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A TREATMENT OF THE EMISSION SPECTRUM FOR A MULTI-PHOTON Ξ-TYPE THREE-LEVEL ATOM DRIVEN BY A SINGLE-MODE FIELD WITH NONLINEARITIES

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Abstract

A treatment of a multi-photon Ξ-type three-level atom interacting with a single-mode field in a cavity, taking explicitly the existence of forms of nonlinearities of both the field and the intensity-dependent atom-field coupling into account. Analytical expression of the emission spectrum is presented using the dressed states of the system. The characteristics of the emission spectrum considering the field to be initially in a squeezed coherent state are exhibited. The effects of the photon multiplicities, mean number of photons, detuning and the nonlinearities on the spectrum are investigated.

Keywords and phrases: emission spectrum, multi-photon, nonlinearities.

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1. Introduction

Since the emission spectrum of the cavity-bound atom contains a great amount of information on the radiation-matter interaction [1, 2], it has received a considerable attention over recent years. The emission spectrum for a two-level atom interacting with a single-mode field exhibits phase sensitivity when the atom is initially prepared in a coherent superposition of its two states [1]. For a certain choice of the relative phase, an asymmetric two-peaked spectrum substitutes for the usual symmetrically placed three peaks [1].

The system of a three-level atom in the Λ-configuration interacting with the electromagnetic field has been extensively studied [3, 4] because of its applications in a number of different contexts such as laser cooling [5], lasing without inversion [6] and electromagnetically induced transparency [7, 8]. The emission spectrum of a Λ-type three-level atom with the two almost degenerate lower levels coupled to a single-mode field in an ideal cavity (cavity adsorptively is zero) [9] has been investigated and non-classical effects including the vacuum Rabi splitting and quantum beats have been revealed. It is also pointed out that the four-peaked spectrum evolves into the three-peaked spectrum when the photon number increases to a certain value. Furthermore, the emission spectra for the three-level atom one-mode [9-12] and two-mode [13, 14] model have been studied.

On the other hand, recently, a great deal of activity has centered on the analysis of the physical properties of nonlinear interaction models describing a localized center coupled to the modes of a quantized bosonic field [15, 16]. There are many physical situations where such models may find applications [17, 18]. For example, it may be of interest in the context of the effective Hamiltonian approach to the two-mode two-photon micromaser [19, 20]. Moreover, it is worthwhile to remark that investigating such models goes beyond an intrinsic theoretical interest in condensed matter systems too, because the development of new and improved materials is expected to lead to the fabrication of three dimensional photonic band gap systems possessing few isolated high-Q resonant field modes [21-23]. The physical origin of these nonlinear interactions may be traced back to the existence of a strong coupling between few levels of the material center and some selected modes of the quantized elastic or electromagnetic field. Very often the problem of interest is investigating the effects of nonlinearity on the quantum dynamics of the system starting from appropriately chosen initial conditions. The starting point is the construction of an effective basic Hamiltonian model which contains the essential ingredients of the microscopic physical situation, at the same time, providing us with an exactly solvable model. The introduction of such Hamiltonian is considered in this paper.

In recent years, there has been tremendous progress in the ability to generate states of the electromagnetic field with manifestly quantum or nonclassical characteristics experimentally [24-26]. Squeezed states of light are nonclassical states for which the fluctuations in one of two quadrature phase amplitudes of the electromagnetic field drop below the level of fluctuations associated with the vacuum state of the field. Squeezed states, therefore, provide a field which is, in some sense, quieter than the vacuum state and hence can be employed to improve measurement precision beyond the standard quantum limits.

The goal of this paper is to shed some light on the emission spectrum for a multi-photon three-level system. The model we shall consider is consisting of a single atom interacting with one mode field in a perfect cavity, including acceptable kinds of nonlinearities of both the field and the intensity-dependent atom-field coupling via multi-photon processes. To reach our goal, it will be more convenient to use exact expression for the unitary operator $U(t)$ in the frame of the dressed state formalism. This will be considered in Section 2. In Section 3, we employ the analytical results obtained and by using the finite double-Fourier transform of the two-time field correlation function, we find an analytical expression for the fluorescence spectrum. We devote Section 4 to numerical results and discussion. Finally, the conclusions are summarized in Section 5.

2. Formulation of the Problem

The Hamiltonian of the system in the rotating-wave approximation is of the form $(\hbar = 1)$:

$$H = H_0 + H_{in}, \tag{1}$$

$$H_0 = \sum_{j=1}^{3} \omega_j \sigma_{j,j} + \Omega \hat{a}^+ \hat{a}, \tag{2}$$

where ω_1, ω_2 and ω_3 are the atomic level energies $(\omega_1 > \omega_2 > \omega_3)$ and Ω is the field frequency, with the detuning parameters Δ_1 and Δ_2 given by

$$\Delta_1 = -k\Omega + (\omega_1 - \omega_2),$$
$$\Delta_2 = -k\Omega + (\omega_2 - \omega_3). \tag{3}$$

The operators $\hat{a}$ and $\hat{a}^+$ are the boson operators for the field satisfying $[\hat{a}, \hat{a}^+] = 1,$ while k refers to the photon multiplicity number.

The interaction part of the Hamiltonian for the multi-photon processes and in the presence of an arbitrary nonlinear medium can be written as

$$H_{in} = \Re(\hat{a}^\dagger \hat{a}) + \lambda_1(\sigma_{12} f_1(\hat{a}^\dagger \hat{a})\hat{a}^k + \hat{a}^{+k} f_1(\hat{a}^\dagger \hat{a})\sigma_{21})$$
$$+ \lambda_2(\sigma_{23} f_2(\hat{a}^\dagger \hat{a})\hat{a}^k + \hat{a}^{+k} f_2(\hat{a}^\dagger \hat{a})\sigma_{32}), \tag{4}$$

$\Re(\hat{a}^\dagger \hat{a})$ and $f(\hat{a}^\dagger \hat{a})$ are Hermitian operator functions of photon number operators such that $\lambda_1 f(\hat{a}^\dagger \hat{a})$ and $\lambda_2 f(\hat{a}^\dagger \hat{a})$ represent arbitrary intensity-dependent atom-field couplings, while $\Re(\hat{a}^\dagger \hat{a})$ denotes the one-mode field nonlinearity which can model Kerr-like medium nonlinearity as will be discussed later. The operators σ_{ij} satisfy the following commutation relations: $[\sigma_{ij}, \sigma_{kl}] = \sigma_{il}\delta_{jk} - \sigma_{kj}\delta_{il}$, $[\hat{a}, \sigma_{ij}] = 0$. The initial state $|\Psi(0)_{AF}\rangle$ of the combined atom-field system may be written as

$$|\,\Psi(0)_{AF}\rangle = |\,\Psi(0)_A\rangle \otimes |\,\Psi(0)_F\rangle, \tag{5}$$

where $|\,\Psi(0)_A\rangle = |1\rangle\langle 1|$ is the initial state of the atom and $|\,\Psi(0)_F\rangle = |\Theta\rangle\langle\Theta|$ is the initial state of the field. The initial state $|\Theta\rangle = \sum p^{(n)}|n\rangle$, where the probability amplitude $p^{(n)}$ is defined in the usual manner as $p^{(n)} = \langle n|\Theta\rangle$.

The time evolution between the atom and the field is defined by the unitary evolution operator generated by H. Thus, $U(t)$ is given by:

$$U(t) \equiv \exp(-iHt).$$

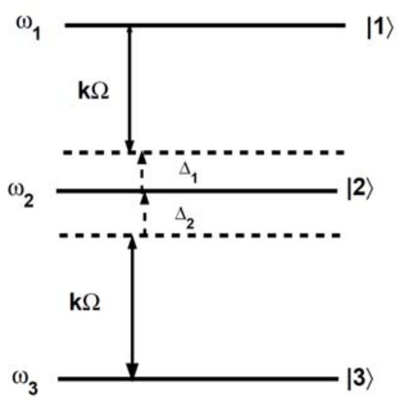

Figure 1. Energy level diagram for a Ξ-type three-level atom with multi-photon detuning Δ_1, Δ_2.

This unitary operator $U(t)$ is written as:

$$\begin{aligned} U(t) = &\sum_{s=0}^{k-1} \exp(-iE_{03}^{(s)}t)|\,\vartheta^{(s)}\rangle\langle\vartheta^{(s)}\,| \\ &+ \sum_{s=0}^{k-1}\sum_{\kappa=1}^{2} \exp(-i\mu_{\kappa}^{(s)}t)|\,\Phi_{\kappa}^{(s)}\rangle\langle\Phi_{\kappa}^{(s)}\,| \\ &+ \sum_{n=0}^{\infty}\sum_{j=1}^{3} \exp(-iE_{j}^{(n)}t)|\,\Psi_{j}^{(n)}\rangle\langle\Psi_{j}^{(n)}\,|, \end{aligned} \tag{6}$$

where the eigenvalues

$$E_{03}^{(s)} = \omega_3 + \Omega s + \Re(s), \quad (s = 0, 1, \ldots, k-1),$$

$$\mu_{1,2}^{(s)} = \frac{R_1 + R_2}{2} \pm \frac{1}{2}\sqrt{(R_1 + R_2)^2 - 4[R_1 R_2 - Q]},$$

$$E_j^{(n)} = -\frac{X_1}{3} + \frac{2}{3}\left(\sqrt{X_1^2 - 3X_2}\right)\cos(\theta_j), \tag{7}$$

where $j = 1, 2, 3$ and

$$\theta_j = \left(\frac{1}{3}\cos^{-1}\left[\frac{9X_1X_2 - 2X_1^3 - 27X_3}{2(X_1^2 - 3X_2)^{\frac{3}{2}}}\right] + (j-1)\frac{2\pi}{3}\right), \tag{8}$$

with

$$R_1 = \omega_2 + \Omega s + \Re(s),$$

$$R_2 = \omega_3 + \Omega(s+k) + \Re(s+k),$$

$$Q = \lambda_2 f_2(s)\sqrt{\frac{(s+k)!}{s!}},$$

$$X_1 = -(r_1 + r_2 + r_3),$$

$$X_2 = -[V_1^2 + V_2^2 - r_1 r_2 - r_1 r_3 - r_2 r_3],$$

$$X_3 = r_3 V_1^2 + r_1 V_2^2 - r_1 r_2 r_3,$$

$$r_1 = \omega_1 + \Omega n + \Re(n),$$

$$r_2 = \omega_2 + \Omega(n+k) + \Re(n+k),$$

$$r_3 = \omega_3 + \Omega(n+2k) + \Re(n+2k),$$

$$V_1 = \lambda_1 f_1(n)\sqrt{\frac{(n+k)!}{n!}},$$

$$V_2 = \lambda_2 f_2(n+k)\sqrt{\frac{(n+2k)!}{(n+k)!}} \tag{9}$$

and $|\,\vartheta^{(s)}\rangle,\ |\,\Phi_{\kappa}^{(s)}\rangle,\ |\,\Psi_{j}^{(n)}\rangle$ are the dressed states of the system associated with the eigenvalues $E_{03}^{(s)},\ \mu_{\kappa}^{(s)}$ and $E_{j}^{(n)}$ $(\kappa = 1, 2,\ j = 1, 2, 3),$ where

$$|\,\vartheta^{(s)}\rangle = |\,s, 3\rangle, \quad (s = 0, 1, ..., k-1),$$

$$|\,\Phi_{\kappa}^{(s)}\rangle = L_{\kappa}^{(s)}|\,s, 2\rangle + q_{\kappa}^{(s)}|\,s+k, 3\rangle,$$

$$|\,\Psi_{j}^{(n)}\rangle = \alpha_{j}^{(n)}|\,n, 1\rangle + \beta_{j}^{(n)}|\,n+k, 2\rangle + \gamma_{j}^{(n)}|\,n+2k, 3\rangle \tag{10}$$

and

$$L_{\kappa}^{(s)} = \frac{-Q}{\sqrt{(R_1 - \mu_{\kappa}^{(s)}) + Q^2}},$$

$$q_{\kappa}^{(s)} = \frac{R_1 - \mu_{\kappa}^{(s)}}{\sqrt{(R_1 - \mu_{\kappa}^{(s)}) + Q^2}}, \tag{11}$$

$$\begin{pmatrix} \alpha_{j}^{(n)} \\ \beta_{j}^{(n)} \\ \gamma_{j}^{(n)} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} -V_1(r_3 - E_{j}^{(n)}) \\ (r_1 - E_{j}^{(n)})(r_3 - E_{j}^{(n)}) \\ -V_2(r_1 - E_{j}^{(n)}) \end{pmatrix}, \tag{12}$$

where

$$M^2 = (r_1 - E_{j}^{(n)})^2(r_3 - E_{j}^{(n)})^2 + V_1^2(r_3 - E_{j}^{(n)})^2 + V_2^2(r_1 - E_{j}^{(n)})^2. \tag{13}$$

Having obtained the explicit form of the unitary operator $U(t)$, the eigenvalues and the eigenfunctions for the system under consideration, we are, therefore, in a position to discuss any property related to the atom or the field.

3. The Physical Transient Spectrum

In this section, we derive the physical transient spectrum $S(\nu)$ by calculating the Fourier transform of the time averaged dipole-dipole correlation function $\langle(\sigma_{12}(t_1)+\sigma_{23}(t_1))(\sigma_{21}(t_2)+\sigma_{32}(t_2))\rangle$. The physical spectrum $S(\nu)$ of radiation field emitted by a cavity-bound atom is given by the expression [10]:

$$S(\nu)=\Gamma\int_0^T dt_1\int_0^T dt_2$$
$$\times\exp[-(\Gamma-i\nu)(T-t_1)-(\Gamma+i\nu)(T-t_2)]$$
$$\times\langle(\sigma_{12}(t_1)+\sigma_{23}(t_1))(\sigma_{21}(t_2)+\sigma_{32}(t_2))\rangle, \tag{14}$$

where T is the interaction time and Γ is the bandwidth of the filter. The Fourier transform of the time averaged dipole-dipole correlation is directly related to the fluorescence spectrum. We shall further analyze the case when the atom is initially prepared in the excited state. After carrying out the various operations, we get

$$S(\nu)=\Gamma\sum_{s=0}^{k-1}\sum_{j=1}^{3}\sum_{\kappa=1}^{2}|p^{(s)}|^2|\alpha_j^{(s)}|^2\Upsilon(\mu_\kappa^{(s)},E_j^{(s)})$$
$$\times[|\alpha_j^{(s)}|^2|L_\kappa^{(s)}|^2+\alpha_j^{*(s)}\beta_j^{(s)}L_\kappa^{(s)}q_\kappa^{*(s)}$$
$$+\alpha_j^{(s)}\beta_j^{*(s)}L_\kappa^{*(s)}q_\kappa^{(s)}+|\beta_j^{(s)}|^2|q_\kappa^{(s)}|^2]$$
$$+\Gamma\sum_{n=k}^{\infty}\sum_{i,j=1}^{3}|p^{(n)}|^2|\alpha_j^{(n)}|^2\Upsilon(E_i^{(n-k)},E_j^{(n)})$$
$$\times[|\alpha_j^{(n)}|^2|\beta_i^{(n-k)}|^2+\alpha_j^{*(n)}\beta_j^{(n)}\beta_i^{(n-k)}\gamma_i^{*(n-k)}$$
$$+\alpha_j^{(n)}\beta_j^{*(n)}\gamma_i^{(n-k)}\beta_i^{*(n-k)}+|\beta_j^{(n)}|^2|\gamma_i^{(n-k)}|^2], \tag{15}$$

where

$$\Upsilon(x,y)=\left[\frac{1+\exp(-2\Gamma T)-2\exp(-\Gamma T)\cos(\nu+x-y)T}{\Gamma^2+(\nu+x-y)^2}\right]. \tag{16}$$

Thus, the time averaged spectrum consists of resonant structures which arise from transitions among different dressed states. The final structure of the time averaged spectrum will depend on the form of the input photon distribution $p^{(n)}$. As the cavity field starts to interact with the atom, the initial photon number distribution $p^{(n)}$ starts to change. Due to the quantum interference between component states, the oscillations in the cavity field become composed of different component states.

4. Numerical Results and Discussion

On the basis of the analytical solution presented above, we shall study numerically the emission spectrum in squeezed coherent states' initial field. The photon number distribution for a squeezed coherent state [27] can be written as

$$|P_n|^2 = \frac{(\tanh r)^n}{2^n n! \cosh r} \left| H_n\left(\frac{\varepsilon}{\sqrt{2\cosh r \sinh r}}\right)\right|^2$$

$$\times \exp[-|\varepsilon|^2 + \tanh r \operatorname{Re}(\varepsilon)^2], \tag{17}$$

where $\varepsilon = \alpha \cosh r + \alpha^* \sinh r$, $\alpha = |\alpha| \exp(i\varsigma)$ and H_n is the Hermite polynomial. We suppose here the minor axis of the ellipse, representing the direction of squeezing, parallel to the coordinate of the field oscillator. The initial phase ς of α is the angle between the direction of coherent excitation and the direction of squeezing. The mean photon number of this field is equal to $\bar{n} = |\alpha|^2 + \sinh^2 r$. Putting $r = 0$, we get the photon distribution for an initial coherent state with $\bar{n} = |\alpha|^2$ whereas for $\alpha = 0$ the photon distribution for an initial squeezed vacuum state with $\bar{n} = \sinh^2 r$ is recovered. The latter distribution is oscillatory with zeros for odd *n*.

By the means of quantum beats and the dressed atom states, we know that when the number *n* changes by one unit, this coupling describes the spontaneous emission of one photon with a frequency close to Ω. Also, there are only six possible allowed spontaneous decays from the three perturbed states of $E^{(n)}$ to lower multiplicities [10, 28]. The two components located at $(\nu + E_3^{(n-k)} - E_1^{(n)})$ and $(\nu + E_3^{(n-k)} - E_2^{(n)})$ join the two components located at $(\nu + E_2^{(n-k)} - E_3^{(n)})$ and $(\nu + E_1^{(n-k)} - E_3^{(n)})$, respectively, to form the two side-band peaks, so that the fluorescence spectrum consists of four peaks, the total splitting being mainly determined by the Rabi nutation frequencies characterizing the coupling of the field with the two transitions [29].

In general, these four peaks are asymmetric, because the intensities of the symmetrically placed side-bands are not equal. When field is strong and exactly resonant (at resonance, the field intensity is, therefore, supposed to be sufficiently high to saturate the atomic transition), the spectra become symmetrical [30, 31]. The peak height is related to the photon number $\bar{n}$ and atom-field coupling constant $\lambda_{1(2)}$. The peak position is associated with not only the photon number multiplicity and the intensity-dependent atom-field coupling constant $\lambda_i f_i$, but also the frequency Ω of the cavity field and the distribution of the photons in the field and their characteristics.

4.1. Effect of multiplicity and mean photon number

In Figure 2, we plot the resonance fluorescence spectrum $S(\nu)$ which describes the spectrum of the light emitted by Ξ-type three-level atom considering the field to be initially in squeezed coherent state, for different multi-photon processes $(k = 1, 2, 3, 4)$ and various values of the coherent parameter $|\alpha|$, the horizontal axis indicates scaled frequency $(\nu - k\Omega)/\sqrt{\lambda_1\lambda_2}$. For small value of a multi-photon process $k = 1$, it is clear that Rabi peaks dominate the spectrum for small mean photon number $\bar{n}$. Since the two peaks located at $(\nu + E_1^{(n-k)} - E_1^{(n)})$ and $(\nu + E_2^{(n-k)} - E_2^{(n)})$ coalesce into each other strongly but not totally to give a double peak at Ω, so there is a central structure containing a deep gap surrounded by two symmetric spikes emerging at the center as shown in Figure 2 (Frame (1)(a)). As we move away from the center, two symmetric side-bands, however, with structure appear. This is due to the two components located at $(\nu + E_3^{(n-k)} - E_1^{(n)})$ and $(\nu + E_3^{(n-k)} - E_2^{(n)})$ joining the two components located at $(\nu + E_2^{(n-k)} - E_3^{(n)})$ and $(\nu + E_1^{(n-k)} - E_3^{(n)})$, respectively, to form the two side-bands where small peaks due to the quantum beats emerge and modify the wings. With the increase in the photon number $\bar{n}$ of the cavity field, more states would contribute to the spectrum and the effect of the vacuum state diminishes. As long as the mean photon number and variance increase, the two wings become two sidebands around Ω and they not only move away from the central spikes and their structure start to wash out, but they are also symmetrically placed because the dressed states in the triplet are equally spaced, see Figure 2 (Frame (1)(b)). As the observation region is elongated by the increase in the mean photon number, all components gain height and become narrower, while the depth and width between the central spikes decrease because the central two peaks coalesce into each other. So, for sufficiently large values of the mean photon number, the two central spikes coalesce into each other fully, then give a single peak at the center leading to a three peak structure [10, 29]. But we have dramatic changes in the spectrum as the photon multiplicities increase $(k = 2, 3, 4)$, where central structure peak disappears completely, and there is a deep gap surrounded by two symmetric spikes' group emerging around the central frequency, which move away from the central frequency as the mean photon number and photon multiplicity increase, see Figure 2 (Frames (2), (3) and (4)). This due to the spectrum components mentioned above are separated and spread out to form a number of symmetric individual peaks. As the observation region is elongated by the increase in multiplicity and the mean photon number, while the number of peaks decreases and all components become narrower.

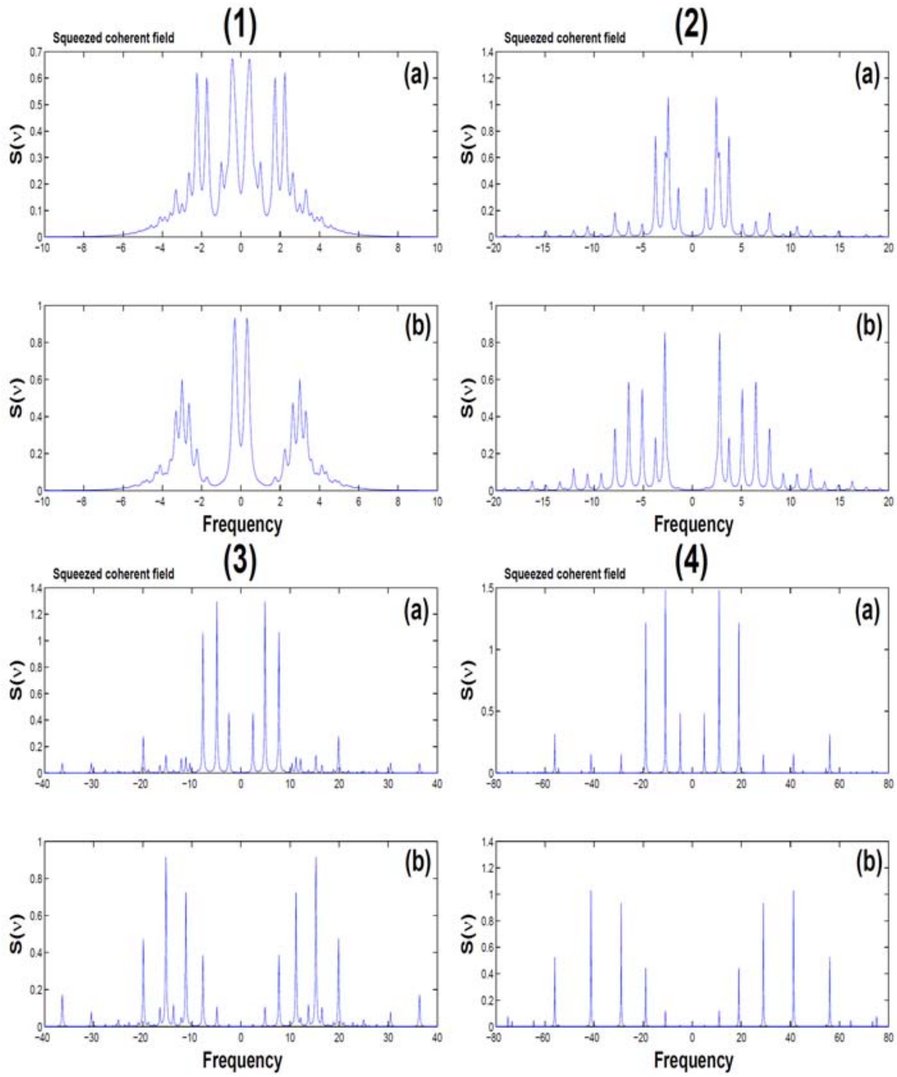

Figure 2. The evolution of the function $S(\nu)$ in a perfect cavity as a function of $(\nu - k\Omega)/\sqrt{\lambda_1\lambda_2}$ with $\lambda_{1,2} = 1$, $\Delta_{1,2} = 0$, $\chi = 0$, $\Gamma = 0.1$, $\varsigma = 0$, $T = 100$, $f_{1,2} = 1$ and $r = 1.1$, (a) $\alpha = \sqrt{1}$, (b) $\alpha = \sqrt{3}$, for all (1) $k = 1$, (2) $k = 2$, (3) $k = 3$, (4) $k = 4$.

4.2. Effect of detuning

Now, we would like to shed some light on the spectrum behavior when the detuning parameters (Δ_1, Δ_2) differ from zero. The Rabi splitting in the non-resonant case not only depends on n and the intensity-dependent coupling as in the resonant case, but also on the detuning parameters, so that, if the frequency of the applied field is detuned from the atomic resonance frequency $(\text{i.e., } \Delta_i \neq 0),$ then the populations of the dressed states are unequal. Thus, the transient spectrum under these conditions can be asymmetric and ultimately the central component and one of the Rabi side-bands can vanish despite the fact that the quadrature-noise spectrum exhibits a significant amount of noise at these frequencies. The asymmetry arises from the stimulated emission induced between the dressed states by the squeezed coherent field. These asymmetries consist of an enhanced sideband on the atomic resonance frequency side of the central frequency and more pronounced oscillations between the central frequency and the enhanced sideband than on the opposite side of the central frequency. Also, one can see that the large value of the detuning parameter results in the disappearance of one peak and large displacement in the location of the maximum value of the spectrum. Finally, we can conclude that detuning adds asymmetry to the spectrum and increase of the heights of the sidebands at the right hand side as we compare the frames of the present figure and Figure 2 (Frame (b)) where the case of absence of detuning is considered. The sideband on the left hand side is suppressed, while the sideband on the right hand side gains height and becomes narrower. Also, the shape of the spectrum is changed on both the sides of the central line until the one on the left almost disappears as shown in Figure 3 (Frame (b)). These phenomena disappear obviously as the value of a multi-photon process increases $(k > 3)$ see Figure 3 (Frame (4)), while at $k = 4$ detuning almost not affects the spectra, and the symmetric spikes still emerge in the two sides of the spectrum. Also, the height of peaks decreases as the values of k increase except $k = 4$ comparing Frames (1), (2) and (3) with Frame (4) in Figure 3.

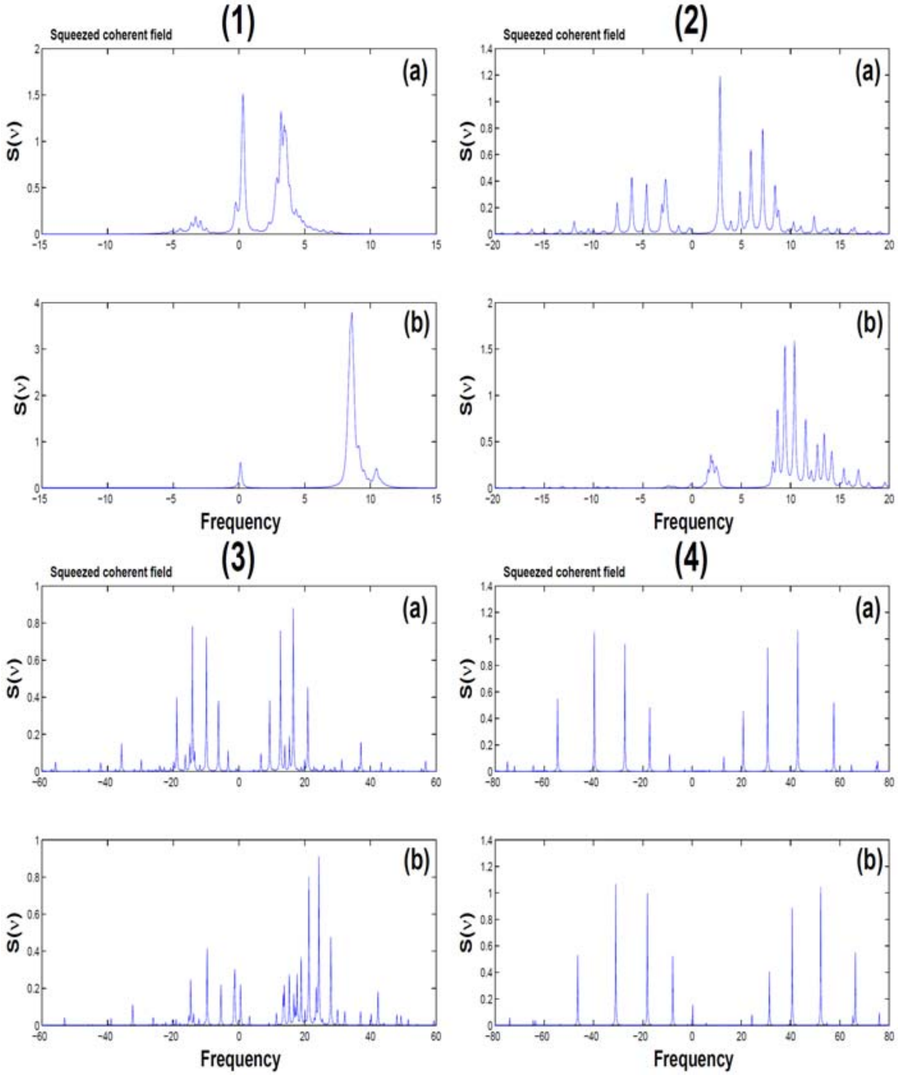

Figure 3. The evolution of the function $S(\nu)$ in a perfect cavity as a function of $(\nu - k\Omega)/\sqrt{\lambda_1\lambda_2}$ with $\lambda_{1,2} = 1,$ $\chi = 0,$ $\Gamma = 0.1,$ $\varsigma = 0,$ $f_{1,2} = 1,$ $T = 100,$ $\alpha = \sqrt{3},$ $r = 1.1$ and (1) $k = 1,$ (2) $k = 2,$ (3) $k = 3,$ (4) $k = 4,$ for all (a) $\Delta_1 = 1,$ $\Delta_2 = 2,$ (b) $\Delta_1 = 8,$ $\Delta_2 = 10.$

4.3. Effect of Kerr medium

Now, we will turn our attention to the effect on the spectrum $S(\nu)$ of the nonlinearity of the field with a Kerr-type medium due to the term $\Re(n)$ being taken in the form $\chi n(n-1)$, where χ is related to the third-order nonlinear susceptibility. In fact, the optical Kerr effect is one of the most extensively studied phenomena in the field of nonlinear optics because of its applications [32-34]. In general, addition of the Kerr-like medium parameter to the problem adds asymmetry to the spectrum and changes the shape of the spectrum. The peaks are redistributed on both sides of the spectrum. Also, the right hand sideband is suppressed while the left hand sideband gains height and becomes narrower gradually as the values of χ increased, see Figure 4. As χ increases, the Kerr-like medium becomes dominant and the atom-field terms become almost ineffective as has been noticed as the two level system by [35]. This can be traced in the dependence of the Kerr term on the second order of the photon numbers which becomes the dominating factor compared with the atom-field term which depends on the square root of these numbers. Therefore, the spectrum becomes essentially the elements due to the Kerr-like medium. Numerical calculation shows that when we take $\chi = 0.8$ the term $\left(\frac{-X_1}{3}\right)$ is almost the effective term in equation (7) which defines the eigenvalues of the system. That means the three eigenvalues close to each other and we finish with an almost single-peak around the central frequency. So that the right sideband peaks almost disappear for the large values of χ. Also, we observe that all those phenomena take place in a fast way as photon multiplicity number k decreases (compare Frames (1)-(4) in Figure 4). So, for large values of k such as $k = 4$ we still note asymmetric peaks are redistributed in both sides of the central line even if for the large values of χ, see Figure 4 (Frame (4)(b)). Finally, we may conclude that the effect of the nonlinear medium on the spectrum of the emitted light is the shift of the spectrum to the left (which is opposite to the effect of detuning) and changing the amplitudes of the peaks depending on the values of χ and k. Also, the maximum height of the peaks increases as χ and k increase.

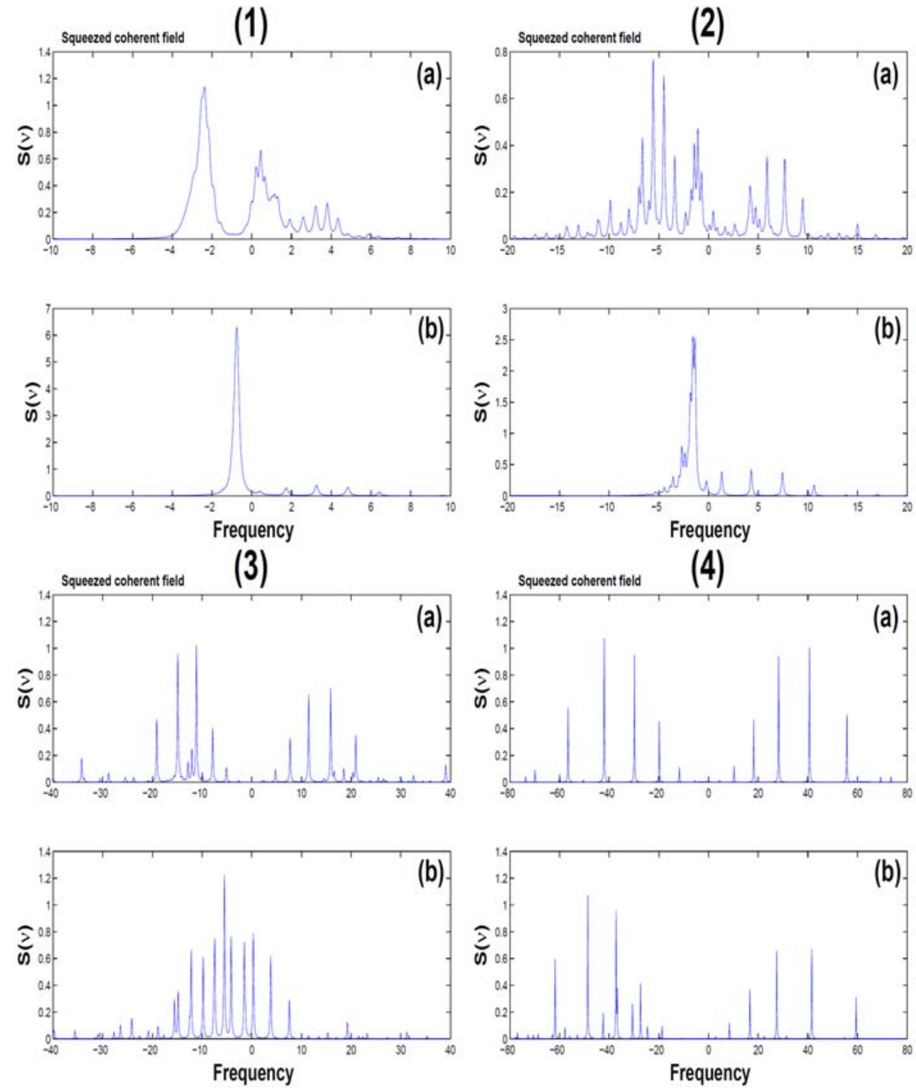

Figure 4. The evolution of the function $S(\nu)$ in a perfect cavity as a function of $(\nu - k\Omega)/\sqrt{\lambda_1\lambda_2}$ with $\lambda_{1,2} = 1$, $\Delta_{1,2} = 0$, $\Gamma = 0.1$, $\varsigma = 0$, $f_{1,2} = 1$, $T = 100$, $\alpha = \sqrt{3}$, $r = 1.1$, and for all (a) $\chi = 0.1$, (b) $\chi = 0.8$ with (1) $k = 1$, (2) $k = 2$, (3) $k = 3$, (4) $k = 4$.

4.4. Effect of intensity-dependent coupling functional

In Figure 5, we study the effect of different functionals of the intensity dependent coupling $f_1(n)$, $f_2(n+k)$ on the resonance fluorescence spectrum $S(\nu)$. Generally, we can observe that the structure shown in Figure 2(b) is no longer evident here. When we take $f_1(n) = \sqrt{n}$, $f_2(n+k) = \sqrt{n+k}$ the stepwise excitation becomes larger than that for $f_1(n) = f_2(n+k) = 1$. Hence the range of the spectrum is extended and the vacuum Rabi splitting appears distinctively more dominant compared with the case of $f_1(n) = f_2(n+k) = 1$. Also, the smaller peaks do not merge in sidebands but show up markedly with their heights subsiding as they spread out. Also, in Figure 5 (Frame (1)(a)), note the appearance of two wall separated central spikes surrounded by two symmetric groups of small spikes, with their heights having a maximum for the small middle spikes. By increasing k (i.e., $k = 2, 3, 4$) we note two symmetric groups of spikes with different heights around the center. Also, the number of these spikes decreases as the values of k increase, see Figures 5 (Frames (2)(a), (3)(a) and (4)(a)). Changing the coupling functional results in a radical change in the spectrum. When we consider $f_1(n) = \frac{1}{\sqrt{n}}$, $f_2(n+1) = \frac{1}{\sqrt{n+k}}$ see Figure 5 (Frame (1)(b)), here only three-peak structure is strikingly apparent. For $k = 2$ we note a deep gap between two higher wall spikes at the center surrounded by two lower sidebands. The number of spikes increases while the maximum heights of these spikes decrease as k increases, see Figure 5 (Frame (c)). While for $k = 3, 4$ we note that the spikes become closer to the center, and the two wall spikes nearest to the center become smaller, while the other sideband spikes gain height.

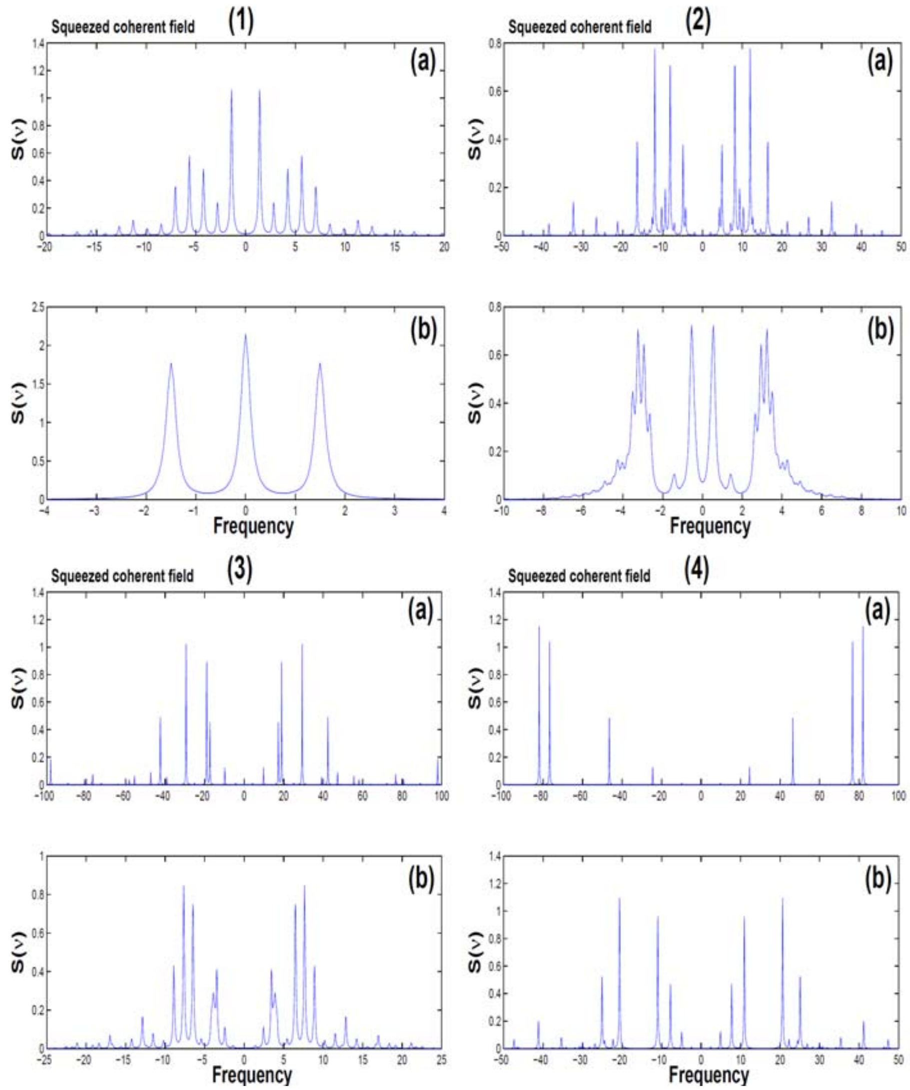

Figure 5. The evolution of the function $S(\nu)$ in a perfect cavity as a function of $(\nu - k\Omega)/\sqrt{\lambda_1\lambda_2}$ with $\lambda_{1,2} = 1$, $\Delta_{1,2} = 0$, $\chi = 0$, $\Gamma = 0.1$, $\varsigma = 0$, $T = 100$, $\alpha = \sqrt{3}$, $r = 1.1$, (1) $k = 1$, (2) $k = 2$, (3) $k = 3$, (4) $k = 4$, and for all (a) $f_1(n) = \sqrt{n}$, $f_2(n+k) = \sqrt{n+k}$, (b) $f_1(n) = \dfrac{1}{\sqrt{n+k}}$, $f_2(n+k) = \dfrac{1}{\sqrt{n+k}}$.

5. Conclusion

In summary, we have investigated the emission spectrum for a general formalism for a multi-photon Ξ-type three-level atom, taking into account arbitrary forms of nonlinearities of both the field and the intensity-dependent atom-field coupling. We have explored the influence of various parameters of the system on the emission spectrum. We have used the finite double-Fourier transform of the two-time field correlation function to find an analytical expression for the spectra. The spectrum in the cavity is calculated for the field to be initially in a squeezed coherent state. It is observed:

• Generally the spectrum of Ξ-type three-level atom for the cavity field has four-peak structure. But for sufficiently large values of the mean photon number, the spectrum tends to a three peak structure.

• The peak position is associated with not only the photon number $(\bar{n})$ and the photon multiplicity number k but also the intensity-dependent atom-field coupling constant $\lambda_i f_i(n)$.

• The heights of the spectrum components becomes shorter and the distances between them is larger as the mean photon number increased.

• The symmetry shown in the standard three-level atom model for the spectra no longer present, once Kerr effect or detuning is considered.

• The effect of detuning on the spectrum of the emitted light is twofold: The first effect is the shift of the spectrum to the right hand side. The second effect is the dependence of the amplitudes and heights of the peaks on Δ_i.

• The Kerr medium has an effect opposite to the effect of the detuning, where the earlier has shorter elements. Also, the heights and widths of the peaks not only depend on the photon multiplicity but also on the value of χ. Consequently, changes in the detuning and the Kerr medium parameters can show in the spectra, and hence the heights of the peaks, their shifts and widths are altered compared the case of resonance.

• The strong field effects can be produced by choosing the right parameters for these nonlinearities.

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