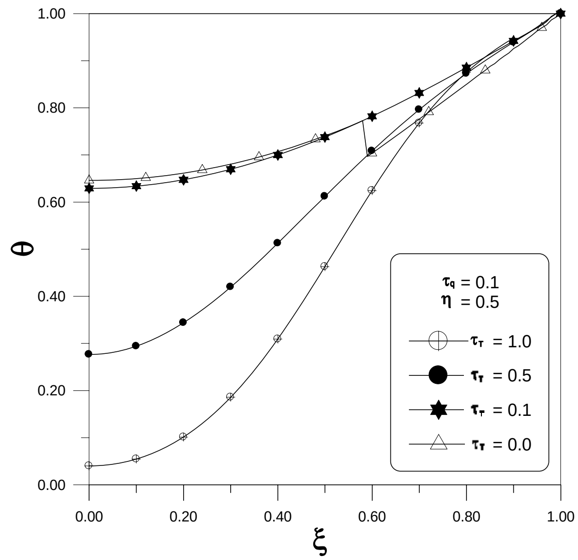


**Figure 3.** Spatial temperature distribution within the domain at different phase-lag in temperature gradient using the dual phase heat conduction model.

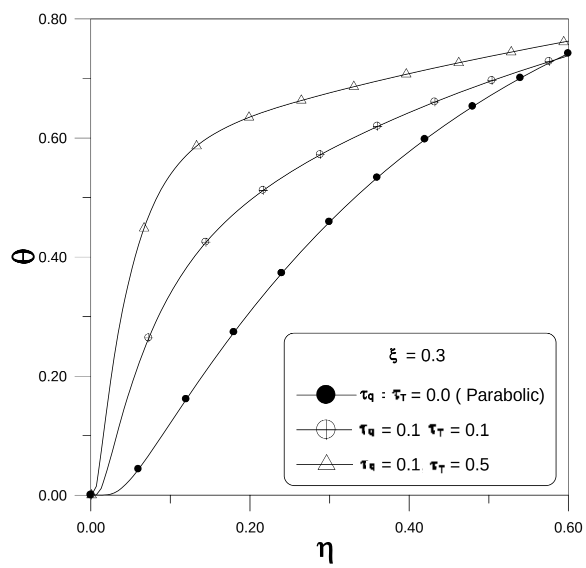


**Figure 4.** Transient temperature distribution for parabolic and dual-phase-lag heat conduction model in the domain.