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THE OPTIMAL MEXICAN HAT WAVELET FILTER DE-NOISING METHOD BASED ON CROSS VALIDATION METHOD

Wenyi Liu

School of Mechanical and Electrical Engineering
Xuzhou Normal University
Xuzhou, 221116, P. R. China
e-mail: liuwenyi1984@yahoo.com.cn

Abstract

A new de-noising method based on parameter optimized Mexican hat wavelet is put forward in this paper. For the similar shape to the mechanical shock vibration signals, the Mexican hat wavelet is chosen as the mother wavelet and improved by the shape parameters optimization. The noise jamming in the raw vibration signals can be filtered by the continue wavelet transform (CWT) using the improved Mexican hat wavelet as the mother wavelet. The shape parameters of the Mexican hat wavelet are optimized by the cross validation method (CVM). In the CWT process, the optimal scale factor is also obtained by the circle CVM. The useful components can be extracted by the CWT with the optimal shape parameters and scale factor. The experimental result shows that the proposed method can not only de-noise the useless noise effectively, but also extracts the fault feature availably.

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Keywords and phrases: wavelet transform, wavelet de-noising, optimal Mexican hat wavelet, cross validation method, fault diagnosis.

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1. Introduction

In mechanical condition monitoring and fault diagnosis, transient signals always contain important information of the monitored objects. Vibration analysis is the best-known technology applied in mechanical condition monitoring, especially for rotating equipments. Signals under considerations are known to be non-stationary, for which the signal parameters are time-varying [1-3]. For spectral analysis of such type signals, the time-frequency analysis techniques have been widely used [4, 5]. However, if the signal feature components' energy is low, the amplitude of the interfering noise will be higher than the signal feature components, and at the same time, the frequency spectrum of the noise and signal will be mixed together. In this situation, traditional time-frequency analysis methods cannot separate the useful signal features from the noise jamming effectively. Therefore, it is important to pre-process raw signals before analysis since the raw data contain some redundant information.

Wavelet analysis, which is the most popular one for non-stationary signal analyzing, overcomes the drawbacks of other techniques by means of analytical functions that are local in both time and frequency. Wavelet transform (WT) method is widely used in mechanical signal de-noising process [6-8]. However, traditional wavelet de-noising method has some difficulties in the analysis process, such as the selection of the mother wavelet function, the decomposition level of signal, the order of the mother wavelet function, etc. Some papers used wavelet de-noising methods in mechanical fault de-noising and feature extraction, but had not given some useful theoretical methods to ensure the decomposition level or the order of mother wavelet function [9, 10]. Most researchers used comparison methods to choose some optimal mother wavelet functions and decomposition level. In this situation, it is a waste of time and energy to do a lot of contrastive experiments [11]. Wavelet transform can be regarded as the inner product of a time domain signal with the translated wavelet-base function. Therefore, the WT can be regarded as the filter process to the signal, by which the noise in the signal can be de-noised effectively. The WT resulting coefficients reflect the different features of the noise and the useful signal, which can be separated by the filter process. Therefore, by choosing suitable wavelet function and take the continue wavelet transform (CWT) process, it is feasible to de-noise the interfering noise in the raw signals and keep the effective feature components.

This paper chooses Mexican hat wavelet, which is in shape similar to the mechanical shocking signal, as the mother wavelet function in CWT process. The parameters of the improved Mexican hat wavelet and the wavelet transform scale factor are optimized by the cross validation method (CVM). The paper is structured as follows: An introduction is given in Section 1. The principle of the Mexican hat wavelet filter de-noising method is discussed in Section 2. The parameters of the improved Mexican hat wavelet are optimized in Sections 3. The application of the proposed de-noising method in the gear fault experiment is presented in Sections 4. And some conclusions are drawn in Section 5.

2. Principle of Mexican Hat Filter De-noising

The wavelet analysis results are series of wavelet coefficients, which indicate the comparability between the signal and the particular wavelet. In order to extract the fault features of the signal more effectively, an appropriate wavelet base function should be selected [12-14]. The corresponding wavelet family consists of a series of daughter wavelets, which are generated by dilation and translation operations from the mother wavelet and shown as follows:

$$\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right), \quad a, b \in R, \quad a \neq 0, \tag{1}$$

where the parameter a is the scale factor, b denotes the time location, which is used to keep energy preservation.

For any function $f(t) \in L^2(R)$, its WT can be expressed as

$$W_f(a, b) = |a|^{-1/2} \int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt, \tag{2}$$

where $*$ is the conjugate of the function.

In particular, the WT can be regarded as the inner product of the signal and mother wavelet function. Therefore, it can be deduced that

$$W(a, b) = \sqrt{a}F^{-1}\{X(f)\psi^*(af)\}, \tag{3}$$

where $X(f)$ is the Fourier transform of signal $x(t)$, $\psi^*(af)$ is the Fourier transform of the wavelet base $\psi^*(at)$, F^{-1} is the inverse Fourier transform.

The above function can be regarded as a filtering process. In other words, the WT of the signal can be regarded as the filter process passing the band-pass filter whose frequency response is $\psi^*(af)$. Therefore, it is feasible to filter de-noising the signal by choosing suitable mother wavelet function to do CWT.

The wavelet transform resulting coefficients reflect the correlation between the signal and the selected wavelet base function. To increase the amplitude of the generated wavelet coefficients related to the fault impulses and enhance the fault detection process, the selected wavelet base function should be similar to the mechanical shock response in characteristics [3-5, 11-14]. From the time domain wavelet of the mechanical shock signal, we notice that the Mexican hat wavelet is similar to the mechanical impulse signal [15-19]. Therefore, wavelet de-noising using a Mexican hat wavelet as a base function can be used to extract the impulses for mechanical faults detection.

Mexican hat wavelet can also be called *Marr wavelet*, is the second derivative of the Gaussian function. If we define the Gaussian function as $\theta(x)$, then we can obtain

$$\theta(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma}}. \tag{4}$$

The wavelet function of the Mexican hat wavelet can be derived as

$$\psi(x) = \frac{d^2\theta(x)}{dx^2}. \tag{5}$$

Then we can obtain,

$$\psi(x) = \frac{2}{\sqrt{3\sqrt{\pi}}}(1 - x^2)e^{-x^2/2}. \tag{6}$$

The Fourier transform of $\psi(x)$ is defined as

$$\hat{\Psi}(\omega) = \sqrt{2\pi}\omega^2 e^{-\omega^2/2}. \tag{7}$$

Then

$$\int_{-\infty}^{+\infty} |\Psi(\omega)|^2 dx = \int_{-\infty}^{+\infty} (1 - x^2)^2 e^{-x^2} dx = \frac{3}{4}\sqrt{\pi}. \tag{8}$$

We can obtain $\Psi(\omega) \in L^2(R)$.

The time domain wavelet is shown in Figure 1(a). Set the shift factor $b = 0$, by changing the scale factor a, we can obtain the filter characteristic of the Mexican hat wavelet under different factors, as shown in Figure 1(b).

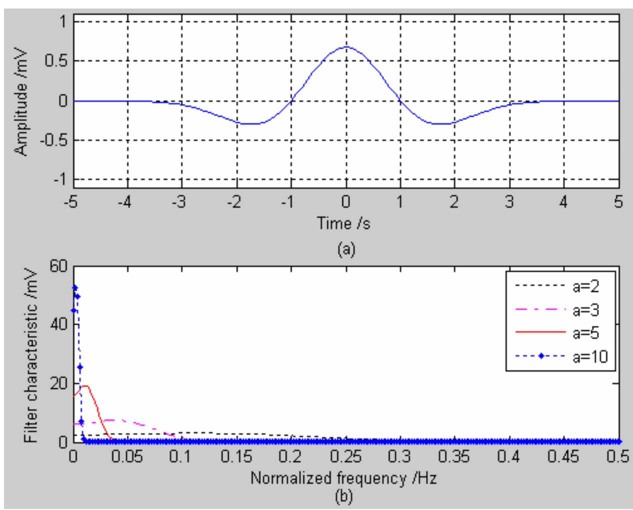

Figure 1. Mexican hat wavelet (a) Time domain wavelet; (b) Filter characteristic in frequency domain.

As shown in Figure 1, the time domain characteristics of Mexican hat wavelet are exponential decay, incompact support, good time-frequency localization and symmetry at the zero point. Figure 1(b) shows the obvious band-pass filter characteristic of Mexican hat wavelet. Compared to other low-pass filters, Mexican hat wavelet is much more suitable to extract the middle frequency signal. We can obtain different band-pass filters by changing the scale factor a. However, the wavelet shape in formula (1) is fixed and cannot fit the fast decay signal or big amplitude signal. The analysis affect cannot be ideal in this manner. Some papers adjust the mother wavelet shape by improving some parameters, but only limited to the inner small changing in the exponential function. In order to expand the application of the Mexican hat wavelet, we improve the parameters in the base of formula (1) and increase two parameters m and n, finally, obtain the improved Mexican hat wavelet as

$$\psi(x) = m(1 - x^2)e^{-nx^2}, \tag{9}$$

where m and n are the wavelet shape adjusting parameters.

Select $m = 0.8$, $n = 0.6$, $b = 0$, by changing the scale factor a, we can obtain the wavelet and filter shape of Mexican hat wavelet, as shown in Figure 2.

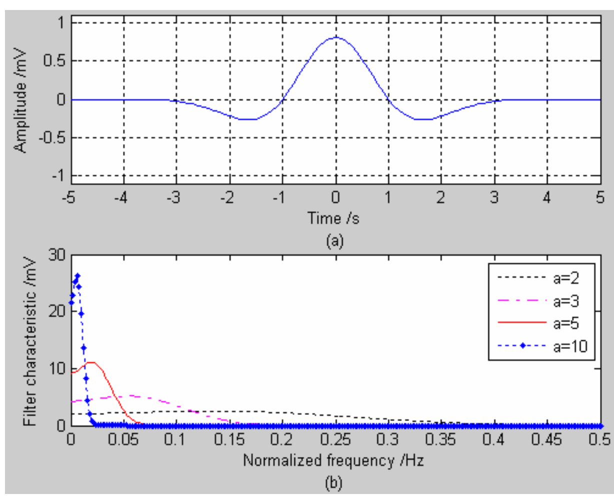

Figure 2. Mexican hat wavelet when $m = 0.8$, $n = 0.6$, $b = 0$ (a) Time domain wavelet; (b) Frequency filter characteristic.

Compared with Figure 1, we can obtain, by changing the shape parameters m and n, the filter characteristic of Mexican hat wavelet can also be changed. Therefore, in practical application, we should choose suitable parameters m and n according to requirement details of the analyzed signal. At the same time, the selection of parameters m and n is difficult in wavelet de-noising process.

After the wavelet transform process, in some scale range, the interfering noise in the raw signal will be suppressed and the useful signal features can be strengthened that can be extracted easily. The Mexican hat wavelet de-nosing can filter some small transient noise and keep the feature transient signal. Therefore, by adjusting the wavelet parameters of Mexican hat wavelet, we can filter the noise jamming in the raw signal effectively.

3. Parameters Confirmation

In the above analysis, the filter de-noising process by CWT has two main steps: one is to select the mother wavelet function parameters m and n, the other is to ensure the scale factor in the CWT process. For the improved Mexican hat wavelet we chosen, we calculate the parameters m and n by cross validation method (CVM) [20].

3.1. The cross validation method (CVM)

In the area of signal inspection and fault diagnosis, the aim is to extract useful feature components and acquire ideal de-noising effect. In other words, the distinction between de-noising signal and raw signal should better exhaustively small. Therefore, the following function can be used to scale the de-noising effect

$$M(t) = \int \{(f'(x) - f(x))^2\} dx. \tag{10}$$

However, the above estimation is hard to realize because the real signal is unknown. The cross validation method (CVM) can be used to estimate the function by constituting a new cost function [26]. So the CVM can be used to acquire the optimized scale factor in WT. For $N\ (N = 2^m)$ point disperse signal with noise,

$$g(k) = f(k) + n(k), \tag{11}$$

where $f(k)$ is the original signal, $n(k)$ is Gaussian white noise which obeys $N(0, \sigma^2)$ distribution. Its specific computation procedures are as follows:

(1) Extract the even part in sequence $g(k)$ and generate a new sequence $g^E(k)(k = 1, 2, ..., 2^{m-1})$, and produce the odd sequence in the same way.

(2) The estimate of the even sequence can be acquired by interpolating the odd sequence:

$$g'^E(k) = \frac{1}{2}[g^o(k) + g^o(k+1)]. \tag{12}$$

(3) Select the wavelet scale factor, perform CWT on the sequence $g'^E(k)$, and obtain a new sequence $g''^E(k)$.

(4) Acquire the odd sequence signal $g''^o(k)$ in the same way. The whole $M'(k)$ can be expressed as

$$M'(k) = \sum_{k=1}^{2^{m-1}} \{(g''^o(k) - g''^E(k))^2\}. \tag{13}$$

3.2. Shape parameters optimizing

We introduce the CVM to optimize the results and obtain the optimal parameters *m* and *n* of the improved Mexican hat wavelet. The detail steps are: at first changing the parameters *m* and *n* in some given range and obtain some different wavelet bases; do the CWT process using these wavelet bases and obtain some results; optimize the results using the CVM and obtain the final optimal parameters *m* and *n*. The parameters selection process is shown in Figure 3:

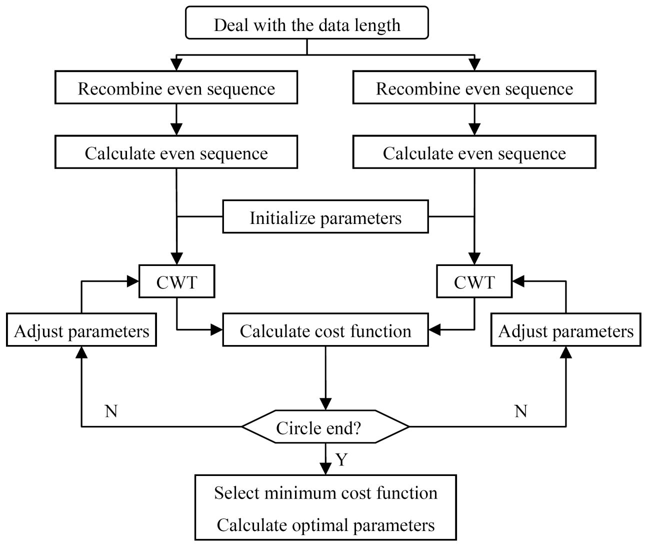

Figure 3. Sketch map of the parameters selection process.

As shown in Figure 3, the even sequential estimation $\bar{f}^{E}(k)$ and the odd sequential estimation $\bar{g}^{O}(k)$ are calculated according to arithmetic in 3.1. Choose the parameters *m* and *n* in some given range and obtain the mother wavelet, do CWT to sequence $\bar{f}^{E}(k)$ and finally obtain the wavelet transform results $\hat{f}^{E}(k)$. The odd sequential estimation $\bar{f}^{O}(k)$ and the corresponding results $\hat{f}^{O}(k)$ can be gotten by the same arithmetic. Then the total cost function $\hat{M}(t)$ can be expressed as

$$\hat{M}(t) = \sum_{k=1}^{2^{j-1}} \{(\hat{f}^{E}(k) - \hat{f}^{O}(k))^2\} dx. \tag{14}$$

If we fix the step and scale factor, adjust the parameters *m* and *n* in some range, then we can obtain an $M \times N$ matrix, in which every value is corresponding to a cost function $\hat{M}(t)$. Every cost function is calculated by a set of data *m* and *n*. Theoretically, the minimum cost function is corresponding to the optimal parameters *m* and *n*.

3.3. Wavelet scale factor a optimizing

We calculate different cost functions $M'(a)$ by changing the wavelet scale factor a in some given range. At last, we can obtain a matrix and the optimal parameter is the one which makes the cost function minimal in the sequence. With the above treatment, the adapt adjustment is realized.

In order to simplify the calculation and improve the computation efficiency, a simplified calculation is designed to choose the scale factor, which can be expressed as follows:

(1) At first, specify the scale range $[m, n]$ and the step i.

(2) Calculate the best scale factor $m + ki$ which makes the cost function smallest through CVM.

(3) Reduce the search range by changing it to $[m+(k-1)i, m+(k+1)i]$ and decrease the step by adjusting it to $0.1i$.

(4) Repeat the above steps 2 and 3 until the final best scale factor a is obtained.

After obtaining the best scale factor, the filter process is performed through equation (2). Then the filter de-noising of the signal is realized. The noise in raw signal is removed through above process.

4. Experimental Analyses

Experiments were conducted using a 2 hp reliance electric motor, and the acceleration data was collected at locations near to and remote from the motor bearings. Motor bearings were seeded with faults using electro-discharge machining. Faults ranging from 0.007 inches in diameter to 0.040 inches in diameter were introduced separately at the inner raceway, outer raceway and the rolling ball. Faulted bearings were reinstalled into the test motor and vibration data was recorded for motor loads of 0 to 3 horsepower (motor speeds of 1797 to 1720 RPM) [29].

Bearing vibration data was collected using accelerometers, which were attached to the housing with magnetic bases. Accelerometers were placed at the 12 o'clock position at both the drive end and fan end of the motor housing. Digital data was collected at 12,000 samples per second for drive end bearing faults. Speed and the horsepower data were collected by the torque transducer/encoder equipment.

One part of vibration data from the drive end bearing is shown in Figure 4:

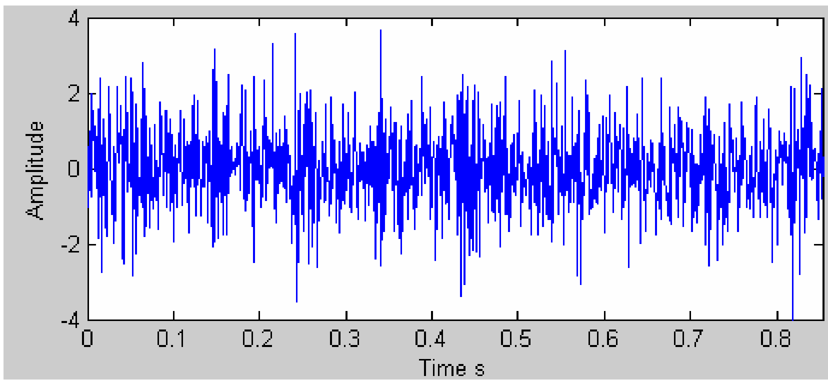

Figure 4. Time domain wavelet of bearing vibration signal.

As shown in Figure 4, the existence of noise disturbed the signal's time domain display. We deal with this signal with the optimal Mexican hat wavelet filter de-noising method. At first, we calculate the mother function parameters m and n. Considering of the operating rate and calculated amount, we set the parameter precision as 0.1 and at last obtain $m = 0.4$, $n = 0.5$. Then we initialize a scale factor range $[1, 64]$ and set as 1, at last obtain optimal scale factor $a = 4$. In the second circle, the scale range is $[1, 5]$ and the final optimal scale factor $a = 3.6$.

Taking CWT process using the optimal Mexican hat wavelet, the scale factor range is $[1, 32]$ and the step is 1. The scale factor map of the signal is shown in Figure 5:

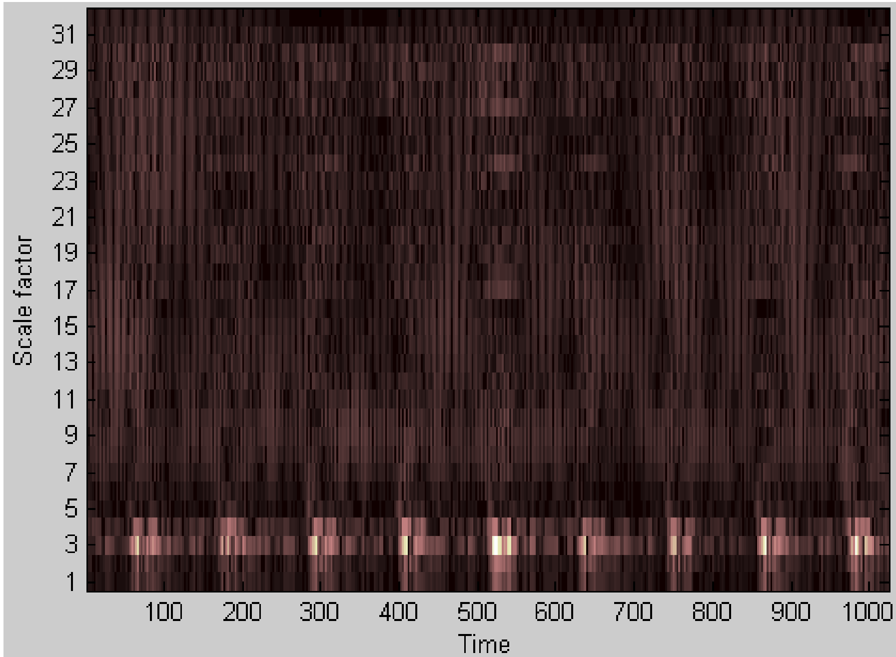

Figure 5. CWT scale factor map of the signal.

As shown in Figure 5, in the scale factor of [3, 5], there exist obvious white points, which are corresponding to some serious shock points in the bearing. These points verify the distribution range of the optimal scale factor.

The filter de-noising results are shown in Figure 6(c) by doing CWT to the analyzed signal with the optimal scale factor $a = 3.6$. In order to compare the de-noising effect of the proposed method, we choose other two traditional wavelet de-noising methods to analysis the same signal. Select wavelet base as Daubechies ‘db8’ and the wavelet decomposition level as 5, stein unbiased risk estimation rule to calculate the threshold value, soft threshold manner, the result is shown in Figure 6(a). Select wavelet base as Symlets ‘sym8’, wavelet decomposition level as 5, ‘sqtwolog’ rule to calculate the threshold value, soft threshold manner, the result is shown in Figure 6(b).

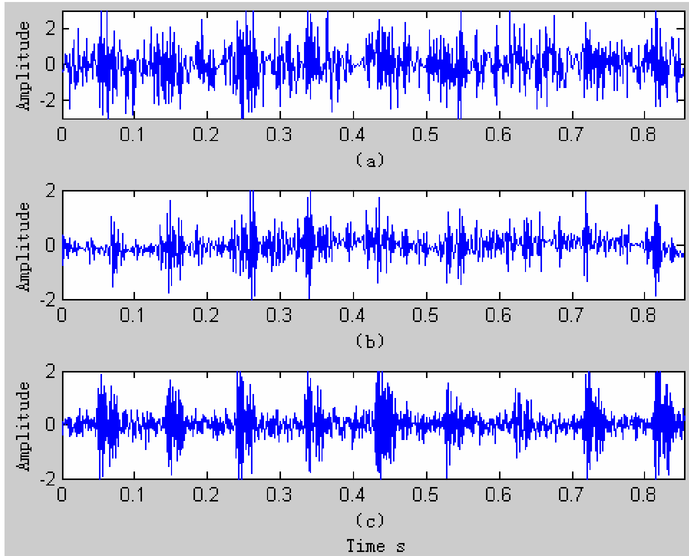

Figure 6. De-noising results (a) Optimal Mexican hat filter de-noising; (b) Traditional wavelet de-noising one; (c) Traditional wavelet de-noising two.

From Figure 6, we can see, the optimal Mexican hat filter de-noising method can filter most of the noise jamming in the raw signal. Both the two compared wavelet de-noising method cannot de-noise the signal ideally. Especially in Figure 6(b), the useful signal is also filtered in the de-noising process. The de-noising effect can be proved in this experiment analysis.

5. Conclusions

Aimed at the shortage of the traditional wavelet de-noising methods, an optimal Mexican hat wavelet filter de-noising method is proposed in this paper. The mother wavelet shape parameters are optimized by the cross validation method to be more similar to the mechanical vibration signals. The scale factor in the CWT process is also optimized using the cross validation method in circle manner. The experimental analysis and comparison to the other two traditional wavelet de-noising methods proved the effect of the proposed method. This method can not only filter most of the noise jamming in the raw signal, but also extracts the feature components in the strong noise background.

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