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MHD AND THERMAL RADIATION OF AN OPTICALLY THIN GRAY FLUID IN THE PRESENCE OF AN INDUCED MAGNETIC FIELD

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Abstract

The aim of this work is the study of the MHD unsteady flow of an incompressible viscous and radiative, optically thin gray fluid past an infinitely vertical plate moving with constant velocity. The magnetic Reynolds number is not small, so that the induced magnetic field is present. The partial differential equations (PDEs) and their initial and boundary conditions, describing the problem under consideration, are dimensionalized and the numerical solution is obtained by using the finite volume method. The numerical results for the velocity, temperature and the induced magnetic field are shown in figures for different parameters of the problem under consideration, such as the

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magnetic parameter, M, the radiation parameter, R, the magnetic Prandtl parameter, P_m and the Grashof number, Gr. The analysis of these results showed that the flow, temperature and induced magnetic fields are noticeable influenced by these parameters. This study will bring out insight in the problem of an optically gray thin fluid flow under the influence of radiation in the presence of an induced magnetic field.

1. Introduction

The flow of an electrically conducting and viscous fluid in the presence of free convection has been extensively studied by many authors. On the other hand, the effects of radiation at high temperatures of an optically thin gray fluid are not fully documented. Such theoretical research in the area could provide a first insight in very significant applications in space technology, space vehicle re-entry, nuclear engineering and several other industrial areas.

England and Emery [1] have studied the radiation effects of an optically thin gray and viscous fluid bounded by a stationary plate. Raptis and Perdikis [2] investigated the unsteady flow under the radiation effect of a thin gray fluid over a moving vertical plate. The influence of the radiation and mass transfer on a thin gray and viscous fluid has been studied by Muthucumarawamy and Chandrakala [3]. The radiation effects on flow past an impulsively started vertical plate with variable temperature and mass flux was studied by Muthucumarawamy and Vijayalakshimi [4].

In the case of an electrically conducting fluid under the influence of a magnetic field the magnetic Reynolds number of the flow is very small $(\mathrm{Re}_m \ll 1)$ and the induced magnetic field is considered negligible.

Magnetohydrodynamics (MHD) and radiation effects on moving isothermal vertical plate with variable mass diffusion of a thin gray fluid was investigated by Muthucumarawamy and Janakiraman [5]. Rajesh [6] has studied the radiation effects of a thin gray fluid on MHD free convective flow near a vertical plate with ramped wall temperature under small magnetic

Reynolds number. Rajput and Kumar [7] have investigated the rotation and radiation effects on MHD flow of a thin gray fluid past an impulsively started vertical plate with variable temperature. All the above authors have studied the MHD case under the assumption that the induced magnetic field is negligible.

The purpose of this study is the investigation of the MHD unsteady flow of an incompressible viscous and radiative, optically thin gray fluid past an infinitely vertical plate moving with constant velocity when the magnetic Reynolds number is not small, so that the induced magnetic field is present.

2. Mathematical Analysis

We consider the unsteady flow of an incompressible and viscous fluid occupying a semi-infinite region of the space bounded by an infinite plate moving with constant velocity. The x'-axis is taken along the plate in the vertical upward direction and the y'-axis normal to the plate, see Figure 1. The magnetic Reynolds number of the flow is not considered small and the induced magnetic field is given by a PDE [8]. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium and an electrically conducting one. It is assumed that the magnetic field in the region of the plate is of the form $\vec{H}' = (H'_{x'}, H_0^*, 0)$, where H_0^* is the y'-component of the magnetic field and it is constant. The radiation heat flux in the x'-direction is considered negligible in comparison to the y'-direction. All the fluid properties are considered constant except the influence of the density variation in the body force term.

Under the above assumptions the flow is governed by the following equations:

Equation of motion

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{\mu_0}{\rho} H_0^* \frac{\partial H'_{x'}}{\partial y'}, \tag{1}$$

Equation of the induced magnetic field

$$\frac{\partial H'_{x'}}{\partial t'} = H_0^* \frac{\partial u'}{\partial y'} + \frac{1}{\sigma\mu_0} \frac{\partial^2 H'_{x'}}{\partial y'^2}, \tag{2}$$

Energy equation

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'}, \tag{3}$$

where u' is the velocity component at the x'-direction, t' is the time, ν is the kinematic viscosity, g is the acceleration due to gravity, β is the coefficient of the volume expansion, μ_0 is the magnetic permeability, σ is electrical conductivity, T' is fluid temperature, T'_∞ is fluid temperature at infinity, c_p is specific heat at constant pressure, k is thermal conductivity and q_r is radiative heat flux.

The initial and the boundary conditions are

$$t' = 0: \quad u'(y') = 0, \;\; T'(y') = T'_\infty, \;\; H'(y') = 0, \tag{4}$$

$$t' > 0: \quad \begin{cases} u' = u_0, \;\; T' = T'_w, \;\; H'_{x'} = c^* H_0^* \;\; \text{at} \;\; y' = 0, \\ u' \to 0, \;\; T' \to T'_\infty, \;\; H'_{x'} \to 0 \;\; \text{as} \;\; y' \to \infty, \end{cases} \tag{5}$$

where u_0 is the velocity of the plate, T'_w is the temperature of the plate and c^* is a constant.

For the case of an optically gray fluid [1]-[7], the local radiant is expressed by

$$\frac{\partial q_r}{\partial y'} = -4\alpha^* \sigma^* (T'^4_\infty - T'^4), \tag{6}$$

where α^* is the absorption coefficient of the fluid and σ^* is the Stefan-Boltzman constant.

We assume that the temperature differences within the flow are sufficiently small that T'^4 can be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'^4_∞ and neglecting higher order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty. \tag{7}$$

By using equations (6) and (7), equation (3) gives

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\alpha^*\sigma^* T'^3_\infty (T'_\infty - T')}{\rho c_p}. \tag{8}$$

We introduce the dimensionless quantities

$$\begin{aligned}
&y = \frac{y' u_0}{\nu}, \quad \text{(distance)}\\
&t = \frac{t' u_0^2}{4\nu}, \quad \text{(time)}\\
&u = \frac{u'}{u_0}, \quad \text{(velocity)}\\
&\Theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \text{(temperature)}\\
&H = \left(\frac{\mu_0}{\rho}\right)^{1/2} \frac{H'_{x'}}{u_0}, \quad \text{(magnetic field)}\\
&Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3}, \quad \text{(Grashof number)}\\
&\Pr = \frac{\rho\nu c_p}{k}, \quad \text{(Prandtl number)}\\
&P_m = \nu\sigma\mu_0, \quad \text{(magnetic Prandtl number)}\\
&M = \left(\frac{\mu_0}{\rho}\right)^{1/2} \frac{H_0^*}{u_0}, \quad \text{(magnetic parameter)}\\
&R = \frac{16\alpha^*\sigma^* T'^3_\infty \nu^2}{k u_0^2}, \quad \text{(radiation parameter).}
\end{aligned} \tag{9}$$

Using the dimensionless quantities in (9), equations (1), (2) and (8) become respectively:

$$\frac{\partial u}{\partial t} = 4Gr\Theta + 4\frac{\partial^2 u}{\partial y^2} + 4M\frac{\partial H}{\partial y}, \tag{10}$$

$$\frac{\partial H}{\partial t} = 4M\frac{\partial u}{\partial y} + \frac{4}{P_m}\frac{\partial^2 H}{\partial y^2}, \tag{11}$$

$$\frac{\partial \Theta}{\partial t} = \frac{4}{\Pr}\frac{\partial^2 \Theta}{\partial y^2} - \frac{4R}{\Pr}\Theta. \tag{12}$$

The initial and boundary conditions in dimensionless form are as follows:

$$t = 0: \quad u(y) = 0, \quad \Theta(y) = 0, \quad H(y) = 0, \tag{13}$$

$$t > 0: \quad \begin{cases} u = 1, \quad \Theta = 1, \quad H = c \quad \text{at} \quad y = 0, \\ u \to 0, \quad \Theta \to 0, \quad H \to 0 \quad \text{as} \quad y \to \infty, \end{cases} \tag{14}$$

where $c = \frac{c^* H_0^*}{u_0}\left(\frac{\mu_0}{\rho}\right)^{1/2}$.

3. Numerical Solution

For the numerical solution of the system of equations (10)-(12) subject to the initial and boundary conditions (13) and (14) an efficient technique is developed based on the trust-region dogleg method. The algorithm is a variant of the Powell dogleg method described in [9, 10]. In this technique, the finite volume method on a collocated grid was used for discretizing the coupled set of PDEs. Due to the nature of the equations we assume that x and z are considered unity. Grid and time independence studies were performed to establish that the results are not time and space dependent. Δy was considered to be equal to 0.05 and Δt was considered to be equal to 0.005 in all presented results.

In the developed numerical method a guess-correct philosophy that gradually improves the guessed solution by repeated use of the discrete governing equations was employed [11]. When the governing equations are non-linear, it is preferable to treat each equation considering it to have one unknown, temporarily treating other variables as known using the current values. The discretized form of the system of equations (10)-(12) is given below:

$$\left(\frac{\Delta y}{\Delta t}+\frac{8}{\Delta y}\right)u_p-\frac{4}{\Delta y}u_{p+1}-\frac{4}{\Delta y}u_{p-1}$$

$$=(4Gr\Delta y)\Theta_p+2M(H_{p+1}-H_{p-1})+\frac{\Delta y}{\Delta t}u_p^0, \tag{15}$$

$$\left(\frac{\Delta y}{\Delta t}+\frac{8}{\mathrm{P}_m\Delta y}\right)H_p-\frac{4}{\mathrm{P}_m\Delta y}H_{p+1}-\frac{4}{\mathrm{P}_m\Delta y}H_{p-1}$$

$$=4M(u_{p+1}-u_{p-1})+\frac{\Delta y}{\Delta t}H_p^0, \tag{16}$$

$$\left(\frac{\Delta y}{\Delta t}+\frac{8}{\Pr\Delta y}+\frac{4R}{\Pr}\right)\Theta_p-\frac{4}{\Pr\Delta y}\Theta_{p+1}-\frac{4}{\Pr\Delta y}\Theta_{p-1}=\frac{\Delta y}{\Delta t}\Theta_p^0, \tag{17}$$

where u_p, H_p, θ_p are the unknown quantities at the center of the control volume as shown in Figure 2, and where all the parameters, $\Pr$, P_m, M, R and Gr are introduced in (9).

4. Results and Discussion

A thorough analysis of the problem under consideration includes the study of the velocity, temperature and induced magnetic fields under the influence of the dimensionless parameters entering in the problem under consideration, such as the magnetic parameter, M, the radiation parameter, R, the magnetic Prandtl parameter, P_m and the Grashof number, Gr. The Prandtl number was retained the same for all cases and equal to 0.71 and $c = 0.5$.

The velocity and induced magnetic fields distributions for different time $(t = 0.05, 0.1, 0.15, 0.2, 0.25)$ are shown in Figure 3. In this case, the velocity increases with increase of time, t, whereas the induced magnetic field decreases with increase of time, t, away from the plate.

The effect of the magnetic parameter, M, on the velocity and induced magnetic field for different time t, ($t = 0.1$ and 0.25) are shown in Figures 4 and 5. For $t = 0.1$ the magnetic parameter, M, decreases the velocity close to the plate and increases the velocity away from the plate, Figure 4. On the other hand for $t = 0.25$ the velocity decreased when the magnetic parameter was increased throughout the boundary layer, Figure 5. The induced magnetic field reduced when the magnetic parameter, M, increased for $t = 0.1$ and 0.25, Figures 4, 5.

Figure 6 represents the effect of Grashof number, Gr, on the velocity profiles for $t = 0.1$. It was observed that the velocity increased with the increase of Grashof number. In Figure 7, the temperature profiles were calculated for different values of thermal radiation parameter, R ($R = 1.0$, 2.0, 4.0, 8.0). The effect of thermal radiation parameter was substantial on the temperature profiles. More precisely, it was observed that the temperature was increased with decreasing radiation parameter.

Finally the effects of magnetic Prandtl parameter, P_m, ($P_m = 0.2$, 0.4, 0.6, 0.8, 1.0) on the velocity and induced magnetic field profiles are shown in Figure 8 for time, $t = 0.1$. The increase of the magnetic Prandtl parameter increases the velocity above the plate and substantially reduces the induced magnetic field.

5. Conclusions

The numerical investigation of the MHD unsteady flow of an incompressible viscous and radiative, optically gray thin fluid past an infinitely vertical plate moving with constant velocity when the magnetic Reynolds number is not small, showed that: (1) the velocity decreased when the magnetic parameter was increased in the boundary layer and the induced magnetic field reduced when the magnetic parameter, M, increased. Additionally, (2) the velocity increased with the increase of Grashof number and the temperature was substantially increased with decreasing radiation parameter. Finally, (3) the increase of the magnetic Prandtl parameter increased the velocity above the plate and substantially reduced the induced magnetic field values. It is hoped that this study will bring out insight in the problem of an optically gray thin fluid flow under the influence of radiation in the presence of an induced magnetic field.

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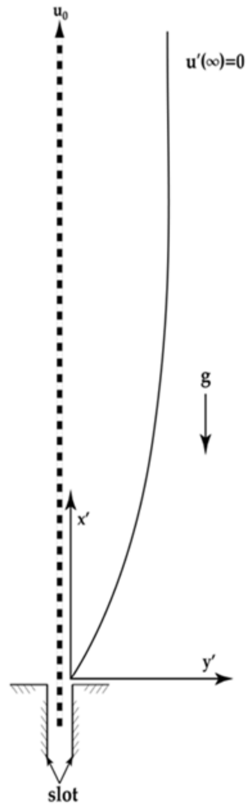

Figure 1. Physical model and co-ordinate system of the problem.

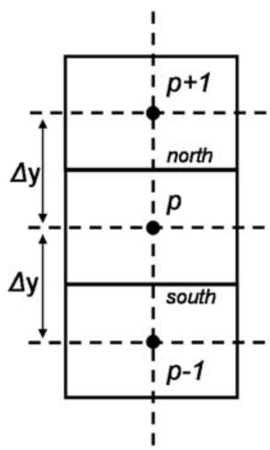

Figure 2. Discretization around the control volume for the unknown parameters using a collocated grid approach.

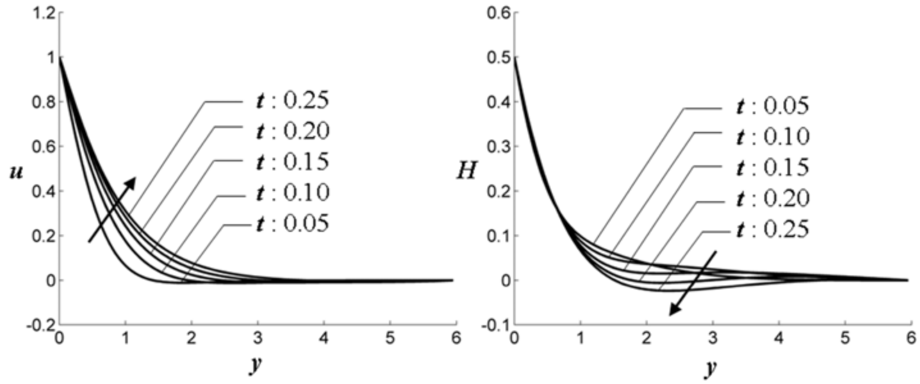

Figure 3. Velocity and induced magnetic field profiles for t: 0.05, 0.1, 0.15, 0.2, 0.25, and M : 2.0, Pr : 0.71, P_m : 0.2, R : 1.0, Gr : 2.0.

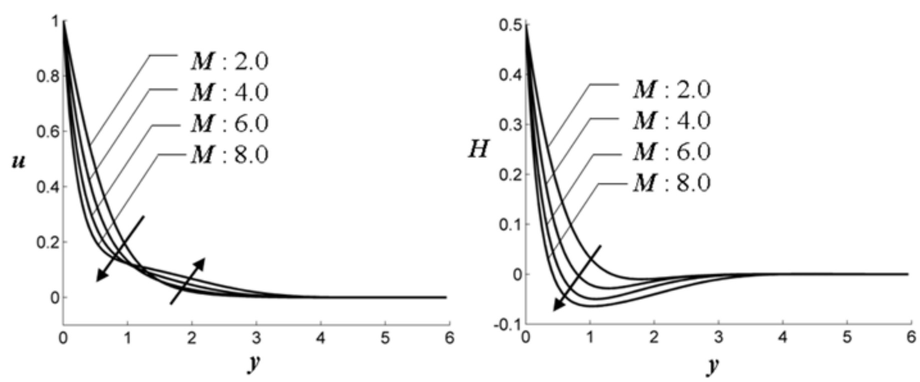

Figure 4. Velocity and induced magnetic field profiles for M : 2.0, 4.0, 6.0, 8.0, and Pr : 0.71, P_m : 0.2, R : 1.0, Gr : 2.0, for $t = 0.1$.

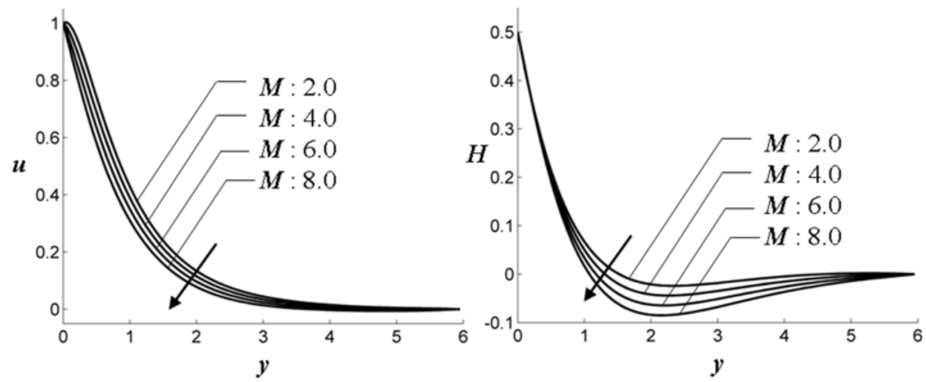

Figure 5. Velocity and induced magnetic field profiles for M : 2.0, 4.0, 6.0, 8.0, and Pr : 0.71, P_m : 0.2, R : 1.0, Gr : 2.0, for $t = 0.25$.

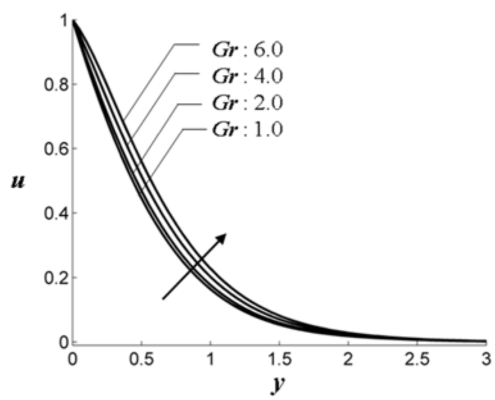

Figure 6. Velocity profiles for Gr : 1.0, 2.0, 4.0, 6.0, and Pr : 0.71, P_m : 0.2, R : 1.0, M : 2.0, for $t = 0.1$.

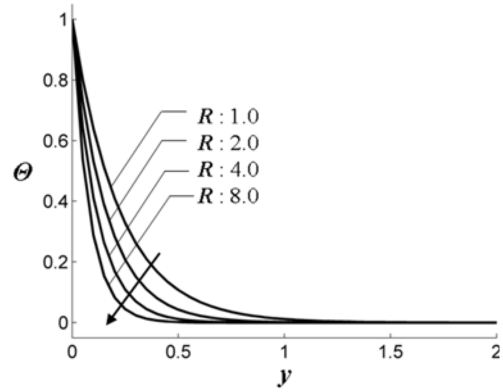

Figure 7. Temperature profiles for R : 1.0, 2.0, 4.0, 8.0, and Pr : 0.71, P_m : 0.2, M : 2.0, Gr : 1.0, for $t = 0.1$.

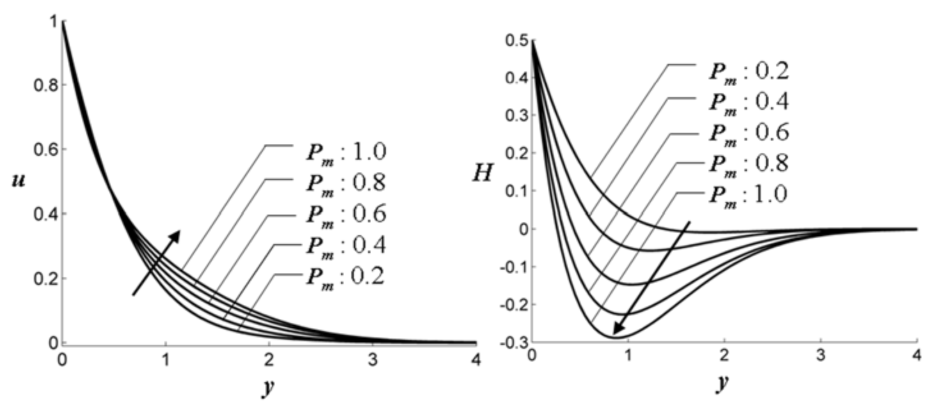

Figure 8. Velocity and induced magnetic field profiles for P_m : 0.2, 0.4, 0.6, 0.8, 1.0, and Pr : 0.71, R : 1.0, M : 2.0, Gr : 1.0, for $t = 0.1$.